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Row of Shear Cracks Moving in Piezoelectric Crystals

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ABSTRACT

The elastic and electric fields created around an infinite row of collinear, antiplane shear cracks moving within a hexagonal piezoelectric crystal are studied using an extended method of dislocation layers. The solutions for a finite piezoelectric plate containing a single crack and a plate with an edge crack are also supplied by this analysis.

Keywords: *Moving antiplane shear cracks; Piezoelectric crystal; Dislocation layers.*

1. INTRODUCTION

Integral transform techniques and complex potential function theory have been used extensively to analyze the deformations created around cracks and inclusions within linear isotropic, and more generally anisotropic, elastic solids. But it is also well-established that continuous distributions of dislocations can be exploited to reproduce the behaviour of slit-like cracks. For example, Bilby and Eshelby [1] and Lardner [2] have adequately reviewed and applied this powerful so-called 'dislocation layer' technique within isotropic materials A general six-dimensional complex eigenvector and eigenvalue formalism was developed by Stroh [3, 4] for studying dislocations in linear arbitrary anisotropic media. This enabled Barnett and Asaro [5] to extend Stroh's method to study cracks in infinite anisotropic solids and thereby to predict their general properties, such as the crack extension forces and the energy of deformation. The dislocation layer technique was observed by Barnett and Asaro [5] "to be more straightforward and to facilitate computational convenience" more easily than the approach involving Fourier transforms and dual integral equations adopted previously by Stroh [3] to investigate the same problem.

However, high technological electromechanical devices, such as sensors, transducers, ultrasonic generators and actuators, make widespread use of 'smart' piezoelectric ceramics and thus the investigation of fracture in such materials is of considerable interest. Tupholme and Harvey [6, and the references therein] introduced a compact extended subscript notation for studying the propagation of surface acoustic waves in a nonlinear piezoelectric solid of arbitrary symmetry. Analogously, Barnett and Lothe [7] generalized Stroh's six-dimensional model for purely elastic media to an eight-dimensional complex framework to account for the effects of piezoelectricity. But the Stroh and integral formalisms necessitate the use of complicated numerical schemes for producing general results. Since the pioneering work of Deeg [8], who employed Green's function methods originating from the investigations of Barnett and Lothe [7], various analyses of stationary piezoelectric cracks and dislocations have been undertaken, as referenced in the investigations of, for example, Pak [9, 10, 11], Sosa [12], Sosa and Pak [13], Tupholme [14] and Yang et al [15].

Recent studies have been devoted to moving cracks in piezoelectric materials. Li and Mataga [16, 17], for example, presented general analyses of a semi-infinite, antiplane crack propagating in a solid with hexagonal symmetry using multiple Laplace transform, Wiener-Hopf and Cagniard-de Hoop methods, and Chen and Yu [18] used Laplace transforms and dual integral equations to find expressions for the crack-tip stress fields around a Yoffe moving crack, in terms of contour integrals involving functions which need to be evaluated numerically. Subsequently, Tupholme [19] used the dislocation layer technique to undertake an explicit investigation of the stress and displacement fields around a mode III loaded moving crack.

Investigations of the stress and displacement fields around an infinite row of collinear, loaded Griffith cracks within an isotropic elastic medium have been undertaken using the extensive complex variable general techniques of Muskhelishvili [20], and others. As part of a research project into "the vibration characteristics of railway coaches" and "the predominant effect of shear deformations in box beams", Koiter [21] presented a solution based on Muskhelishvili's general theory for equally-spaced, loaded cracks having equal lengths. Further, England and Green [22] employed an alternative method for analyzing the situation when each crack is opened by equal and opposite normal pressures. They used suitable integral representations of the complex potentials, and thereby reduced the problem to the solution of appropriate integral equations.

The purpose of the present paper is to demonstrate that the dislocation layer technique is especially suitable for extension to study the elastic and electric fields created around a row of collinear Yoffe cracks moving within a hexagonal piezoelectric material. This physically important situation has not been analyzed previously by any technique. The foundations of the current analysis are provided in the study by Wang and Zhong [23] of the fields created in a piezoelectric material around a moving screw dislocation having finite discontinuities across the slip plane in both the displacement and the electric potential.

Simultaneously, as discussed in Section 5, this analysis also provides the solutions to the problems of a single central crack within a finite piezoelectric plate and a finite plate having an edge crack.

2. PHYSICAL AND THEORETICAL FORMULATION

A homogeneous piezoelectric crystal which is hexagonally symmetrical of class 6 mm in its response is supposed to be everywhere at rest and stress-free in a natural reference state initially with a uniform density ρ.

