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Closed Form of the Elementary Coaxial Cable Parameters in a Longitudinal Thermal Perturbation in Combination with the Skin Effect

Hatim G. Zaini
Taif University, College of Engineering, Electrical Engineering Department, Taif, Saudi Arabia

ABSTRACT

An Analytical calculation of the primary parameters, resistance and inductance especially, has been established in the case where both the skin effect and the linearly varying temperature along the coaxial transmission line are taken into account. The mathematical formulations are derived by first calculating the total resistance and inductance of the coaxial cable conductors within which the effective cross section due to the skin effect is considered. The effect of the temperature variation along the coax is also taken into account. The use of these primary parameters can be of paramount importance in the simulation using PSPICE or MULTISIM to calculate both frequency and time-domain responses such as the attenuation and phase shift.

Keywords: *Coaxial cable, Skin Effect, Distributed Parameters, thermal variation.*

1. INTRODUCTION

When ac current flows in a cylindrical conductor the charge carriers distribute themselves across the diameter of the wire. As frequency increases these carriers move away from the centre and so less is available for conduction. The wire's resistance, therefore, increases with frequency (skin effect). In addition to being frequency dependent the skin effect, due to its dependence on the conductor's resistivity, varies with thermal variations.

The skin effect has been treated using the well-known Bessel functions approximations [1] for the transient analyses [2][3] where the authors have substituted analytically the skin effect resistance formulation in the propagation constant and the characteristic impedance which permits an analytical formulation for the time-domain response using the inverse Laplace transform. These investigations have been made using the assumption the coax cable is of uniform temperature distribution, in the transverse and longitudinal directions.

The paper we are presenting deals with the computation of both resistance and inductance of the coaxial cable in the case where the transmission line thus studied lies in a medium whose temperature increases linearly with the longitudinal distance. The transverse losses are neglected in our investigation as their effect is minimal. The utilized model concerns a range of frequencies that are sufficiently high so as the skin effect in the conductors is taken into account. Combining analytically the two physical effects, namely the skin effect and the thermal longitudinal variations, we first calculate using the total differential equations the resistance and inductance of finite cable length. The whole coaxial cable is regarded as a discrete transmission line as in [4] and [5]. We then develop recurrent equations for each elementary cell according to the previously calculated total resistance and inductance for both Inner and Outer conductors. Our work is based upon the assumption that the Inner conductor is solid for the sake of simplicity. Similar calculations can apply for the case of hallow Inner conductor. The capacitance is assumed to be constant along the whole cable, whereas conductance is neglected for the sake of simplicity. The knowledge of the R, L, C, and G distributed parameters at uniform temperature allows an accurate and efficient calculation of the attenuation and phase using the Telegrapher's equation. However, in the case of linearly varying temperature; a discrete model of the whole coaxial cable is required. The simulation programs PSPICE or MULTISIM can conveniently be used for both transient analysis and frequency domain attenuation and phase computations as in [6] and [7].

2. CLOSED FORM MODEL

As attempted, the model will be first based upon the analytic equations of the infinitesimal resistances and inductances of the above mentioned Inner and Outer conductors. In [8] the authors have derived expressions for the inner and outer conductor impedance of a coaxial cable using Bessel functions but only for uniformly distributed temperature along the coax. In our paper we do consider both effects but using the "thick cable" approximations. In other words we assume that the skin depth is much smaller than the cable dimensions (radii), otherwise the calculations become too cumbersome and tedious to solve. For the case of "thin cable" approximations we recommend the use of the numerical models provided in reference [7]. The advantage of our model, which will be detailed in the subsequent sections, is that the closed forms are simple to implement in PSPICE and there is no need to generate a code as in [7], besides this the computation time becomes very reasonable.

The skin effect for a uniform temperature distribution is illustrated in the figure below.

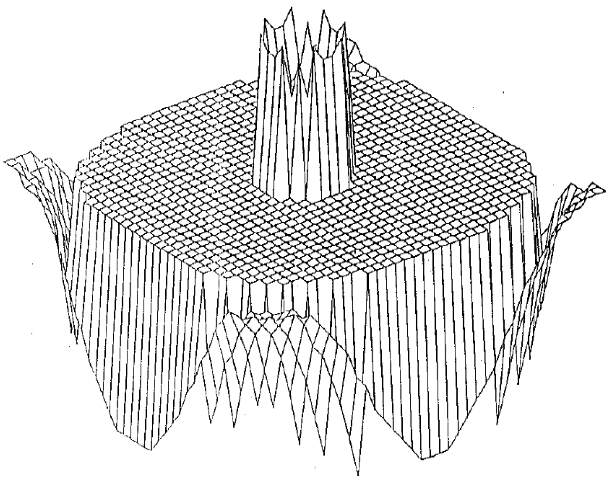

Figure 1: 3D Representation of the skin effect in a coax cable

2.1 Inner Conductor Resistance

For a solid Inner conductor the charges are flowing to the surface and figure 2 illustrates the phenomenon:

