International Journal of Engineering & Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-3444
International Journal of Engineering and Technology Volume 1 No. 3, December, 2011

Design and Analysis of Smart Structures for Active Vibration Control using Piezo-Crystals

Deepak Chhabra[1]*, Pankaj Chandna[2], Gian Bhushan[3]

[1*]Department of Mechanical Engineering, University Institute of Engineering &Technology
Maharshi Dayanand University, Rohtak,Haryana, INDIA
[2,3] Department of Mechanical Engineering, National Institute of Technology Kurukshetra,Haryana, INDIA

ABSTRACT

The present work considers the active vibration control of beam like structures with laminated piezoelectric sensor and actuator layers bonded on top and bottom surfaces of the beam. A finite element model based on Euler-Bernoulli beam theory has been developed. The contribution of the piezoelectric sensor and actuator layers on the mass and stiffness of the beam has been considered with modeling of entire structure in a state space form. The designing of state/output feedback control by Pole placement technique and LQR optimal control approach are demonstrated to achieve the desired control. From the analysis, it has been observed that the LQR control scheme is very effective in controlling the vibration. Numerical simulation shows that including and varying the location of the sensor / actuator mass and stiffness from the free end to the fixed end on the beam produces a considerable change in the system's structural vibration characteristics. The study illustrates that sufficient vibration suppression can be attained by means of the proposed methods.

Key words: *Piezoelectric material, FEM, LQR optimal control, Pole placement*

1. INTRODUCTION

Smart structures consist of highly distributed active device which comprises sensors and actuators either embedded or attached with an existing passive structure coupled by controller. The piezoelectric sensor senses the disturbance and generates an electric charge due to the direct piezoelectric effects. The piezoelectric actuator in turn produces a control force/moment due to the converse piezoelectric effects. If the control force is appropriate, the structural vibration may be suppressed. This technology has several applications such as active vibration and buckling control, shape control and active noise control. The finite element method is powerful tool for designing and analyzing smart structures. Both structural dynamics and control engineering need to be dealt to demonstrate smart structures. A three-dimensional finite element model of a smart structure with embedded piezoelectric sensors and actuators is developed for analyzing the response of active damping structures to steady state input. The closed loop numerical simulation in the frequency domain and in the time domain is attempted (Lim et al., 1997& 1999). Beam, plate and shell type elements have been developed incorporating the stiffness, mass and electromechanical coupling effects of the piezoelectric laminates (Narayanan S., Balamurugan V., 2003). Xu S.X, Koko T.S. (2004) proposed the design method for intelligent structures with finite element code and control design is carried out in state space form established on finite element modal analysis. Karagulle et al. (2004) introduced the active vibration control using APDL (ANSYS Parametric Design language) in ANSYS. They analyzed the results obtained from the APDL for the two-degrees of freedom system and found superiority with the Laplace transform method. Further, application of APDL to smart structures has also been investigated. Xing-Jian Dong et al., (2006) proposed a general analysis and design scheme of piezoelectric smart structures by using ANSYS and observer/Kalman filter identification (OKID) approach, and to experimentally study the feasibility and efficiency of OKID approach in the vibration control of piezoelectric smart structures. Vasques C.M.A., Rodrigues J. Dias (2006) studied the effectiveness of control strategies, constant gain and amplitude velocity feedback, and optimal control strategies, linear quadratic regulator (LQR) and linear quadratic Gaussian (LQG) controller, in order to suppress vibrations in smart beams. Kumar, K.R. and. Narayanan S, (2008) used a model-based linear quadratic regulator (LQR) controller for the optimal placement of collocated piezoelectric actuator–sensor pairs on a thin plate. They have also formulated the problem using the finite element method (FEM) as multi-input–multi-output (MIMO) model control..Manjunath T.C, Bandyopadhyay B. (2009) presented the modeling and design of multirate output feedback based sliding mode control scheme for the vibration control. Malgaca Levent (2010) integrated the control methods into the finite element solutions (ICFES) with ANSYS. The author analyzed the active control of free and forced vibrations for a smart laminated composite structure (SLCS) using ICFES simulation and compared with the experiment results.

In most of present researches, FEM formulation of smart cantilever beam usually done in ANSYS and design of control laws are carried out in Mat LAB control system toolbox. Hence, for designing piezoelectric smart structures with control laws, it is necessary to develop a general design scheme of actively controlled piezoelectric smart structures. The objective of this work is to address a general design and analysis scheme of piezoelectric smart structures with control laws. The classical control law, pole placement technique and LQR optimal control approach using state feedback and arbitrary value of gain by output feedback has analyzed to achieve the desired control. Numerical examples are presented to demonstrate the validity of the proposed design scheme. This paper has organized in to three parts, FEM formulation of piezoelectric smart structure with designing control laws, Numerical simulation and Conclusion.

2. MODELING OF SMART CANTILEVER BEAM WITH CONTROL LAWS

2.1 Finite Element Formulation of beam element

A beam element is considered with two nodes at its end. Each node is having two degree of freedom (DOF). The shape functions of the element are derived by considering an approximate solution and by applying boundary conditions. The mass and stiffness matrix is derived using shape functions for the beam element. Mass and stiffness matrix of piezoelectric (sensor/actuator) element are similar to the beam element. To obtain the mass and stiffness matrix of smart beam element which consists of two piezoelectric materials and a beam element, all the three matrices added. The cantilever beam is modeled by FEM assembly of beam element and smart beam element. The last two row's two elements of first matrix are added with first two row's two element of next matrix. The global mass and stiffness matrix is formed. The boundary conditions are applied on the global matrices for the cantilever beam. The first two rows and two columns should be deleted as one end of the cantilever beam is fixed. The actual response of the system, i.e., the tip displacement *u*(*x*, *t*) is obtained for all the various models of the cantilever beam with and without the controllers by considering the first two dominant vibratory modes.

