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Experimental Study of Multistage Coupled Oscillators

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ABSTRACT

To gain high power in a desirable higher frequency, combining a number of identical elements have been a good practical solution. In recent years significant effort has been carried out to combine independent oscillators coupled together. When these oscillators are coupled together the complexity of the whole system increases due to the fact that interaction between the elements occur in several ways with exchange of power, frequency, phase, amplitude or other. This paper presents the necessary conditions to be met while coupling group of independent oscillators together. A number of identical oscillators will be designed and implemented to couple them in variety of ways. The experimentally observed relationships will be compared with the existing theoretical results to advance the understanding of this important issue.

Keywords— *Coupled Oscillator, Locked Frequency, Phase tuning, Normal Mode*

1. INTRODUCTION

The increasing need of high power systems at microwave and millimetre frequencies for the communication market motivated the research in the fields of power combining techniques for the last two decades. The rapid expansion of communication technology such as radar to transportation, industrial and scientific application, it has become more and more important to achieve high power in high frequency region. In accordance with a long-term trend towards system operating at higher and higher frequencies, however the power combining has not been very well developed at the present time. In recent years, to gain high power in a desirable higher frequency, combining a number of identical elements have been a good practical solution and several interesting properties of power combiners have been found, such as phase noise reduction, graceful degradation, and overall cost reduction by the possible use of smaller devices with better power added efficiency [1].

Moreover, in the development of terahertz source, it becomes an important issue to develop techniques that enable a large number of oscillators to operate in phase. Part of this scenario is oscillator arrays, where injection locking and/or phased-locked loop techniques are used to achieve synchronous operation. But the behaviour of significant number of independent oscillators that are coupled to each other is a more challenging problem in both theory and practice.

We have presented some important behaviours of the coupled oscillator system in the work. We have built eight independent oscillators coupled together in variety of ways up to six stages and investigated the amplitude, frequency and phase relationship. Some analysis and characteristics of the implemented oscillator will be discussed prior to the synchronization of the oscillators. The experimentally observed relationships have been compared with the theoretical results and good qualitative agreements between theory and experiment is observed. The work establishes a better understanding toward Coupled Oscillator system and experimental measurement on them.

2. RELATED WORK

As observed in nature, the dynamic behaviours of real signal sources can be affected by nearby sources close enough to interact [2]. The partial transfer of the signal between sources – determines how much the mode of operation is influenced. In this process a very strong interaction may lead the signals to lock together into a common state, while a very weak interaction may leave the sources in free-running condition. In between these two limits several other behaviours may occur based the system dynamics and characteristics. When the characteristic of the uncoupled source is nonlinear, the dynamics governing the overall system is very complex and explains that coupled nonlinear systems have so many different behaviours.

The search for nonlinearities of high frequency oscillators started in the early twentieth century. Van der Pol [3] was first to accurately model the oscillation phenomena using a second order nonlinear differential equation, referred as the VDP equation by mathematicians and engineers. He analysed both the free running and the injection locked cases .The VDP model describes some of the important features of real oscillating systems, such as the amplitude dependent negative resistance. Adler [4] carried on the Van der Pol investigation and derived a differential equation that relates the oscillator phase to the injection signal parameters.

Mackey [5] included the effect of phase modulation in Adler's analysis to and also showed that the injection locked oscillator had some significant advantage. The work of Van der Pol was then applied and refined by Kurokawa [6],[7] to the injection of microwave oscillators, which present a more complicated negative resistance that was pictured in VDP and Adler's equations.

Till then a lot of coupling schemes had been proposed in the microwave engineering community but none of them combined reliability and simplicity. The important work of York and others [8] [9] extended Kurokawa's analysis to coupled oscillators systems deriving a set of coupled nonlinear differential equations for phase and amplitude dynamics.

3. DESIGN AND CHARACTERISTICS

As it is known the number of oscillator circuit that are used in practice is overwhelming, and the main consideration to design oscillators suitable for this work was to choose, simple circuitry that are easy to fabricate and possible to make identical element with very small difference in output characteristics such as frequency, amplitude etc.

