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Simulation of Turbulent Flow Using FEM

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ABSTRACT

An analysis of developing and fully developed turbulent flow was carried out when different techniques have been used in a zone close to the solid wall. In this paper, wall element technique based on the finite element method (FEM) has been adopted and compared with other techniques for determination of confined turbulent flow with a one equation model used to depict the turbulent viscosity is applied to a smooth straight channel.

Keywords*:* *Developing and fully developed turbulent flow, Pressure flow, Coupled and iterative methods.*

1. INTRODUCTION

Fluid dynamics has many important applications. The varieties of these applications represented in wide range of applied computer science as well as engineering problems. Due to the growth of technology, these applications take advantage of the increasing speed of computers and hardware capabilities. The fluid motion can be represented by the well known Navier-Stokes (N-S) equations.

Due to the complexity of these equations an analytical solution is intractable and the last three decades, with the development and availability of more powerful digital computers much attention has been focused on the numerical simulation for solving the resulting set of non linear partial equations which dominate the flow behavior processes, the so-called computational fluid dynamics (CFD). This has been developed and used with confidence to solve a large range of flow problems and at the same time can offer a cost-effective to many fluid problems. Indeed, under conditions where experimentation is extremely difficult, CFD may be the only methodology available. Numerous theoretical and experimental works are available on laminar flow [1-2], but this is not the case of turbulent flow. Since it has not been possible to obtain exact analytical solutions to such flows, an accurate numerical approach would be very beneficial to researchers. The finite element method (FEM) is one of these methods that have recently emerged as a powerful tool for solving the N-S equations. Within the computational domain (i.e. main domain), the finite element method is used to discretise the equations governing the fluid motion.
It is well known that when a fluid enters a prismoidal duct the values of the pertinent variables change from some initial profile to a fully developed form, which is thereafter invariant in the downstream direction. The analysis of this region, which is known as developing region, has been the subject of extensive studies.

An effective technique is required to model the variation of the pertinent variables near a solid boundary, where the variation in velocity and kinetic energy, in particular, is extremely large near such surfaces since the transfer of shear form the boundary into the main domain and the nature of the flow changes rapidly. A significant grid refinement would be required, if a conversational finite element is used to model the near wall zone (N.W.Z.). Therefore, several solution techniques have been suggested in order to avoid such excessive refinement [3-5]. A more common approach is to terminate the actual domain subject to discretisation (main domain) at some small distance away from the wall, where the gradients of the independent variables are relatively small, and then a technique is a required to model the flow behavior in the near wall element. In this paper, different techniques were used. One of them was the use of an element technique, which is based on the use of the one dimensional finite element technique (i.e. one dimensional element in one direction normal to the solid wall). The validity of the wall element technique has been tested and proved in the previous work for fully developed turbulent flow [6-7]. Presently, the validity of the technique has been tested for developing flow, along with other techniques to simulate turbulent flow in a smooth straight channel.

2. MATHEMATICAL DESCRIPTION

The current investigation relates to steady - state incompressible two dimensional turbulent flow of a Newtonian viscous fluid with no body forces acting. For such a situation, the Navier-Stokes (N-S) equations associated with this type are,

$$\rho u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu_e\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right)\right] \quad \text{……... (1)}$$

Where i,j= 1,2. u_i, p are the time - averaged velocities and pressure respectively, ρ is the fluid density, μ_e is the effective viscosity which is given by $\mu_e = \mu + \mu_t$, μ and μ_t are the molecular viscosity and turbulent viscosity, respectively. The flow field must satisfy the continuity equation, which may be written as:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

Equation (1) and (2) cannot be solved unless a turbulence closure model can be provided to evaluate the turbulent contribution to μ_e. In the present work, a one equation model has been adopted so that,

$$\mu_t = C_\mu \rho k^{1/2} 1_\mu \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