A beam element of length l_b with two DOFs at each node i.e. translation and rotation is considered.

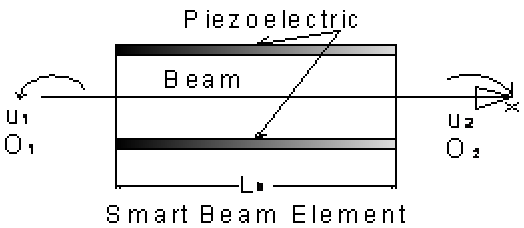


Fig. 1.

The displacement u is given by (x)=$[N]^T[p]$ (1)

$$= [N_1(x) \quad N_2(x) \quad N_3(x) \quad N_4(x)] \begin{bmatrix} u_1 \\ \theta_1 \\ u_2 \\ \theta_2 \end{bmatrix}$$ (2)

$N_1(x)$, $N_2(x)$, $N_3(x)$, $N_4(x)$ are the shape functions and u_1, θ_1 and u_2, θ_2 are the DOF's at the node1 and node2 respectively

Where $N_1(x) = 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3}$ (3)

$$N_2(x) = x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2}$$ (4)

$$N_3(x) = \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3}$$ (5)

$$N_4(x) = \frac{-x^2}{l_b} + \frac{x^3}{l_b^2}$$ (6)

The kinetic energy and bending strain energy of the element can be expressed as:

$$T=\frac{1}{2}\int_0^{l_b} \rho_b A_b \left[\frac{\partial u(x,t)}{\partial t}\right]^2 dx=\frac{1}{2}\dot{u}^T [m]\, \dot{u}$$ (7)

$$V=\frac{1}{2}\int_0^{l_b} E_b I_b \left[\frac{\partial^2 u(x,t)}{\partial t^2}\right]^2 dx=\frac{1}{2}\ddot{u}^T [k]\, \ddot{u}$$ (8)

Where, ρ_b is the density of beam, E_b is the Young's modulus, I_b is the moment of inertia of cross-section, A_b is the area of cross-section.

The governing differential equation of motion for the beam element can be represented as:

$$M^b \ddot{p} + C\dot{p} + K^b p = q$$ (9)

where M^b, C, K^b, q are the mass, damping, stiffness, and the force co-efficient vectors of beam element.
The consistent mass matrix and stiffness matrices are obtained as:

$$[M^b] = \rho_b A_b \int_0^{l_b} [N]^T [N]\, dx$$ (10)

$$[K^b] = E_b I_b \int_0^{l_b} [\ddot{N}]^T [\ddot{N}]\, dx \quad (11)$$

$$[M^b] = \rho_b A_b \int_0^{l_b} \begin{bmatrix} 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3} \\ x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2} \\ \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3} \\ -\frac{x^2}{l_b} + \frac{x^3}{l_b^2} \end{bmatrix} \begin{bmatrix} 1 - \frac{3x^2}{l_b^2} + \frac{2x^3}{l_b^3} & x - \frac{2x^2}{l_b} + \frac{x^3}{l_b^2} & \frac{3x^2}{l_b^2} - \frac{2x^3}{l_b^3} & -\frac{x^2}{l_b} + \frac{x^3}{l_b^2} \end{bmatrix} dx \quad (12)$$

$$[M^b] = \frac{\rho_b l_b A_b}{420} \begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix} \quad (13)$$

$$[K^b] = E_b I_b \int_0^{l_b} \begin{bmatrix} \frac{6}{l_b^2} + \frac{12x}{l_b^3} \\ -\frac{4}{l_b} + \frac{6x}{l_b^2} \\ \frac{6}{l_b^2} - \frac{12x}{l_b^3} \\ -\frac{2}{l_b} + \frac{6x}{l_b^2} \end{bmatrix} \begin{bmatrix} \frac{6}{l_b^2} + \frac{12x}{l_b^3} & -\frac{4}{l_b} + \frac{6x}{l_b^2} & \frac{6}{l_b^2} - \frac{12x}{l_b^3} & -\frac{2}{l_b} + \frac{6x}{l_b^2} \end{bmatrix} dx \quad (14)$$

$$[K^b] = \frac{E_b I_b}{l_b^3} \begin{bmatrix} 12 & 6l_b & -12 & 6l_b \\ 6l_b & 4l_b^2 & -6l_b & 2l_b^2 \\ -12 & -6l_b & 12 & -6l_b \\ 6l_b & 2l_b^2 & -6l_b & 4l_b^2 \end{bmatrix} \quad (15)$$