3.1 Practical Oscillator Circuit

As we are going to couple several similar oscillators together, the single oscillator unit leads the implementation of other units (with similar characteristics) as all the experiments of this work depend on the properties of the single element. After careful investigation of the entire oscillator it has been found that the pierce type LC oscillator meets our design objective. So we have chosen Pierce type oscillator as the basic circuit for our experiment. The practical oscillator circuit looks like:

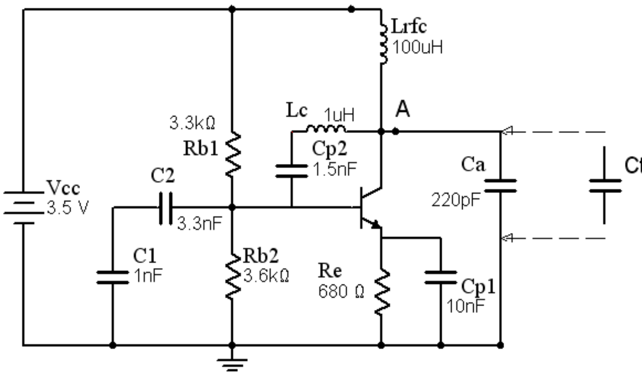

Figure 1: Practical oscillator circuit

In figure 1 the option for tuning capacitor C_t, parallel to Ca gives the necessary tuning and the marked node A, where we are going to put the one end of the coupling capacitors to couple similar oscillator unit.

3.2 Output Waveform

For this work we have built eight single oscillator unit and each unit is numbered as 1, 2, 3…8. From here to the end we will use the same notation i.e. Osc1, Osc2 etc to explain and compare different oscillator unit.

In this section we will analyse some important characteristics of the single oscillator unit (Osc1) which are relevant for these experiments and then move on to the comparison of different oscillator unit in the foregoing sections.

The oscillator unit that corresponds to number 1 has an output frequency of nearly 10.9 MHz. As we are using tuned circuit the capacitance effect has a large impact on the output frequency. But when constructing circuit we were very careful to minimize the wiring capacitance effect by connecting components very close to each other and we have used a very strong common ground plane for all the oscillators.

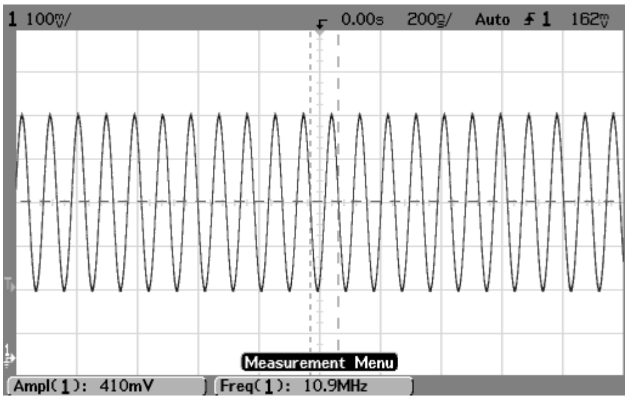

Figure 2: Output waveform at node A (Oscillator1)

Figure 2 shows the output at node A for Osc1, which is taken from the oscilloscope trace. The output shows a pure sine wave with no distortion and output voltage of 410mv at 3.5V supply voltage.

3.3 Output Amplitude

The output amplitude at node A in Figure 1 for all the oscillators with different supply voltage without tuning capacitor is given in table 1.

We should note that, the output voltage is proportional to the value of the capacitance at the output terminal. It is seen that, for supply voltage of 2.50 volts the output amplitude is 138(pk-pk) mV if there is no tuning capacitor and 164(pk-pk) mV for tuning capacitor 6.8pF. It is also found that at 5.50volts supply the output amplitude is nearly equal for all the value of tuning capacitor. With higher supply voltages the transistor gain limits the output voltage and nearly equal for all the values of tuning capacitance.