1_μ is the length scale of turbulence which is given by

1_μ = 2.5 1_m, 1_m is the mixing length based on the prandtl hypothesis which has been specified algebraically for the present purposes as 0.4 times the normal distance from the nearest wall surface, C_μ is a constant and k is the time-averaged turbulence kinetic energy. The μ_t given by equation (3) requires that k to be known. This can be evaluated via a further transport equation given by:

$$\rho u_j \frac{\partial k}{\partial x_j} = \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + \mu_t \frac{\partial u_i}{\partial x_j}\left[\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right] - E \,.. \text{(4)}$$

Where $E = C_D \rho k^{3/2} / 1_\mu$, μ_t / σ_k is the turbulent diffusion coefficient, σ_k is the turbulent prandtl or Schmidt number and C_D is a constant.
The turbulence model based on equations (1),(2) and (4) are called the one-equation (k-l) model.

The above governing equations have been solved using a finite element method [8]. Within the near wall zone either universal laws concept [9], or one dimensional parabolic elements in a direction normal to solid wall is adopted. Within the main domain, conventional two dimensional isoparametric elements domain (i.e. 2-D up to the wall), are used to discretise the flow domain.

3. NEAR WALL ZONE TREATMENT

Within the near wall zone (Figure 1) different techniques were used, these are as follows,

i. Conventional finite elements (i.e. 2-D elements up to the wall) are used to discretise the N.W.Z. and the variable values, following analysis, are used as reference data. However an excessive mesh refinement was needed which is expensive in computer time and memory.

ii. In order to avoid such excessive refinement, semiempirical equations, known as "Wall Functions" or the so-called "Universal Laws", are used to bridge from a solid boundary to the main domain.

iii. In the present work, a finite elements technique has been adopted, using one-dimensional (3-noded elements) normal to the wall as shown in Figure 2.
In (iii) the momentum equations in direction normal to the wall surface, together with pressure equation and the kinetic energy equation are solved in the near wall zone.

4. BOUNDARY CONDITIONS

The boundary conditions applied are as follows:

1. Velocities and pressure

i. Forced boundary conditions such as,

$\phi = \phi_\Gamma$ on the boundary Γ_1 (ϕ = velocity)

ii. Traction boundary conditions, where the traction's are either defined or updated on boundary,

$$\tau_{x_1} = -p + \frac{\mu_e}{\rho}\left(\frac{\partial u_1}{\partial x_1}\right) \qquad x_1\text{- parallel to walls}$$

$$\tau_{x_2} = \frac{\mu_e}{\rho}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right) \qquad x_2\text{- normal to walls}$$

2. Turbulent kinetic energy (k) as per (i) above or updated Neumann conditions.

In this work, poisiuille flow is considered only and the boundary conditions were imposed as shown in Figure 1.
Compatible fully developed velocity and kinetic energy profiles which look like parabolic curve were imposed at the upstream section when fully developed turbulent flow was considered at the first stage and the traction's were updated at downstream. These profiles were obtained by using the outlet values form each iteration as new approximations to the values at the inlet until a convergent condition is satisfied.

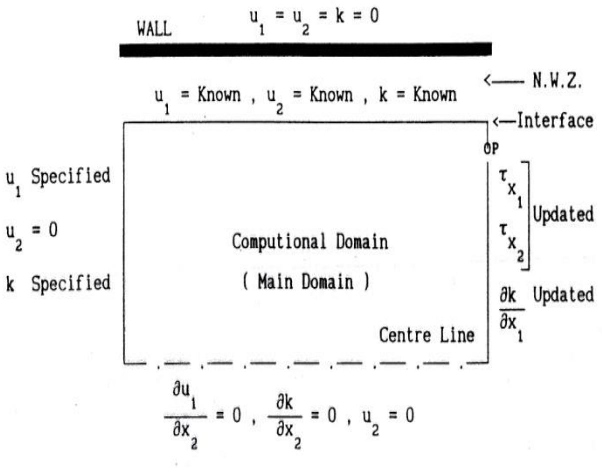

Figure 1: Boundary conditions when the mesh is terminated at small distance away from the wall