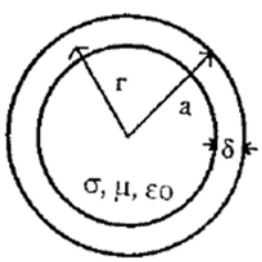

Figure 2: Inner Conductor geometry

According to figure 2 the infinitesimal resistance of a piece of wire of length dz is given by:

$$dR_i(z) = \frac{\rho_i(z)}{s(z)}.dz \quad (1)$$

Where $\rho_i(z)$ represents the distance-dependent Inner conductor resistivity and s(z) the effective cross section within which the charges carriers distribute.

It is important to point out that since this cross section depends explicitly on the skin depth; its value should be markedly affected by the temperature variations. Therefore the skin depth can reasonably be written as a z-dependent function as follows:

$$\delta_i(z) = \sqrt{\frac{\rho_i(z)}{\pi\mu_i f}} \quad (2)$$

Where μ_i represents the Inner conductor magnetic permeability constant, whereas f is the carrier frequency assumed to be sufficiently high. The effective cross section s(z) in eqn. (1) can be calculated according to figure 2 by the following :

$$s(z) = \pi\delta_i(z)(\delta_i(z) - 2a) \quad (3)$$

Substituting (3) in (1) leads to the infinitesimal resistance of the Inner conductor:

$$dR_i(z) = \frac{\rho_i(z)}{\pi\delta_i(z)(\delta_i(z)-2a)}.dz \quad (4)$$

Eqn. 4 is the simplified using the assumption that frequencies are sufficiently high to consider the following approximation:

$$\delta_i(z) \ll 2a \quad (5)$$

This relation is known as the “thick cable” approximation and, regardless of the cable length, eqn. 5 remains always valid. Therefore it is clearly seen that eqn. 1 can be written as follows:

$$dR_i(z) = \frac{1}{2\pi a}.\sqrt{\frac{\rho_{i0}\mu_i f}{\pi}}.\sqrt{1 + k_i z}.dz \quad (6)$$

Where the parameters in (6) are defined as follows:

ρ_{i0} : is the Inner conductor resistivity at room temperature
f: ac carrier frequency
k_i : Temperature-depth-dependent coefficient
a: Inner conductor radius

Integrating (6) over a given length, say Z, the total Inner conductor resistance takes the form:

$$R_i(Z) = \frac{1}{3\pi a k_i}.\sqrt{\frac{\rho_{i0}\mu_i f}{\pi}}.\left\{(1 + k_i.Z)^{\frac{3}{2}} - 1\right\} \quad (7)$$

Equation provides the resistance when both the skin depth and the temperature gradient are combined. We may notice the dependence of this resistance upon the $\sqrt{f}$, which denotes the skin effect. In addition the Z dependence confirms the dependence upon the longitudinal variation of the temperature.

2.2 Outer Conductor Resistance

The outer conductor is hallow and isolated from the inner conductor with low dielectric loss material such as Ceramic, Neoprene-Teflon, or Polypropylene. As the charges flow close to the surface on the inner part of the outer conductor, we do only consider the skin effect that is above the radius b as in figure 2 below.

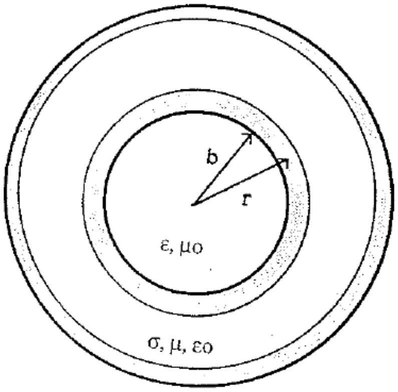

Figure 3: Outer Conductor geometry

The calculation method in this case is quite similar to the Inner conductor except that the integration boundaries do change because of the geometrical structure that differs. The obtained results for the outer conductor are given, as attempted, in the functional form as follows:

$$R_o(Z) = \frac{1}{3\pi b k_o} \cdot \sqrt{\frac{\rho_{o0}\mu_o f}{\pi}} \cdot \left\{(1 + k_o . Z)^{\frac{3}{2}} - 1\right\} \quad (8)$$

Where the parameters in (8) are defined as follows:

ρ_{o0} : is the outer conductor resistivity at room temperature
f: ac carrier frequency
k_o : Temperature-depth-dependent coefficient
b: Inner radius of the Outer conductor

The total resistance of a portion of coaxial cable of length Z is then given by the following closed form relation:

$$R(Z) = R_i(Z) + R_o(Z) \quad (9)$$

When the two conductors are made of the same material (e.g. copper) the coefficients k_i and k_o are equal.