Table 1: Supply voltage Vs Output amplitude (all oscillators)

Supply Voltage(V)	Output Amplitude(pk-pk) mV							
	Osc1	Osc2	Osc3	Osc4	Osc5	Osc6	Osc7	Osc8
3.00	291	250	160	140	110	238	272	344
3.50	410	372	338	340	300	356	397	431
4.00	472	453	444	450	420	444	485	491
4.60	538	535	538	545	510	522	563	563
5.00	585	590	588	595	600	560	610	610
5.50	641	650	632	660	640	619	663	663

3.4 Frequency Tuning

For synchronization of oscillators it is very important to know the tuning properties of all the oscillators. We measure all the frequencies for different values of C_t for all the oscillators. By using the figure 1 the output frequency of the Pierce Oscillator is written as,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{L_C(C_a + C_t)}}$$

So, if we change the value of the tuning capacitor C_t the output frequency will be changed as well. The relationship is inversely proportional i.e. if the capacitance is increased the frequency will be decreased.

All the efforts had been done to make the different oscillator units same in characteristics. But in practice it is not the case. It is important to note that, we intentionally made two oscillators which are in large extent different from the other oscillators. We numbered these two oscillators as Osc4 and Osc5. A graph showing the tuning behaviour is given on figure 3.

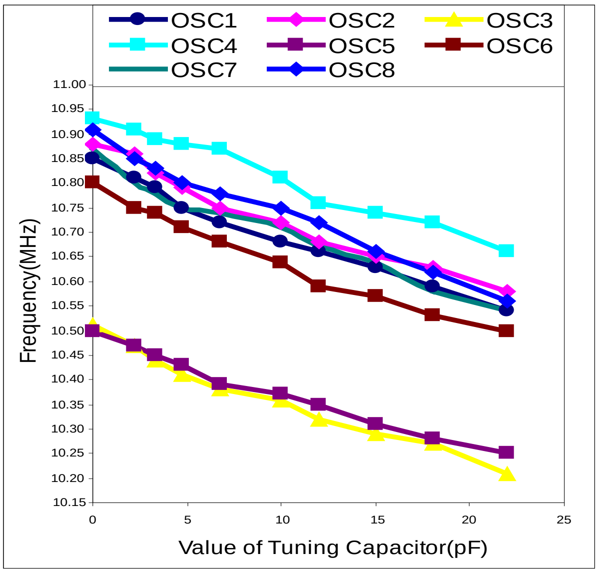

Figure 3: Frequency tuning Behaviour (All Oscillators)

For convenience, we make a note of frequency of all the Oscillators without the tuning capacitance as this data will be necessary at the later stages.

Table 2: Oscillator Frequencies (without Tuning Capacitor)

Oscillator No	Frequency(MHz)
Osc1	10.85
Osc2	10.88
Osc3	10.51
Osc4	10.93
Osc5	10.50
Osc6	10.80
Osc7	10.87
Osc8	10.91

4. SYNCHRONIZATION OF OSCILLATORS

The concept of synchronization is extremely important in nature. This synchronization effect has a long history dating back to observations of synchronized mechanical pendulums by Huygens who was the first to describe it mathematically .The theory of synchronization of electrical oscillators has been pioneered by Van der Pol [3] in the early twentieth century followed by Krylov and Bogoliubov [10], Huntoon and Weiss [11]. Kholchlov [12] has also given some explanation over the mutual synchronization of oscillators.

As deeply explored by Van der Pol the dynamics of nonlinear oscillating electrical circuits is affected by the injection of an external signal. Several behavioural changes have being observed, but the case of most interest occurs when the period of the oscillations becomes the same of the external source and thus the oscillator's frequency is locked to the injection signal. Adler [4] was one of the first to study the locking phenomena. He developed an expression for the frequency range over which an oscillator will be locked to an external signal .When the injection is mutual, then the sources influence each other and synchronization occurs when they reach a common frequency.

4.1 Two Oscillator Case

In our experiments we coupled Osc1 and Osc2 by the following figure 4. It is seen that the coupling capacitor C_C is connected between node A1 of Osc1 and node A2 of Osc2.