2.2 Finite Element Formulation of Smart Beam Element

The mass and stiffness matrix for the smart beam element with piezoelectric patches placed at the top and bottom surfaces as a collocated pair is given by:

$$M^p \ddot{p} + C\dot{p} + K^p p = \{f_a\}\{\emptyset_a(t)\} \quad (16)$$

Where M^p , C, K^p, are the mass, damping, stiffness, and $\{f_a\}$, is the force co-efficient vectors which maps the applied actuator voltage to the induced displacements of smart beam element, $\emptyset_a(t)$ the voltage applied to the actuator, develops effective control forces and moments.
The mass matrix of smart beam element is given by:

$$[M^p] = \frac{\rho_b l_b A_b}{420} \begin{bmatrix} 156 & 22l_b & 54 & -13l_b \\ 22l_b & 4l_b^2 & 13l_b & -3l_b^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_b & -3l_b^2 & -22l_b & 4l_b^2 \end{bmatrix} + 2 * \frac{\rho_p l_p A_p}{420} \begin{bmatrix} 156 & 22l_p & 54 & -13l_p \\ 22l_p & 4l_p^2 & 13l_p & -3l_p^2 \\ 54 & 13l_b & 156 & -22l_b \\ -13l_p & -3l_p^2 & -22l_p & 4l_p^2 \end{bmatrix} \quad (17)$$

$$[K^P]=\frac{EI_{eq}}{l_p^3}\begin{bmatrix} 12 & 6l_p & -12 & 6l_p \\ 6l_p & 4l_p^2 & -6l_p & 2l_p^2 \\ -12 & -6l_p & 12 & -6l_p \\ 6l_p & 2l_p^2 & -6l_p & 4l_p^2 \end{bmatrix} \quad (18)$$

$$EI_{eq}=E_b I_b+2E_p I_p \quad (19)$$

$$I_p=\frac{1}{12}bt_a^3+bt_a\left(\frac{t_a+t_b}{2}\right)^2 \quad (20)$$

2.3 Control Laws

The various control laws such as one control law, which is based on output feedback by assuming arbitrary value and one optimal control law Linear quadratic regulator (LQR) based on state feedback and one control law, which is based on pole placement by state feedback has been explained as:-

2.3.1 LQR optimal control by state feedback

LQR optimal control theory is used to determine the active control gain. The following quadratic cost function is minimized:

$$j=\frac{1}{2}\int_0^{\infty}(x^T\,Q\,x+u^T Ru)dt \quad (21)$$

Q and R represent weights on the different states and control channels and their elements are selected to provide suitable performance. They are the main design parameters. J represents the weighted sum of energy of the state and control.

Assuming full state feedback, the control law is given by:

$$u=-Kx \quad (22)$$

with constant control gain

$$K=R^{-1}B^T S \quad (23)$$

Matrix S can be obtained by the solution of the Riccati equation, given by

$$A^T S+SA+Q-SBR^{-1}B^T S=0$$

The closed loop system dynamics with state feedback control is given by:

$$\dot{x}=(A-BK)x+Er(t) \quad (25)$$

2.3.2 Control by output feedback

Output feedback control provides a more meaningful design approach in practice. Measured outputs (ε) from sensors are directly feed back to actuators through

$$u=-K\varepsilon \quad (26)$$

The closed loop system dynamics with output feedback control is given by:

$$\dot{x}=(A-BKC)x+Er(t) \quad (27)$$

$$A_c=(A-BKC)$$

2.3.3 Pole Placement by State Feedback

Stability and optimality of a system are closely related to the location of poles or eigenvalues of the system. Pole placement can be achieved by feedback control. The poles of this system are eigenvalues of A. We use state feedback

u = Kx, where Kx is linear state feedback.

Under this feedback control, the controlled system is given by:

$$\dot{x}=(A-BK)x+Er(t) \quad (28)$$

The poles of the controlled system are $(A-BK)$.

To find out the value of gain K, A characteristic equation of the system is considered as:

$$\varphi(s)=s^n+a_{n-1}s^{n-1}+\dots\dots\dots+a_1 s+a_0 \quad (24)(29)$$

The poles of the controlled system is in the desired locations represented by the desired characteristic equation as:

$$\varphi(s)=s^n+a'_{n-1}s^{n-1}+\dots\dots\dots+a'_1 s+a'_0 \quad (30)$$

This can be achieved by letting the feedback matrix be:

$K = [k_0 \; k_1 \dots \dots \dots \dots k_{n-1}] =$
$[a_0 - a_0' \; a_1 - a_1' \dots \dots \dots a_{n-1} - a_{n-1}']$ (31)

2.4 Linear piezoelectricity

The linear piezoelectric coupling between the elastic field and the electric field can be expressed by the direct and the converse piezoelectric equations respectively:

$$D = dT + \epsilon^T E \quad (32)$$

$$S = s^E T + dE \quad (33)$$

d relates the electric displacement D to the stress under zero electric field, The piezoelectric constant d relates the strain to the electric field E in the absence of mechanical stress and s^E refers to the compliance when the electric field is constant.

2.5 Laminar Sensor Equation

The sensor voltage of piezoelectric element is given by,(Manjunath T.C, Bandyopadhyay B., 2009)

$$V^s(t)=K_c\, G_c\, d_{31}\, E_p \left(\frac{t_b}{2} + t_a\right) b[\,0 \; -1 \; 0 \; 1 \; 0 \; 1][\dot{p}] \quad (34)$$

Where G_c is gain, t_b, t_a are the thickness of beam and actuator, K_c is the controller gain

$$V^s(t)=g^T[\dot{p}]$$

2.6 Controlling Force from Actuator

Similar to the sensor, the piezoelectric layer which acts as actuator bonded to the structure. The geometrical arrangement is such that the useful direction of expansion is normal to that of the electric field. Thus, the activation capability is governed by piezoelectric constant d_{31}. With standard engineering notation, the equation of stress for piezoelectric material given by Premont is:

$$\sigma_{11}=E_p\epsilon_{11} - e_{31}\frac{V}{t_p} \quad (35)$$

where, V is the voltage applied to the piezoelectric material.
The controlling force equation given by:

$$f_a = E_p d_{31} b\{-1 \; 0 \; r_a \; 1 \; 0 \; -r_a\}^T \, w(t) \quad (36)$$

where, r_a is the distance measured from the neutral axis of the beam to the mid plane of actuator layer, E_p Young's modulus of piezoelectric material, $d_{31}$is Piezo strain constant, b is the width of material.