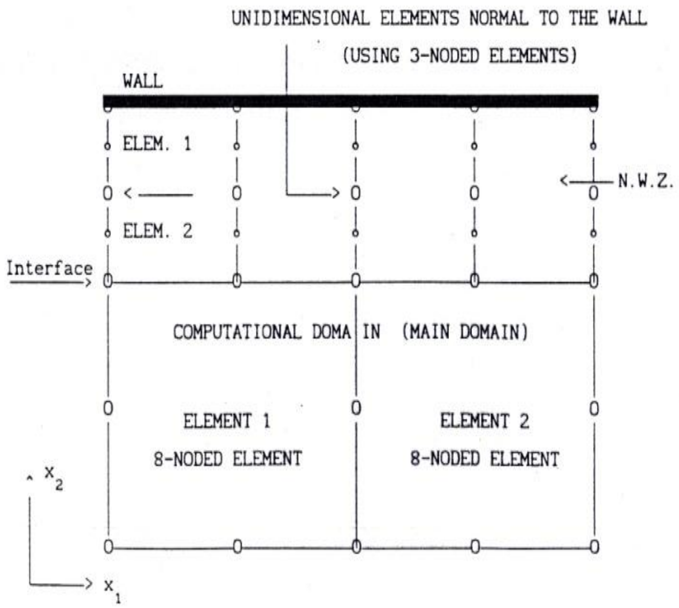

Figure 2: One-dimensional elements in one-direction normal to the wall used in the N.W.Z

5. RESULTS AND DISCUSSION

The validity of the adopted wall element technique (1-D elements in one direction) has been tested for two stages developing and fully developed flow, and compared with other accepted techniques and with experimental results in a parallel-sided duct of width D, which is taken as 1.0 in the present work, and length L. Different Reynolds number based upon the width of the channel of 50.000 and 12.000 were considered when pressure flow was considered only.

In the first stage, Full developed turbulent flow was considered, in this compatible fully developed velocity and kinetic energy profiles were imposed as initial upstream values. These values obtained by using the outlet values as inlet for the next iteration until a converged condition is satisfied.

Figures 3 clearly shows that the velocity values obtained by universal profiles have some discrepancy from those obtained from the advocated wall element technique (i.e. 1-D in 1 direction normal to the wall), and also shows an excellent agreement with the correct solution which resulted from the complete mapping (i.e. 2-D up to the wall). In fact, these are, superior to those obtained using universal laws. Figure 4 shows excellent agreement between the adopted technique and experimental results [10].

Figures 5 and 6 refer to the kinetic energy and the turbulent viscosity. These clearly demonstrate that the results obtained from the adopted wall element technique (1-D elements in one direction) and 2-D elements up to the wall are identical.

The conclusion of this stage is that, the wall functions (universal laws) have been shown to have limited application. Also, the use of 2-D elements up to the wall is not economically viable since it needs an excessive refinement which is very costly in computer time and memory size. The wall element technique has been applied successfully to fully developed flow. Since each 1-D string of elements is analysed individually, this saved computer memory and time required. For example, converged solution was obtained in 71 sec when 1-D elements in one direction employed; and 193 sec when universal laws employed. However, a considerable saving in both can result if a comparison is made with analysis when 2-D elements are used up to the wall

The second stage was concerned with developing turbulent flow in a channel. The velocity imposed upstream was constant at 2.0 m/sec and the kinetic energy imposed as 0.02 m^2/sec^2 and tractions updated downstream. The results are assumed to be converged when the relative change in any variables is less than 1%. A very fine mesh distribution was used such that with further refinement no increase in a accuracy was apparent.

Converged velocity profiles for developing turbulent flow are illustrated in Figure 7, which shows that the results obtained from the adopted wall element technique corresponds to that when the whole domain is mapped but differ from those when the universal laws was used.

The velocity, kinetic energy and turbulent viscosity profiles presented in Figures 8,9 and 10, indicates that for this case the correspondence between the results obtained when full mapping is used and 1-D element in one direction are not as good as that experienced previously, compared to those obtained in figures 5 and 6.

The conclusion of this stage is that, the use of the universal laws technique is not acceptable any longer for developing flow and the use of 2-D elements up to the wall is not economically viable. Also, the accuracy of the wall element technique when used in one direction is clearly not valid for developing flow.