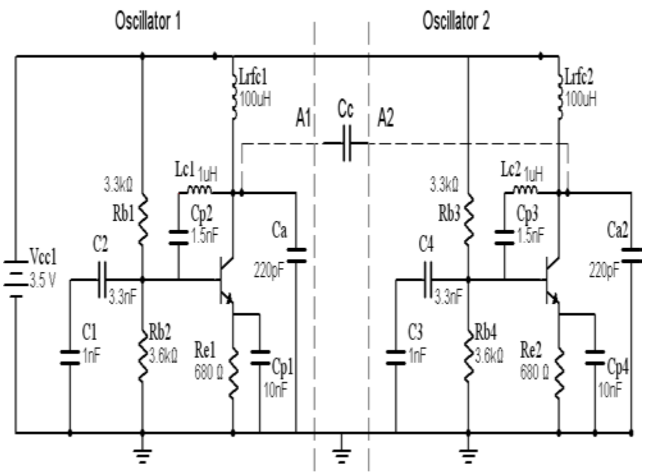

Figure 4: Mutual synchronization of Osc1 and Osc2

To make the two oscillators oscillating nearly in the same frequency we use figure 3, which shows with tuning capacitor 3.3pF at Osc2 terminal does the job. The supply voltage was kept to 2.60 volts and we note that the amplitude A_1 equals to 178mV and the amplitude A_2 equals to 168mv with the capacitor 3.3pf. So we can say that oscillator1 and oscillator2 has nearly the same power and by the theory explained by Kholchlov [12] the amplitude, phase and frequency will be change (compared to free funning condition) after they come to synchronization.

Table 3: Synchronization behaviour for two oscillators

C_C (pF)	A_1 (mV)	A_2 (mV)	Phase, Φ_{12}	Frequency (MHz)
3.3	153	172	-160˚	10.71
4.7	183	184	-170˚	10.64
6.8	195	192	-175˚	10.61
10	206	210	-180˚	10.52
12	213	213	-180˚	10.45
15	222	222	-180˚	10.39
18	231	228	-180˚	10.31
22	238	231	-180˚	10.20

From table 3 we see that, if we change the coupling capacitor the synchronization amplitudes A_1 and A_2 changes from the free running state. But the important characteristics we should note that, when the phase is -180˚ the amplitudes are nearly same for the two oscillation which shows that they are strongly coupled at this stage. The phase is ranged between $-180^\circ < \Phi < 90^\circ$ for stable mode of operation. The output waveforms for the two oscillators are given in the following figure 5 (data used, C_C =22pf)

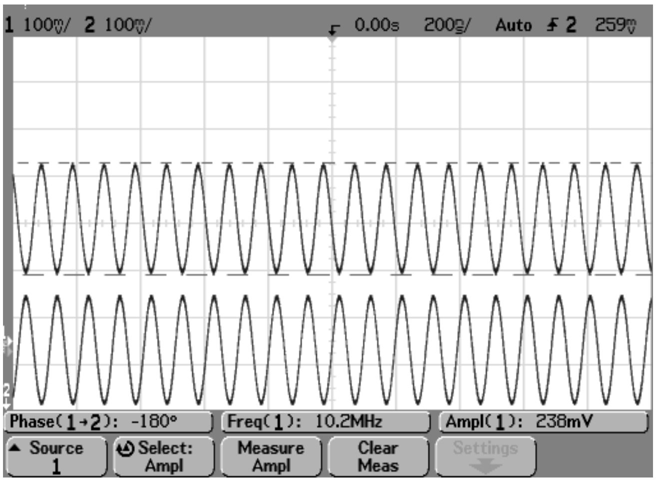

Figure 5: Output waveform for synchronized Oscillators

For the two oscillators we discussed here, we make use of the fact that two oscillators are nearly in the same frequency. We now want to see what happens if the two oscillators are not in nearly in the same frequency. To find out what happens we kept varying the tuning capacitor C_t for one oscillator to make large difference between frequencies keeping the frequency of other oscillator same. The behaviour will then be observed by varying the coupling capacitor. The use of Figure 3 will be useful in this case.