$$f_a = h\, w(t) \quad (37)$$

where, $h=E_p d_{31} b\{-1 \; 0 \; r_a \; 1 \; 0 \; -r_a\}^T$

2.7 Model Reduction

After assembly of each element of beam, the final equation for the smart cantilever beam with piezoelectric patches placed at the top and bottom surfaces as a collocated pair is given by:

$$M\,\ddot{p} + C\dot{p} + K\,p = q + \{f_a\}\{\emptyset_a(t)\} \quad (38)$$

The external force is taken as unit impulse force.
Where M , C, K, are the global mass, Rayleigh damping, stiffness, and further Rayleigh damping co-efficient,

$$C=\alpha[M]+\beta[K] \quad (39)$$

where, α and β are the damping constants

In active vibration control of flexible structures, the use of smaller order model has computational advantages. Therefore, it is necessary to apply a model reduction technique to the state space representation. The reduced order system model extraction techniques solve the problem by keeping the vital properties of the full model only. The frequency range is selected to span first two frequencies of the smart beam in order to find the reduced order model of the system.

Consider a generalized co-ordinate for reduction as

$$p=Vz \quad (40)$$

where V is the modal vectors corresponding to the first two eigen values. After reduction eqn (38) becomes:

$$M_{red}\,\ddot{z} + C_{red}\dot{z} + K_{red}\, z = f_{ext} + f_{red} \quad (41)$$

2.8 State space formulation

In state space formulation, the second order differential equations are converted to first order differential equations.

First order dynamical system is:

$$\begin{bmatrix}\dot{x_1}\\ \dot{x_2}\\ \dot{x_3}\\ \dot{x_4}\end{bmatrix}=\begin{bmatrix}0 & I\\ -M_{red}^{-1}\,\text{Kred} & -M_{red}^{-1}\,\text{Cred}\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix}+\begin{bmatrix}0\\ M_{red}^{-1}V^T h\end{bmatrix}\text{w(t)} + \begin{bmatrix}0\\ \text{M}_{\text{red}}^{-1}\text{V}^{\text{T}}\text{f}\end{bmatrix}\text{r(t)} \quad (42)$$

$$X=A\ x(t)+B\ w(t)+E\ r(t) \quad (43)$$

where, A is known as the system matrix, x(t) is the state vector, matrix B is input matrix, w(t) is a column vector formed by the voltages applied to the actuators and acting as a control force, E is the external force acting on the beam.

& $Y=C\ x(t)+D\ w(t)$ (44)

Where C is the output matrix, and D is the direct transmission matrix

$$Y=V^s(t)=g^T[\dot{p}]= g^T\ V\dot{z}= g^T\ V\begin{bmatrix}x_3\\ x_4\end{bmatrix} \quad (45)$$

$$=[0 \quad gT\ V]\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} \quad (46)$$

Here:

$$A=\begin{bmatrix}0 & I\\ -M_{red}^{-1}\,\text{Kred} & -M_{red}^{-1}\,\text{Cred}\end{bmatrix} \quad (47)$$

$$B=\begin{bmatrix}0\\ M_{red}^{-1}V^T\{\text{f}_\text{a}\}\end{bmatrix} \quad (48)$$

$$C=[0 \quad gT\ V] \quad (49)$$

D=null matrix (50)

$$E=\begin{bmatrix}0\\ \text{M}_{\text{red}}^{-1}\text{V}^{\text{T}}\text{f}\end{bmatrix} \quad (51)$$

3. NUMERICAL SIMULATION

Table 1: Material properties and dimensions of Smart beam

Physical Parameters	Beam Element	Piezoelectric sensor/actuator
Length(m)	l_b=0.226	l_b=0.075
Breath(m)	b=0.025	b=0.025
Thickness(m)	t_b=0.965e-3	t_a=0.75e-3
Elastic Modulus(GPa)	E_b=68	E_p=61
Density(Kg/m^3)	ρ_b=2800	ρ_p=7500
Piezo strain constant(m/V)		d_{31}=274e-12
Piezo stress constant(Vm/N)		g_{31}=10.5e-3
Damping constants	α =0.001, β =0.0001	

EXPLORE
IJET International Journal of Engineering & Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-3444
International Journal of Engineering and Technology Volume 1 No. 3, December, 2011