2.3 Inner Inductance of the Inner Conductor

The Inner inductance of both Inner and Outer conductors depends on the range of frequencies especially when they are markedly high because of the skin effect that affects the effective cross section within which the current flows. Obviously the magnetic field behaves differently and so does the magnetic flux that is proportional to the inductance. As attempted, the current flowing inside the Inner conductor, at higher frequencies, produces a proportional magnetic field. According to fig. 1, let us write the total differential equations for the induced magnetic flux:

$$d\phi_i(r,z) = B_i(r,z)drdz \quad (10)$$

Noting in (9) the additional z-dependent magnetic field due to the existence of the thermal variation gradient (spatially dispersive current density in the conductors):

Using the Ampère's theorem, the magnetic field can easily be calculated as follows:

$$B_i(r,z) = \frac{\mu_i}{2\pi r} i(r,z) \quad (11)$$

The Inner current $i(r,z)$can be written as:

$$i(r,z) = j_i(r,z). s_i(r) \quad (12)$$

Where the current density:

$$j_i(r,z) = \frac{I_t}{\pi\delta_i(z)(\delta_i(z) - 2a)} \quad (13)$$

I_t being the total flowing current. The effective cross section $s_i(r)$ is given by:

$$s_i(r) = \pi(r^2 - a^2) \quad (14)$$

The radius r lies within the interval $[a - \delta_i(z), a]$ according to figure 2.

It is clear for instance that the total inner conductor magnetic flux for a given finite cable length Z takes the form:

$$\phi_i^t = \int_0^Z dz \int_{a-\delta_i(z),}^{a} B_i(r,z)dr \quad (15)$$

Using the same approximation as in (5) regarding the skin effect asymptotic behaviour the total magnetic flux of the inner conductor is given by the following:

$$\phi_i^t = \frac{\mu_i}{4\pi a} . I_t . \left(Z + a . \int_0^Z \xi(z)dz\right) \quad (16)$$

With $\xi(z) = \frac{ln\left(1 - \frac{\delta_i(z)}{a}\right)}{\delta_i(z)}$ (17)

The skin effect can conveniently be written in the functional form as follows:

$$\delta_i(z) = \delta_{i0}\sqrt{1 + k_i z} \quad (18)$$

Substituting (17) into (16) and then into (15) and integrating over z, with the assumption that the skin depth is much smaller than the inner conductor radius, the inductance reads:

$$L_i(Z) = \frac{\mu_i}{4\pi a}\left(Z - \frac{2a^2}{k_i \delta_i^2(0)}(F(Z) - F(0))\right) \quad (19)$$

With the dimensionless function $F(z)$ given by

$$F(z) = \left(1 - \frac{\delta_i(z)}{a}\right) . ln\left(1 - \frac{\delta_i(z)}{a}\right) \quad (20)$$

And

$\delta_i(0) = \delta_{i0} = \sqrt{\frac{\rho_{i0}}{\pi\mu_i f}}$ is nothing but the conventional room temperature skin depth.

2.4 Inner Inductance of the Outer Conductor

The calculation method in this case is quite similar to the inner conductor study. Only integration boundaries do change because of the geometry of the structure of the outer conductor (hallow conductor). The radius r lies within the interval $[b, b + \delta_o(z)]$ according to figure 3.

The integral becomes then:

$$\phi_o^t = \int_0^Z dz \int_b^{b+\delta_o(z)} B_o(r,z) dr \quad (21)$$

The outer conductor Inner inductance is thus given by the following:

$$L_o(Z) = \frac{\mu_o}{4\pi b}\left(Z - \frac{2b^2}{k_o \delta_o^2(0)}(G(Z) - G(0))\right) \quad (22)$$

With the dimensionless function $G(z)$ given by

$$G(z) = \left(1 + \frac{\delta_o(z)}{b}\right) . ln\left(1 + \frac{\delta_o(z)}{b}\right) \quad (23)$$

The total inductance of a portion of coaxial cable of length Z is then, including the mutual inductance, given by the following closed form relation:

$$L(Z) = L_i(Z) + L_o(Z) + Z.\frac{\mu_o}{2\pi}.ln\left(\frac{b}{a}\right) \quad (24)$$

3. CONCLUSION

We have provided a simple and accurate model for both resistance and inductance of the coaxial cable submerged in a linear temperature gradient using the "thick cable" approximations where the skin effect is present (i.e. high frequency domain). The advantage of this model is that it can easily be implemented in PSPICE to simulate the loss and phase in the frequency domain from one hand, and from the other hand to study the impulse response of such a cable in the time-domain. Further investigations could equally suggest a closed form for these primary parameters R and L in the case of "thin cable" approximations.

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