To explain these behaviours we will simplify the figure 4 and change it as follows

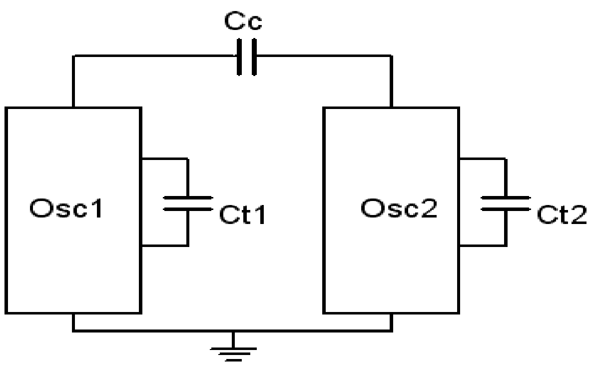

Figure 6: Simplified diagram of Figure 3

From figure 6 we can easily see that capacitor C_C is the coupling capacitor which is connected between node A1 and node A2. C_{t1} and C_{t2} represent tuning capacitor for Osc1 and Osci2 respectively. The whole oscillator circuit is represented by the square block Osc1 and Osc2.

Table 4a: Locking state for Osc1

C_{t2}=0 pF, Frequency of Osc2= 10.88 MHz				
C_{t1}(pF)	Frequency Osc1 (MHz)	Locked?	Locked Frequency (MHz)	Phase
0	10.85	Yes	10.68	162°
2.2	10.81	Yes	10.68	148°
3.3	10.79	Yes	10.68	147°
4.7	10.75	No	-	-
6.8	10.72	No	-	-
10	10.68	No	-	-

Table 4b: Locking state for Osc2

C_{t1}=0 pF, Frequency of Osc1= 10.85 MHz				
C_{t2}(pF)	Frequency Osc2 (MHz)	Locked?	Locked Frequency (MHz)	Phase
2.2	10.88	Yes	10.68	176°
3.3	10.86	Yes	10.68	168°
4.7	10.82	Yes	10.68	157°
6.8	10.79	Yes	10.48	145°
10	10.75	No	-	-
12	10.72	No	-	-

From the table it is seen that, if the frequency separation between the two oscillators is large, then they can't be locked together. For example, at C_{t1}=10 pF the frequency of oscillator1 is 10.68 MHz and at C_{t2}=0 pF the frequency of oscillator2 is 10.88 MHz. The frequency separation is nearly 200 KHz and it falls outside the synchronizing band and they are not locked. So to synchronize oscillators together it is important that the frequency separation should be in the synchronizing band.

4.2 Three Oscillator Case

To better understanding of the mutual synchronization we are now going to consider three oscillators, Osc1, Osc2 and Osc3. For the three oscillator system we are going to precede same as the two oscillator system. We kept the frequency of three oscillators same by providing the necessary tuning.

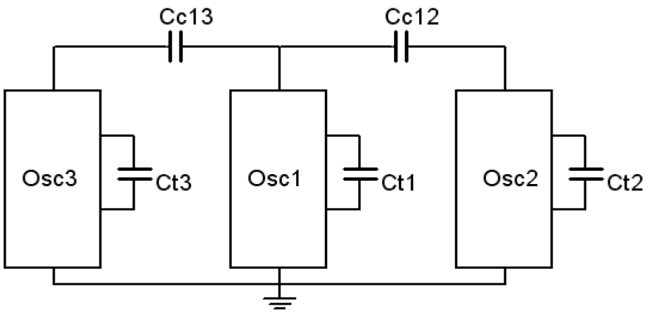

Figure 7: Mutual synchronization of three oscillators

To make all the frequency same for the three oscillator the value of the tuning capacitor can be found as C_{t1}=0, C_{t2}=3.3pF and C_{t3}=6.8 pf. This make all the frequency nearly 10.85 MHz. (The capacitance formed by different measurement probe has been taken into account).

As the frequencies of all the oscillators are nearly same the oscillators should be locked. They are locked in practice and at some higher supply voltage i.e. 5.50 volts. (We will explain the cause when we discuss modes of oscillation). Table 5 gives the results for different values of coupling capacitance between 1, 2 and 2, 3.