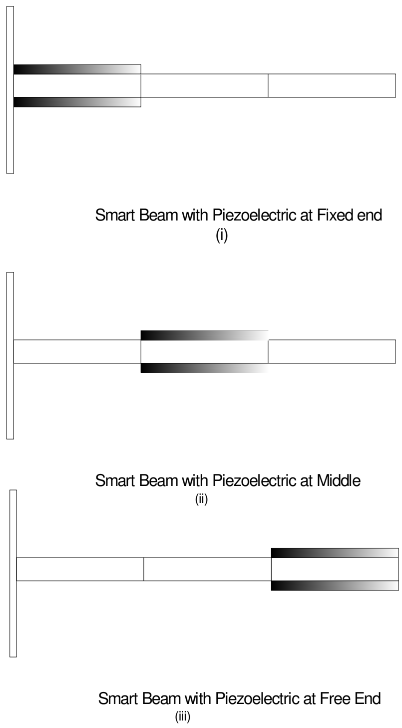
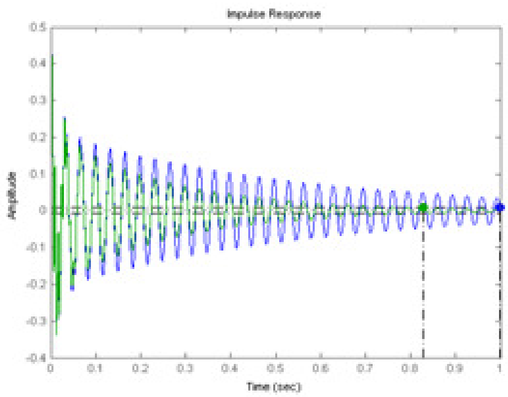


Fig. 2: Position of sensor/actuator on cantilever Beam

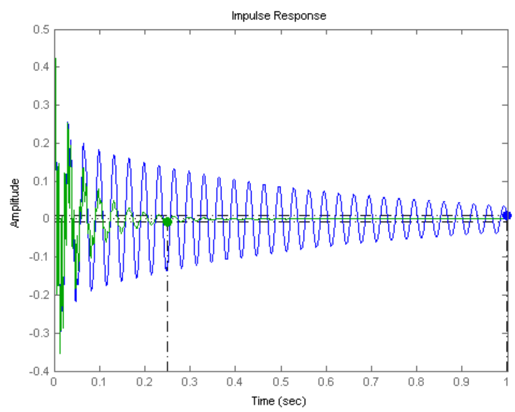
A cantilever beam with three elements of equal length is considered here .The piezoelectric sensor and actuator are placed at three different positions .i.e. at fixed end, middle and free end. The structure consists of an aluminum beam with PZT-5H sensor and actuators patches. The material properties and dimensions of the beam and piezo patches are similar to the experiment performed by Xu and Koko(2004). For analysis, only collocated positions are considered. The physical properties of sensor and actuator have been given in table 1.

Case1:

In the first case, collocated patches are extending near the base of the beam. The responses are taken by giving impulse input. The output feedback and state feedback controller are designed by taking the arbitrary value of gain. In practical designs problems all the states are always, not known for feedback. On the other hand, output feedback control provides a more consequential design. The responses are plotted by changing the value of arbitrary gain for state feedback and output feedback control. Frequency response diagrams are also plotted for controlled and uncontrolled case. The responses are also plotted by changing the position of sensor and actuator on the beam i.e. at fixed end, the middle and free end.



(a)



(b)

Fig. 3: Tip displacement with and without control, when the sensor/ actuator are placed at the fixed end (a) State Feed back (b) Output Feedback

IJET EXPLORE International Journal of Engineering & Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-3444

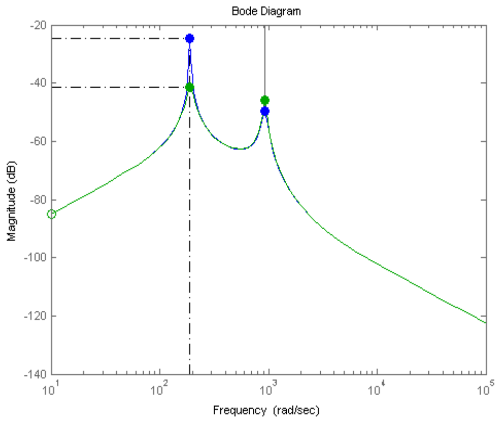


Fig. 4: Frequency response Diagram with open loop and closed loop, when the sensor/ actuator are placed at the fixed end

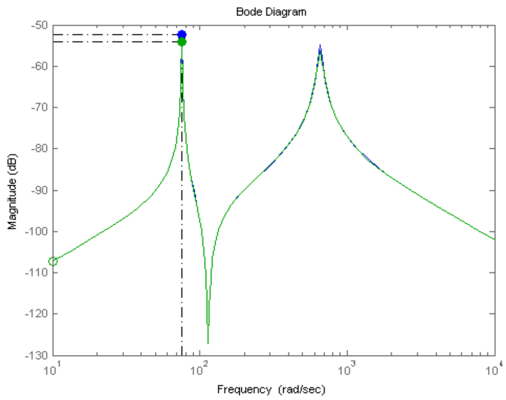
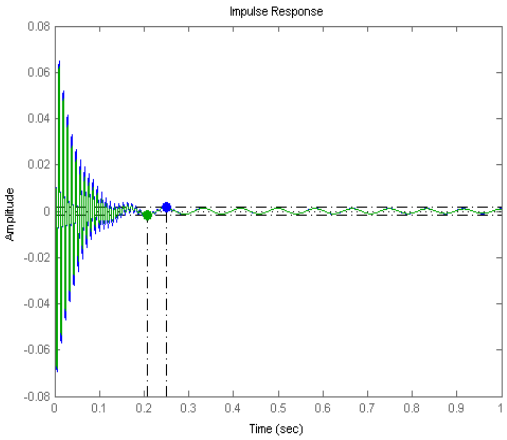
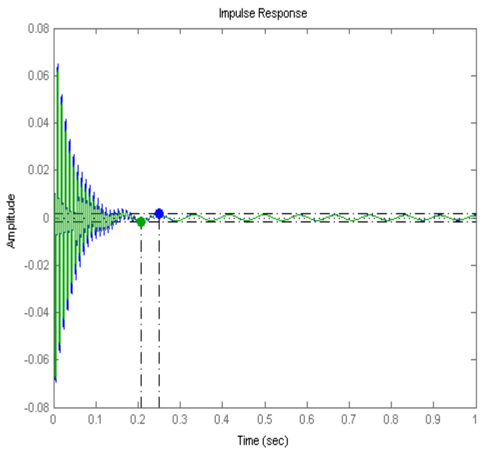