The final stage was concerned with extension of NWZ further into main domain in order to prove the validity of the technique when developing flow is considered. Again, two different methods of solutions (coupled and uncoupled) were used to obtain the velocity and kinetic energy as presented in Figures 11, 12 and 13 when 1-D elements normal to the wall are employed in the NWZ. Clearly, Figure 12, the velocity plots became smoother when all element types are embodied within one overall matrix, especially when the mesh is terminated at 0.47D as compared to the curves in Figure 11 when an uncoupled method based on an iterative technique was employed. Figure 13 confirmed that the use of coupled method gave smoother results when he NWZ was extended up to 0.47D and corresponding to $Y^+ = 31$ at this location.

The conclusion of the final stage is that, the use of coupled method was better than the iterative method when the NWZ was extended. This applies to both developing and fully-developed flow. However, for low Y^+ values the uncoupled method was still applicable.

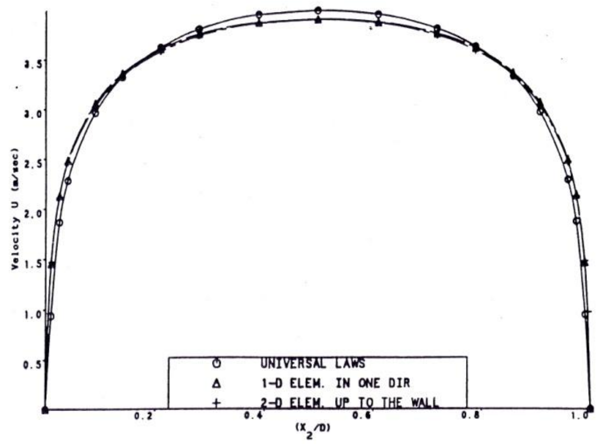

Figure 3: Turbulent velocity profiles for fully-developed flow, at 8D downstream, L=8D, Re=12.000

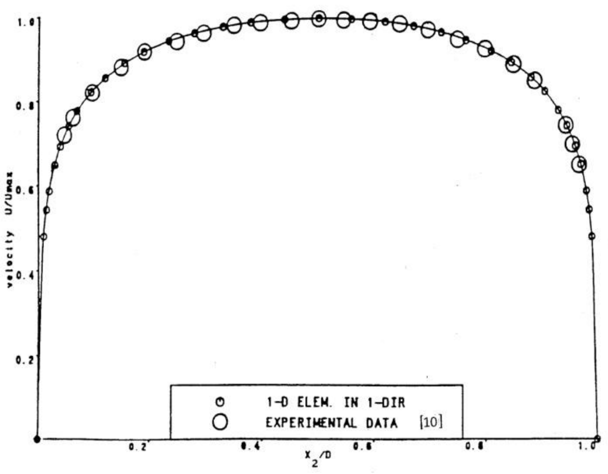

Figure 4: Turbulent velocity profiles for fully-developed flow, at 8D downstream, L=8D, Re=50.000

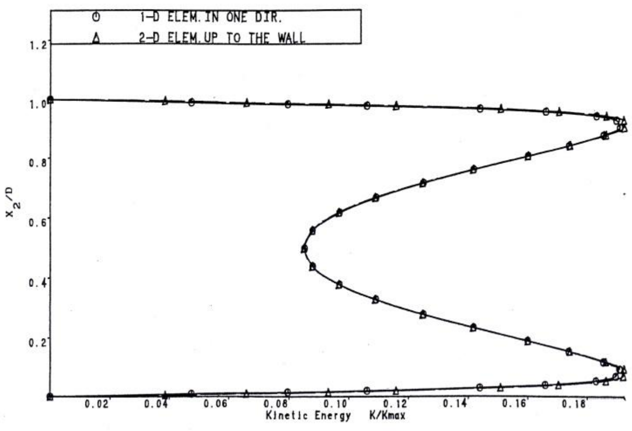

Figure 5: Kinetic energy profiles for fully-developed turbulent flow, at 8D downstream, L=8D, Re=12.000