Table 5: Synchronizing behaviour for three oscillators

Cc12 (pF)	Cc13 (pF)	Amp1 (mV)	Amp2 (mV)	Amp3 (mV)	Phase, Φ_{12}	Phase, Φ_{13}	Locked Frequency (MHz)
2.2	2.2	619	650	632	-27°	-10°	10.81
3.3	3.3	619	650	632	-25°	-8°	10.81
4.7	4.7	619	650	632	-14°	-4°	10.75
6.8	6.8	619	650	632	-6°	0°	10.75
10	10	619	650	632	-2°	0°	10.68
12	12	619	650	632	0°	0°	10.68
15	15	625	644	632	0°	2°	10.68
18	18	625	644	632	0°	4°	10.64

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Comparing table 1 and table 5 we see that the synchronizing amplitude reduces for Osc1 and Osc2 compared to the free running amplitude. But the amplitudes of all the oscillators remain nearly constant with the change of coupling capacitance. The phase behaviour is similar to two oscillator case. Again as we have seen that the frequency of oscillation should be same for all the oscillators after synchronization and after synchronization all the oscillators operate in a single frequency.

We note that, the behaviour of Osc4 is different from other oscillators. In our experiment we replace Osc3with Osc4 in figure 7 and the similar experiments was done. But it was seen that the oscillators never locked.

4.3 Four Oscillator Case

To study the behaviour for four oscillators' case, we used Osc1, Osc2, Osc7 and Osc8 as we remember from Section 3.4 that Osc4 and Osc5 has frequency lower than other six oscillators. The circuit configuration is as following figure 8:

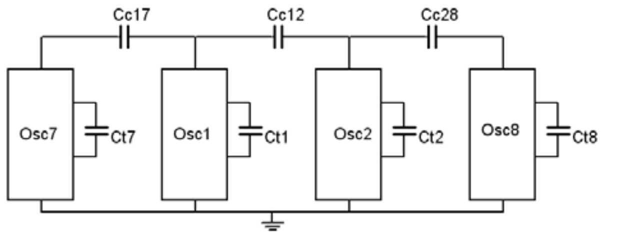


Figure 8: Capacitive coupling of four oscillators

Using figure 3 we found that, there require no tuning capacitor to make all the oscillator oscillate nearly in the same frequency. For this experiment the supply voltage was 5.50 volts .Again it is found that over the tuning rage the oscillators operates nearly in same phase and amplitude and frequency varies in a relatively insensitive way.

We found that the all the oscillators operate in the nearly in the same phase for the configuration of figure 8. We found a way to study the phase tuning behaviour more clearly. The circuit configuration was changed to following configuration:

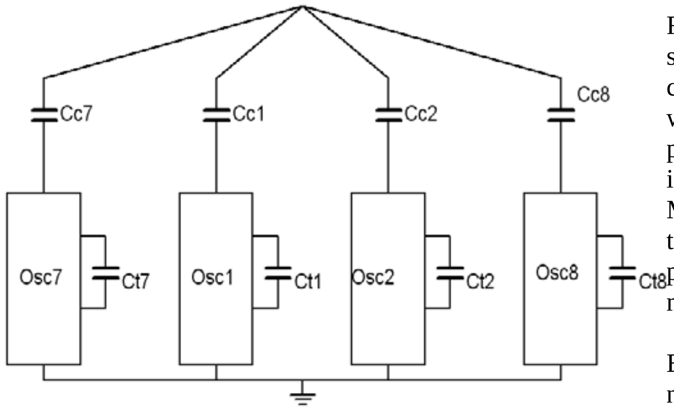


Figure 9: Four Oscillators connected as star

In our experiment we found that the amplitude and frequency tuning behaviour are same for the above configuration. The supply voltage is 5.50 volts and no tuning capacitor was used.

The phase tuning behaviour for four oscillators can be easily described with figure 10. Oscillator1's phase taken as the reference and different colour of arrow indicates the phases of other oscillators with respect to the phase of Osc1. The curved arrow indicates the increment of coupled capacitor .We note the figure is not drawn to scale and it indicates that, with the increment of coupled capacitor all the oscillators are tend to move towards 0̊ and oscillate in same phase of Osc1.