Fig. 6: Frequency Response Diagram with open loop and closed loop, when the sensor/ actuator are placed at the middle

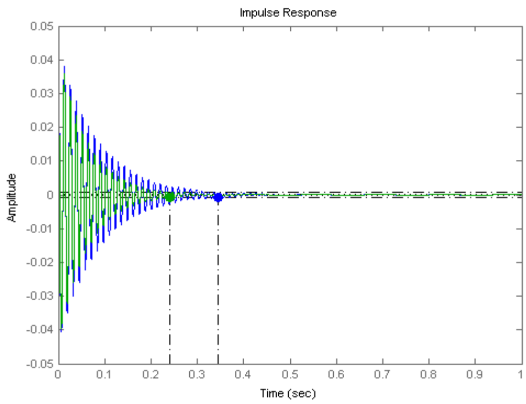


(a)

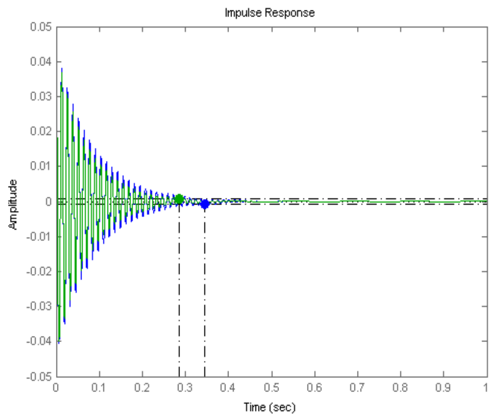


(b)

Fig. 5: Tip displacement with and without control, when the sensor/ actuator is placed at the Middle (a) State Feed back (b) Output Feedback



(a)



(b)

Fig. 7: Tip displacement with and without control, when the sensor/ actuator are placed at the free end (a) State Feed back (b) Output Feedback

EXPLORE
IJET International Journal of Engineering & Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-3444
International Journal of Engineering and Technology Volume 1 No. 3, December, 2011

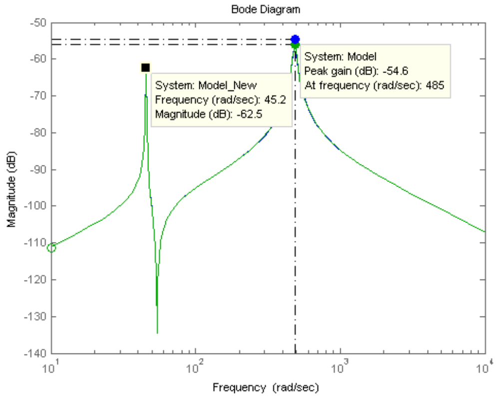
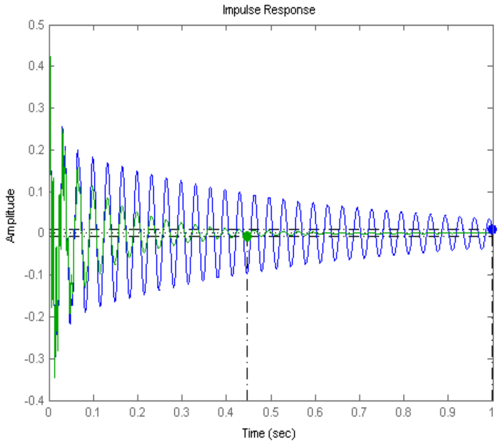
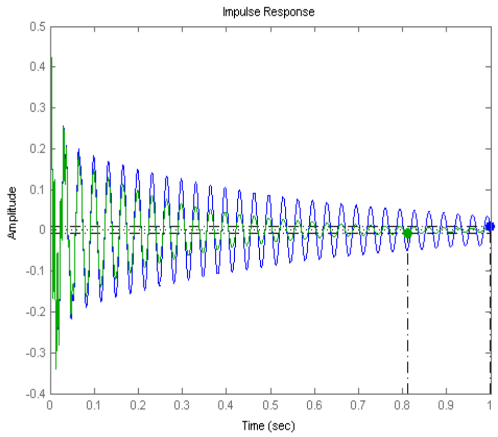


Fig. 8: Frequency Response Diagram with open loop and closed loop, when the sensor/ actuator are placed at the free end



Gain=50



Gain=20

Fig. 9: Tip displacement with and without control, when the sensor/ actuator is placed at the fixed end by taking different value of gain

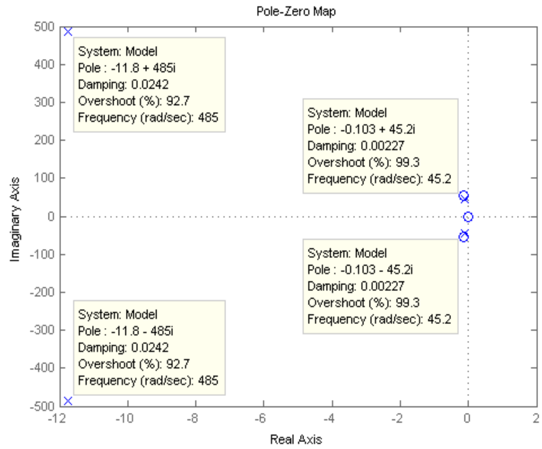
Case 2:

In second case, the closed loop system is designed according to the required eigen values. The eigen values of the open loop system are
-1.03e-001 + 4.52e+001i, -1.03e-001 - 4.52e+001i, -1.18e+001 + 4.85e+002i , -1.18e+001 - 4.85e+002i

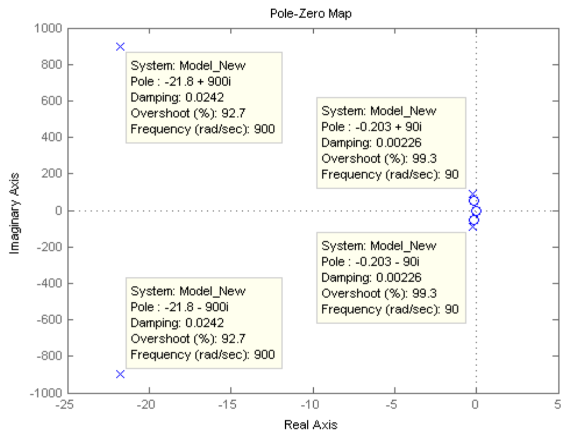
The required eigen values of closed loop system are:

-2.03e-001 + 9e+001i, -2.03e-001 - 9e+001i,-2.18e+001 + 9e+002i,-2.18e+001 - 9e+002i

, for that we find out the optimal gain by pole placement method. The responses are drawn for open loop and closed loop system

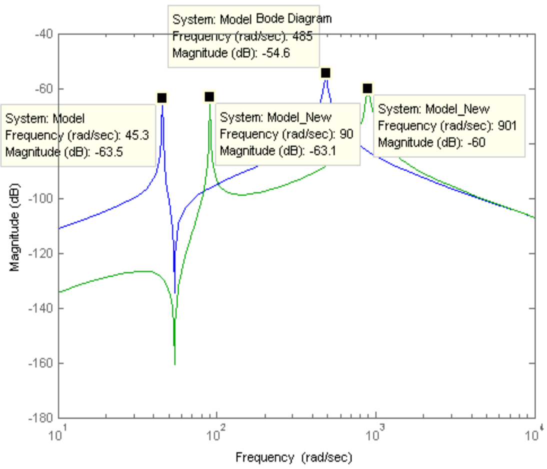


(a)

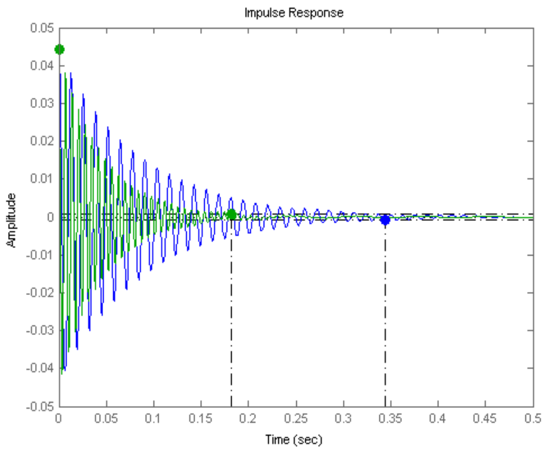


(b)

Fig. 10: (a) Pole zero map of open loop system & (b) Pole zero map of closed loop system



(a)



(b)

Fig 11. (a) Frequency response diagram & (b) Tip displacement without and with control with pole placement method

Case 3:

In third case, an optimal control is designed to minimize the cost function j

$$j = \frac{1}{2}\int_0^{\infty} (x^T Q x + u^T Ru)dt$$

Q=1e8*[20 10 0 0
0 10 0 0
0 0 20 0
0 0 0 10] R=100

For this an optimal value of gain is find out by solving Riccati equation.

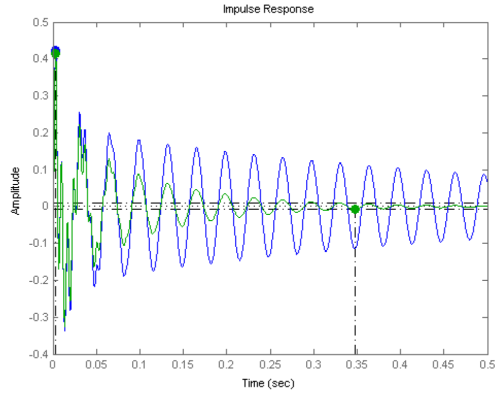


Fig. 12: Tip displacement without and with control with LQR approach, when the sensor/ actuator are placed at the fixed end

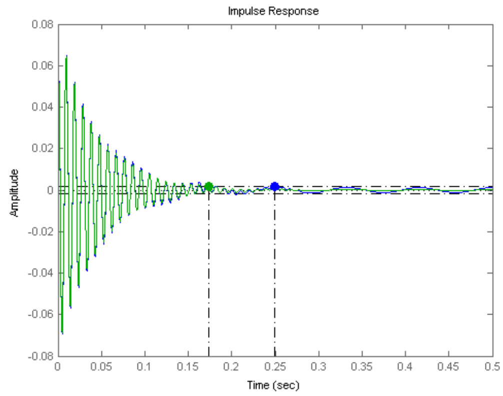


Fig. 13: Tip displacement without and with control with LQR approach, when the sensor/ actuator are placed at the Middle