The material contains an infinite periodic array of equal, plane, collinear, Griffith-type, strip cracks of constant width $2c$. Each of these is supposed to be moving with uniform speed v in its own plane parallel to its axis, as adopted in Yoffe's model [24].

At a time t, it is assumed that the cracks are centred at $x = vt$, $vt \pm 2h$, $vt \pm 4h$, … on the $y = 0$ plane, relative to a fixed system of rectangular Cartesian coordinates (x, y, z) with the z-axis chosen to be directed along the six-fold axis of symmetry of the hexagonal crystal. The cracks thus occupy the regions

$$I_t = \{\ (x, y, z):\ vt - c + 2nh < x < vt + c + 2nh,\quad y = 0,\quad -\infty < z < \infty\ \} \tag{1}$$

of the x-z plane, with $n = 0, \pm 1, \pm 2, \ldots$.

The components σ_{ij}, ε_{ij}, E_i, and D_i, where $i, j = x, y$ or z, of the stress tensor $\boldsymbol{\sigma}$, strain tensor $\boldsymbol{\varepsilon}$, electric field vector $\boldsymbol{E}$, and electric displacement vector $\boldsymbol{D}$, respectively, within the medium are related by the constitutive equations, which in matrix notation take the forms

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{yz}\\ \sigma_{xz}\\ \sigma_{xy}\end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0\\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0\\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0\\ 0 & 0 & 0 & 2c_{44} & 0 & 0\\ 0 & 0 & 0 & 0 & 2c_{44} & 0\\ 0 & 0 & 0 & 0 & 0 & c_{11}-c_{12}\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{xz}\\ \varepsilon_{xy}\end{bmatrix} - \begin{bmatrix} 0 & 0 & e_{31}\\ 0 & 0 & e_{31}\\ 0 & 0 & e_{33}\\ 0 & e_{15} & 0\\ e_{15} & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}E_x\\ E_y\\ E_z\end{bmatrix}, \tag{2}$$

$$\begin{bmatrix}D_x\\ D_y\\ D_z\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 2e_{15} & 0\\ 0 & 0 & 0 & 2e_{15} & 0 & 0\\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \varepsilon_{yz}\\ \varepsilon_{xz}\\ \varepsilon_{xy}\end{bmatrix} + \begin{bmatrix}\varepsilon_{11} & 0 & 0\\ 0 & \varepsilon_{11} & 0\\ 0 & 0 & \varepsilon_{33}\end{bmatrix}\begin{bmatrix}E_x\\ E_y\\ E_z\end{bmatrix}. \tag{3}$$

Here, in the usual contracted Voigt's notation with i and j taking integer values, c_{ij}, e_{ij} and ε_{ij} are the second-order elastic moduli measured at a constant electric field, the second-order piezoelectric constants and the second-order dielectric moduli measured at constant strain of the medium, respectively.

An electric potential, ϕ_e, can be utilized to express $\boldsymbol{E}$ as

$$\boldsymbol{E} \;=\; -\nabla\phi_e \tag{4}$$

and it is convenient to define a moving coordinate ξ by

$$\xi \;=\; x - vt\,. \tag{5}$$

A mode III deformation is created by applying mechanical and electric loads so that $\sigma_{yz} \rightarrow \mathcal{T}$ and $D_y \rightarrow \mathcal{D}$ at infinity, where $\mathcal{T}$ and $\mathcal{D}$ are prescribed constants.

The physical implications of the frequently-adopted electric displacement condition $\mathcal{D}=0$ (and others, for which the corresponding analyses could be given similarly) have been discussed by, for example, Kuo and Barnett [25], Suo et al. [26] and Wang and Mai [27].

3. MOVING PIEZOELECTRIC SCREW DISLOCATION

The stimulation for the present study of the cracks prescribed in Section 2 is provided by considering first a single "piezoelectric screw dislocation" moving in the material, which has not only the traditional discontinuity (the Burgers vector, **b**, of the elastic dislocation) in the elastic displacement, **u**, but also a discontinuity (the strength, b_4, of the charge dipole line) in the electric potential, ϕ_e. These jumps are given, respectively, for $\xi > 0$, by

$$\boldsymbol{u}^{\mathrm{III}}(\xi, 0+) - \boldsymbol{u}^{\mathrm{III}}(\xi, 0-) \quad = \quad (0, 0, -b),$$

$$\phi_e^{\mathrm{III}}(\xi, 0+) - \phi_e^{\mathrm{III}}(\xi, 0-) \quad = \quad -b_4 \tag{6}$$

with the superscript III referring throughout to quantities associated with this mode III antiplane strain deformation. The elastic and electric fields created around such a dislocation have been studied recently by Wang and Zhong [23]. It is these which enable the current analysis to be undertaken.