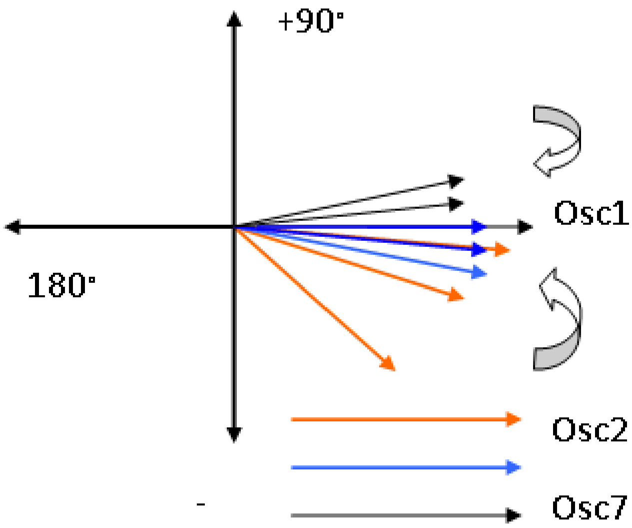


Figure 10: Phase tuning behaviour for four oscillators

The experiments to find out the amplitude, phase and frequency tuning behaviour for five and six stage oscillators had been carried and it was found that all the behaviour are relatively insensitive to the change of tuning capacitor. We haven't included the results for those stages as they gave the same results.

5. MODES OF OSCILLATION

From the analysis of coupled oscillation [13] in we have seen that there are two normal modes in a simple coupled circuit with two oscillators. In the two normal modes it was shown that, the oscillators oscillate either in same phase (0˚) or out of phase (180˚). Oscillators can oscillate in the combination of either of these two modes as well. Mode analysis done by Huntoon and Wesis[14] reveals that if oscillators oscillate in same frequency and same phase powers can be combined and it is called the desire modes.

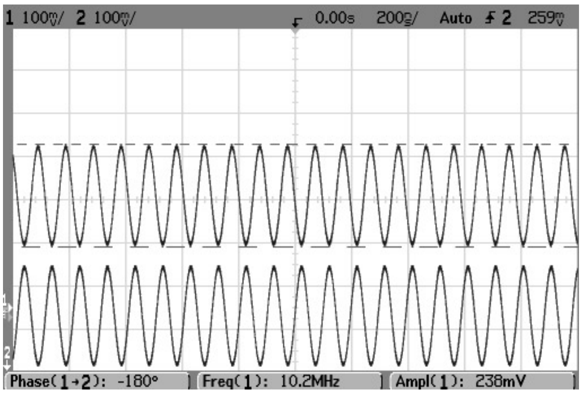
Endo and Mori [15] have found that when the network is nonlinear it is complicated to analyse the behaviour except the modes which are called simultaneous oscillation modes.

In our analysis, we are interested in the desire modes and we will use two separate notations for the two normal modes.

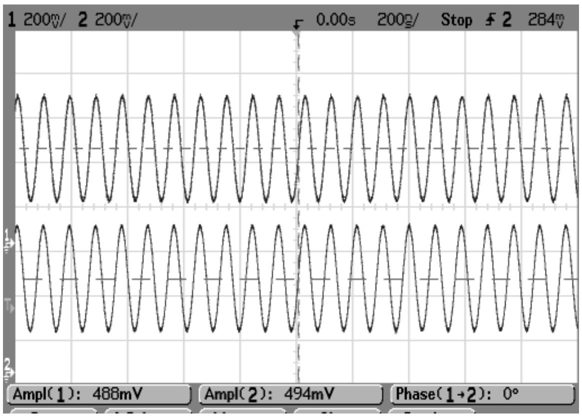
Normal mode1= If Oscillators oscillate in same phase (0˚)
Normal mode2= Oscillators oscillate in out of phase (±180˚)

In our work, we have found that it is possible to make all the oscillators oscillate in the desire mode. For the two oscillator system like figure 6, normal mode1 and normal mode 2 found by varying the power supplied to the oscillator. Experiment result shows that, for 3.3pF coupled capacitance:

- Supply voltage, Vcc= 2.60 Volts, Phase= -180˚ i.e. Normal mode2
- Supply voltage, Vcc= 4.19 Volts, Phase=0˚ i.e. Normal mode1