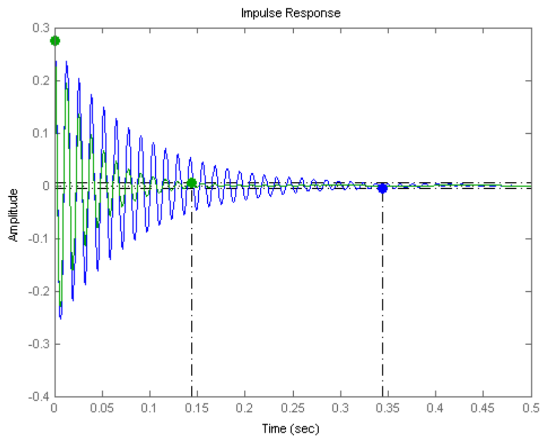


Fig. 14: Tip displacement without and with control with LQR approach, when the sensor/ actuator are placed at the Free End

4. CONCLUSIONS

Present work deals with the mathematical formulation and the computational model for the active vibration control of a piezoelectric smart structure. A general scheme of analyzing and designing piezoelectric smart structures with control laws is successfully developed in this study. The present scheme has the flexibility of designing the system as collocated and non-collocated and user-selected a feedback control law. The active vibration control performance of piezoelectric cantilever structure is studied using pole placement by state feedback, by taking arbitrary value of gain with output feedback and, the linear quadratic regulator (LQR) scheme, which is an optimal control theory based on full state feedback. It has been observed that without control the transient response is predominant and with control laws, sufficient vibrations attenuation can be achieved. Pole placement technique is used to obtain the desired Eigen values of controlled system. The study revealed that the LQR control scheme is very effective in controlling the vibration as the optimal gain is obtained by minimizing the cost function. Numerical simulation showed that modeling a smart structure by including the sensor / actuator mass and stiffness and by varying its location on the beam from the free end to the fixed end introduced a considerable change in the system's structural vibration characteristics. From the responses of the various locations of sensor/actuator on beam, it has been observed that best performance of control is obtained, when the piezoelectric element is placed near the fixed end.

REFERENCES

[1] Dong Xing-Jian, Meng Guang, Peng Juan-Chun, 2006. Vibration control of piezoelectric smart structures based on system identification technique: Numerical simulation and experimental study. *Elsevier Journal of Sound and Vibration* 297 680–693

[2] Hatch, Michael R. 2001. Vibration simulation using MATLAB and ANSYS. CRC Press, LLC, Boca Raton, FL.

[3] Karagulle H., Malgaca L. and Oktem H F, 2004. Analysis of active vibration control in smart structures by ANSYS. *Smart Materials and Structures,* Vol. 13, pp. 661-667

[4] Kumar, K.R. and S. Narayanan,2008. The optimal location of piezoelectric actuators and sensors for vibration control of plates *Smart Materials and Structures*,. **16**(6): p.2680-2691.

[5] Lim Young-Hun,2003. Finite-element simulation of closed loop vibration control of a smart plate under transient loading. *Smart Materials and Structures,* 12, 272-286.

[6] Lin Feng, 2007. Robust Control Design An Optimal Control Approach, ISBN 978-0-470-03191-9©2007 John Wiley and sons Ltd

[7] Malgaca Levent, 2010. Integration of active vibration control methods with finite element models of smart laminated composite structures Elsevier, Composite Structures 92 1651–1663.

[8] Manjunath T.C. , Bandyopadhyay B. 2009, Vibration control of Timoshenko smart structures using multirate output feedback based discrete sliding mode control for SISO systems, *Elsevier Journal of Sound and Vibration* 326 50–74.

[9] Naidu, Desineni *S.,* 2003.Optimal control systems. CRC Press LLC.

[10] Narayanan S, Balamurugan V., 2003. Finite element modelling of piezolaminated smart structures for active vibration control with distributed sensors and actuators. *Elsevier Journal of Sound and Vibration* 262 529–562.

[11] Paraskevopoulos P.N., Modern Control Engineering. Marcel Dekker, INC.

[12] Premont Andre, 2004. Vibration control of active structures An Introduction. Kluwer Academic Publishers.

[13] Qiu Zhi-cheng, Zhang Xian-min, Wu Hong-xin, Zhang Hong-hua, 2007. Optimal placement and active vibration control for piezoelectric smart flexible cantilever plate. *Elsevier Journal of Sound and Vibration* 301 521–543.

[14] Stavroulakis G.E. , Foutsitzic G., Hadjigeorgiou E., Marinova D., Baniotopoulos C.C. (2005). Design and robust optimal control of smart beams with application on vibrations suppression. *Elsevier Advances in Engineering Software* 36 806–813

[15] Vasques C.M.A., Rodrigues J. Dias, 2006. Active vibration control of smart piezoelectric beams: Comparison of classical and optimal feedback control strategies. *Elsevier Computers and Structures* 84 1402–1414

[16] William S. Levine,1999. The Control Handbook. CRC Press, INC

[17] Wodek k. Gawronski, 2004. Advance structural dynamics and active control of structures. Springer-Verlag New York, Inc

[18] Xu S.X.,. Koko T.S, 2004. Finite element analysis and design of actively controlled piezoelectric smart structures. *Elsevier Finite Element in Analysis and Design* 40 (3) 241–262.