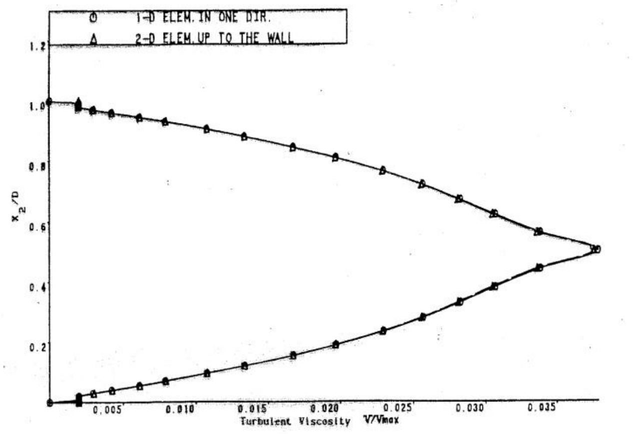

Figure 6: Viscosity distribution profiles for fully-developed turbulent flow, at 8D downstream, L=8D, Re=12.000

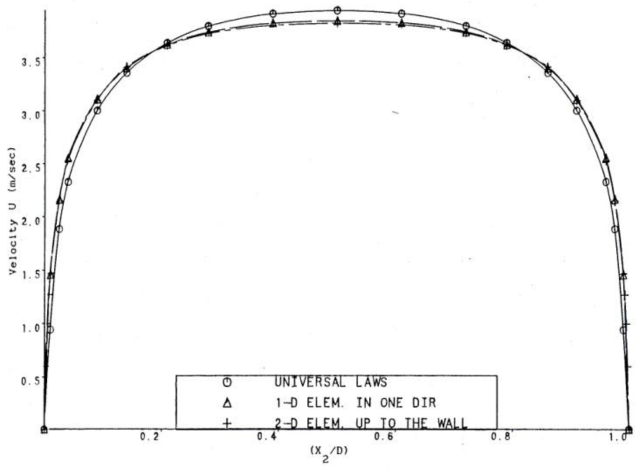

Figure 7: Developing velocity profiles for turbulent flow, at 10D downstream, L=10D, Re=12.000

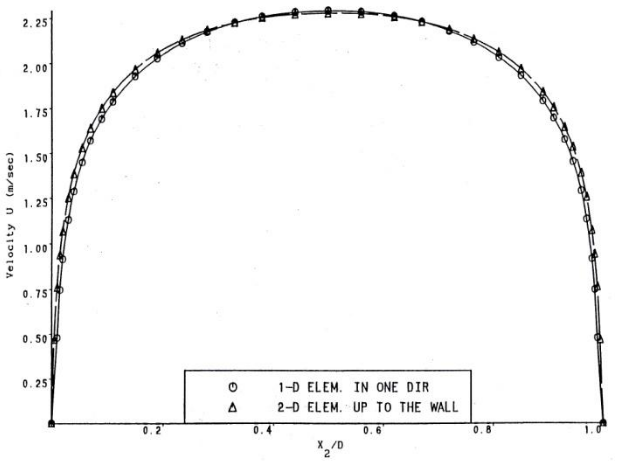

Figure 8: Developing velocity profiles for turbulent flow, at 10D downstream, L=10D, Re=12.000

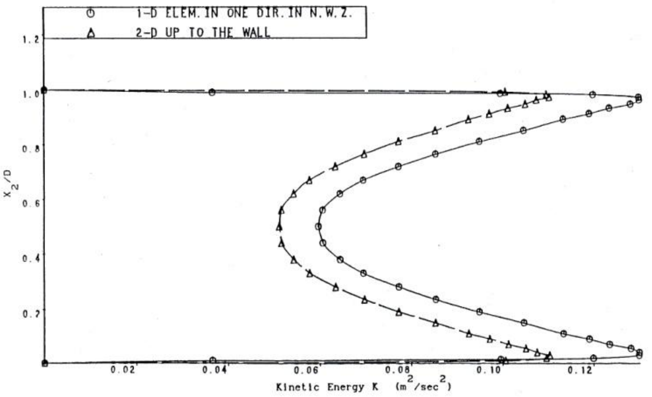

Figure 9: Developing kinetic energy profiles for turbulent flow, at 10D downstream, L=10D, Re=12.000