(a)



(b)

Figure 11: (a) Normal Mode2, (b) and Normal mode1 for two oscillators

For the five stage oscillator we are going to investigate one particular behaviour where we used Osc4 and Osc5 (those are quite different to other oscillators). The oscillators are star connected as the figure 12

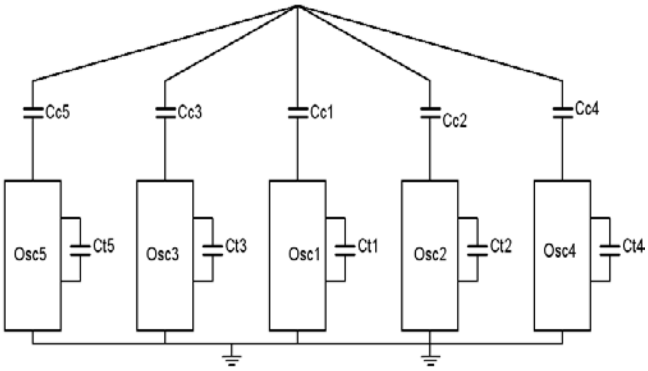


Figure 12: Five stage oscillators connected as star

It is found that Osc1, Osc2 and Osc3 were locked together and oscillate but the other two oscillators didn’t lock to this group. More over, Osc4 and Osc5 locked together in other mode forming a separate group with different frequency of oscillation. So there were two groups of oscillators oscillating in two different frequencies.

Table 6: Oscillation modes variation (five stage oscillator)

Cc(pF)	Phase, Φ_{12}	Phase, Φ_{12}	Phase, Φ_{45}	F_1 (MHz)	F_2 (MHz)
2.2	-180˚	140˚	180˚	10.85	10.45
3.3	-175˚	140˚	180˚	10.80	10.40
4.6	-175˚	130˚	180˚	10.74	10.34
6.8	-170˚	130˚	180˚	10.65	10.32
10	-160˚	120˚	180˚	10.60	10.30
12	-150˚	110˚	180˚	10.55	10.29
15	-150˚	120˚	180˚	10.53	10.28

In table 6, we denoted F_1 as the synchronizing frequency of the first group with oscillator 1, 2 and 3 while F_2 represent the synchronizing frequency of other group with Osc4 and Osc5. Synchronizing frequency F_2 is lower than F_1 in this case as from the figure 3 we find the free running frequency of Osc4 and Osc5 much lower than other oscillators. So they form a group where the free running frequency is nearly same.

6. CONCLUSION

We have presented some important behaviour of the coupled oscillator system in the work. We have built eight independent oscillators coupled together in variety of ways up to six stages and investigated the amplitude, frequency and phase relationship. In our work it was found that, when these oscillators are coupled together the complexity of the whole system increases and for all the stages from two to six, it is possible to make the entire oscillator group oscillate in same phase. We called this mode as normal mode 1.There are other modes of oscillation but only at normal mode 1 the oscillators can be used to combine power. The multimode problem increases as the number of oscillator increases.

We found, the frequency of all the oscillators should be nearly same to make them synchronous, as there are certain frequency band over which the do not lock. When oscillators are, locked they all oscillate in the same frequency and the amplitude will be varied according to the power level of the independent oscillators in free running state and this correlates with the theory presented by Kholchlov [12].

The results include that, the tendency to lock increases with increased phase difference and maximum for 0˚ and 180˚. It is caused due to the fact that, in phase oscillation eliminates current flow through coupling network. If the phase is 90˚ the coupling between independent oscillators approaches zero since the coupling current between adjacent oscillators cancels each other.

In our work, we have found little discrepancy between the theory and experimental observations. The theory presented by Humphrey and Fusco [16] states that, the amplitude and frequency variation are relatively insensitive to variation of coupled capacitance. But our experimental result suggests that, this case is true only when the oscillator locking bandwidth is high i.e. when the input power is high but for low input power it's not the case. They are sensitive to the change of coupled capacitance when the input power is low because the locking bandwidth is low for small input power which corresponds to the theory of Kurokawa.

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