It is advantageous to rename several of the material constants and parameters that they use and to regroup them to make more apparent the important fundamental roles of, for example, μ_e, κ and v_e. Then it can be deduced from the results of Wang and Zhong [23] that for such a moving piezoelectric screw dislocation situated at the origin in a hexagonal piezoelectric crystal

$$u_z^{\mathrm{III}}(\xi, y) \quad = \quad \frac{b}{2\pi} \tan^{-1}\left(\frac{\kappa y}{\xi}\right), \tag{7}$$

$$\phi_e^{\mathrm{III}}(\xi, y) \quad = \quad \frac{1}{2\pi}\left[\frac{b e_{15}}{\varepsilon_{11}} \tan^{-1}\left\{\frac{(\kappa-1)\xi y}{\xi^2+\kappa y^2}\right\} + b_4 \tan^{-1}\left(\frac{y}{\xi}\right)\right] \tag{8}$$

where

$$\kappa = \sqrt{1-\delta_e^2}, \;\; \delta_e^2 = \frac{\rho v^2}{\mu_e}, \tag{9}$$

with the piezoelectrically stiffened elastic constant, μ_e, given by

$$\mu_e \; = \; c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}. \tag{10}$$

The corresponding non-zero components of the elastic and electric fields then follow from the constitutive Eqs. (2) and (3) in the forms

$$\sigma_{xz}^{\mathrm{III}}(\xi, y) \quad = \quad -\frac{by}{2\pi\varepsilon_{11}}\left(\frac{\varepsilon_{11}\kappa\mu_e}{\xi^2+\kappa^2 y^2} - \frac{e_{15}^2}{\xi^2+y^2}\right) - \frac{b_4 y}{2\pi}\frac{e_{15}}{\xi^2+y^2}, \tag{11}$$

$$\sigma_{yz}^{\mathrm{III}}(\xi, y) \quad = \quad \frac{b\xi}{2\pi\varepsilon_{11}}\left(\frac{\varepsilon_{11}\kappa\mu_e}{\xi^2+\kappa^2 y^2} - \frac{e_{15}^2}{\xi^2+y^2}\right) + \frac{b_4\xi}{2\pi}\frac{e_{15}}{\xi^2+y^2}, \tag{12}$$

$$D_x^{\text{III}}(\xi, y) \quad = \quad -\frac{be_{15} - b_4\varepsilon_{11}}{2\pi}\frac{y}{\xi^2 + y^2}\,, \tag{13}$$

$$D_y^{\text{III}}(\xi, y) \quad = \quad \frac{be_{15} - b_4\varepsilon_{11}}{2\pi}\frac{\xi}{\xi^2 + y^2}\,. \tag{14}$$

4. SOLUTION USING AN EXTENDED DISLOCATION LAYER METHOD

The basic procedure of the classical "dislocation layer method", which was devised originally for facilitating studies of cracks within elastic media, involves simulating a strip crack by an equivalent continuous planar distribution of dislocations. Here this fundamental technique is extended and exploited correspondingly for the above row of moving mode III cracks in the piezoelectric material by spreading piezoelectric screw dislocations, which incorporate not only elastic dislocations but also collinear charge dipole lines of the type specified by Eqs. (6) – (10).
To the right of each crack the screws are taken to be positive and to the left they are negative. The densities, $f(\xi)$ and $f_4(\xi)$, respectively, of the proposed distributions of the elastic dislocations and charge dipole lines, which model the cracks under consideration here, are therefore chosen to be odd functions of ξ.

It then follows from Eqs. (12) and (14) that the corresponding components of the stress and electric displacement at a point on the ξ-axis are given by

$$\sigma_{yz}^{\text{III}}(\xi, 0) \quad = \quad \frac{b(\varepsilon_{11}\kappa\mu_e - e_{15}^2)}{2\pi\varepsilon_{11}}\int_{-\infty}^{\infty}\frac{f(\xi')}{\xi - \xi'}d\xi' + \frac{b_4 e_{15}}{2\pi}\int_{-\infty}^{\infty}\frac{f_4(\xi')}{\xi - \xi'}d\xi' \tag{15}$$

$$D_y^{\text{III}}(\xi, 0) \quad = \quad \frac{be_{15}}{2\pi}\int_{-\infty}^{\infty}\frac{f(\xi')}{\xi - \xi'}d\xi' - \frac{b_4\varepsilon_{11}}{2\pi}\int_{-\infty}^{\infty}\frac{f_4(\xi')}{\xi - \xi'}d\xi'\,. \tag{16}$$

Clearly, from the Plemelj formulae, the integrals involved in the above equations must be interpreted as Cauchy principal value integrals.