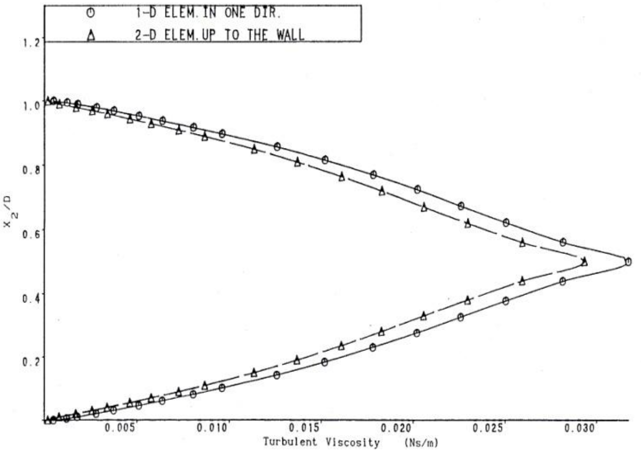

Figure 10: Developing Viscosity distribution profiles for turbulent flow at 10D downstream, L=10D, Re=12.000

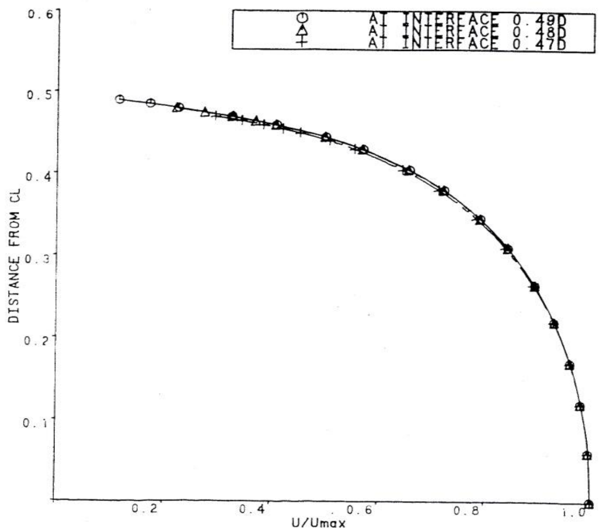

Figure 11: Developing velocity profiles for turbulent flow, at 10D downstream, L=10D, Re=12.000. Equations solved using iterative technique.

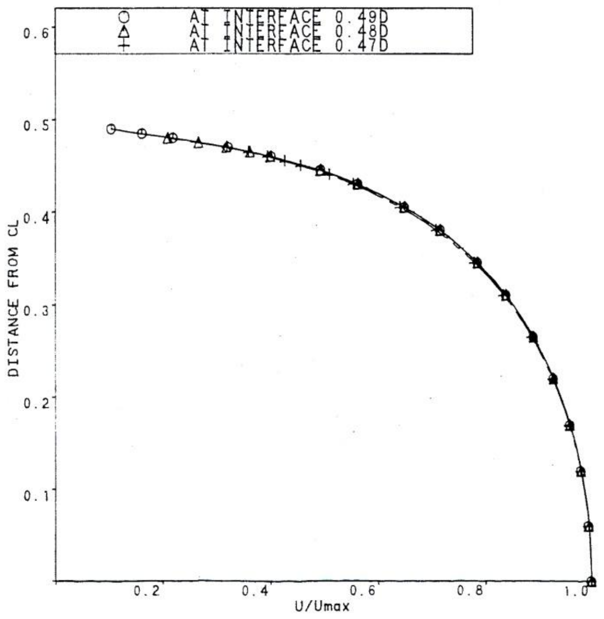

Figure 12: Developing velocity profiles for turbulent flow, at 10D downstream, L=10D, Re=12.000. Equations solved in one matrix.

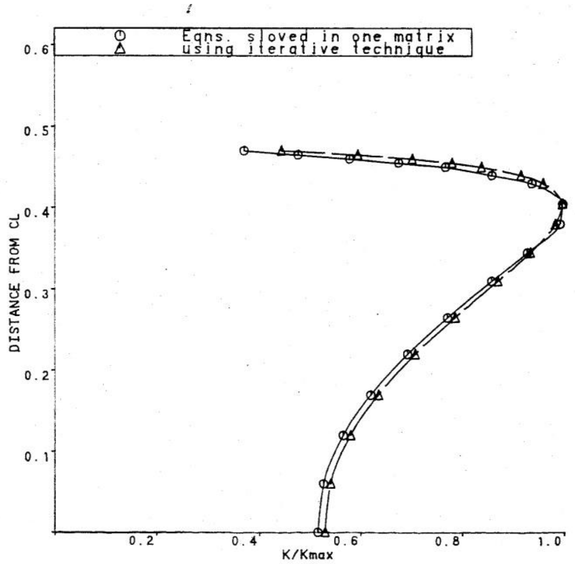

Figure 13: Developing kinetic energy profiles for turbulent flow, at 10D downstream, L=10D, using 1-D in one Dir, Re=12.000, at interface 0.47D

6. CONCLUSIONS

1. The use of the universal laws technique is not acceptable any longer for either developing or fully developed flow.
2. The general use of 2-D elements up to the wall is not economically viable. Therefore to avoid such an excessive refinement, these methods have been replaced by introducing a wall element technique, based on the use of the finite element methods.

3. The adopted wall element technique has been applied successfully and proved to be superior to other techniques and, can be used with confidence for fully developed turbulent flow. Therefore, this technique is valid for fully developed flow but not for developing flow since the assumption of unidirectional flow is unacceptable. Therefore, this technique can be used with confidence for turbulent fully-developed flow only, but not for developing flow.
4. When the NWZ was extended. The use of coupled method shows better and smoother results than those obtained when the iterative method was used. This applies to both developing and fully-developed flow.

REFERENCES

[1] D.M. Hawken, H.R. Tamaddon-Jahromi, P.Townsend and M. F. Webster, A Taylor-Galerkin based algorithm for viscous incompressible flow, *Int. Journal Num. Meth. Fluids,* (1990)

[2] Mehrotra, A. K. and Patience. G. S., Unified Entry Length for Newtonian and power law fluids in Laminar pipe flow, *J. Chem. Eng*., Vol. 68, pp.529-533, (1990)

[3] Launder, B. E. and Shima, N., Second moment closure for near wall sublayer: Development and Application, *AIAA Journal*, Vol. 27, pp. 1319-1325, (1989)

[4] Haroutunian, and Engelman, S., On modeling wall-bound turbulent flows using specialized near-wall finite elements and the standard k-ε turbulent model. Advances in Num. simulation of Turbulent flows, *ASME*, Vol. 117, pp. 97-105, (1991)

[5] Graft T, Gerasimov A, Iacovides H, Launder B., Progress in the generalization of wall-function treatments, *Int. Journal for heat and fluid flow*.

[6] P. 148-160, (2002)

[7] Sabah Tamimi, Validation of wall element technique of turbulent flow, GSTF International Journal on Computing (JoC), Volume 2, No. 1, pp.177-181, (2012).

[8] Sabah Tamimi, An Effective Technique Using Finite Element Methods, International Journal of Electrical & Computer Science IJECS-IJENS, Vol.12, No. 2, pp. 1-5, (2012).

[9] Taylor, C. and Hughes, T. G., Finite element programming of the Navier-Stokes equation, Pineridge press, (1981)

[10] Davies, T. J., Turbulent phenomena, *Academic Press*,(1972)

[11] Nayak, U.S.L. and Stevens, S.J, An experimental study of the flow in the annular gap between a long vehicle and a low close-fitting tunnel, *Report: Dept. of Technology*, *Loughborough University of Technology*, (1973)