The equilibrium equations for the dislocation array are that

$$\sigma_{yz}^{\text{III}}(\xi, 0) \quad = \quad -\mathcal{T}, \quad D_y^{\text{III}}(\xi, 0) \quad = \quad -\mathcal{D}, \quad \text{in} \quad I_t \tag{17}$$

Substitution of the expressions (15) and (16) into these yields a pair of simultaneous equations that can be solved to give the two singular integral equations

$$\int_{-\infty}^{\infty}\frac{f(\xi')}{\xi - \xi'}d\xi' \quad = \quad -\frac{2\pi}{b\varepsilon_{11}\kappa\mu_e}(\varepsilon_{11}\mathcal{T} + e_{15}\mathcal{D})\,, \tag{18}$$

$$\int_{-\infty}^{\infty}\frac{f_4(\xi')}{\xi - \xi'}d\xi' \quad = \quad -\frac{2\pi}{b_4\varepsilon_{11}\kappa\mu_e}\left(e_{15}\mathcal{T} - \frac{\varepsilon_{11}\kappa\mu_e - e_{15}^2}{\varepsilon_{11}}\mathcal{D}\right) \tag{19}$$

for the densities $f(\xi)$ and $f_4(\xi)$.

Now, the infinite integrals involved in Eqs. (18) and (19) can be expressed as the sum of the contributions from each of the cracks. Recalling that $f(\xi) = -f(-\xi)$ and noting that $f(\xi + 2nh) = f(\xi)$ together with the result

$$\sum_{n=1}^{\infty}\frac{1}{z^2 - n^2} = -\frac{1}{2z^2} + \frac{\pi}{2z}\cot \pi z\,, \tag{20}$$

it follows, for example, that the left-hand side of Eq. (18) can be rewritten (cf. Leibfried [28]) in the alternative form

$$\frac{\pi}{2h}\int_{-c}^{c}\frac{\cos\,(\pi\xi'/2h)}{\sin\,(\pi\xi/2h)-\sin\,(\pi\xi'/2h)}\,f(\xi')\,d\xi'\,. \tag{21}$$

Moreover, the introduction of new variables, defined by

$$\xi_1 \;=\; \sin\,(\pi\xi/2h), \qquad c_1 \;=\; \sin\,(\pi c/2h), \qquad \xi_1' \;=\; \sin\,(\pi\xi'/2h), \tag{22}$$

enables the integral equation for $f(\xi)$ to be conveniently transformed into

$$\int_{-c_1}^{c_1}\frac{f_1(\xi_1')}{\xi_1-\xi_1'}\,d\xi_1' \;=\; -\frac{2\pi}{b\varepsilon_{11}\kappa\mu_e}(\varepsilon_{11}\boldsymbol{7}+e_{15}\mathcal{D}) \quad \text{for} \quad |\xi_1|<c_1 \tag{23}$$

with $f_1(\xi_1)=f_1(\sin\,(\pi\xi/2h))\equiv f(\xi)$. By adaptation of the techniques discussed by, for example, Gakhov [29] and Muskhelishvili [30], the appropriate solution of this is given by

$$\begin{aligned} f_1(\xi_1) \;&=\; -\frac{2}{\pi b\varepsilon_{11}\kappa\mu_e}(\varepsilon_{11}\boldsymbol{7}+e_{15}\mathcal{D})\frac{1}{\left(c_1^{\,2}-\xi_1^{\,2}\right)^{1/2}}\int_{-c_1}^{c_1}\frac{\left(c_1^{\,2}-\xi_1'^{2}\right)^{1/2}}{\xi_1'-\xi_1}\,d\xi_1' \\ &=\; \frac{2}{b\varepsilon_{11}\kappa\mu_e}(\varepsilon_{11}\boldsymbol{7}+e_{15}\mathcal{D})\frac{\xi_1}{\left(c_1^{\,2}-\xi_1^{\,2}\right)^{1/2}}\,. \end{aligned} \tag{24}$$

Thus finally, by recalling Eqs. (22), the required dislocation density can be written as

$$f(\xi) \;=\; \frac{2}{b\varepsilon_{11}\kappa\mu_e}(\varepsilon_{11}\boldsymbol{7}+e_{15}\mathcal{D})\frac{\sin\,(\pi\xi/2h)}{\left[\sin^2(\pi c/2h)-\sin^2(\pi\xi/2h)\right]^{1/2}}\,. \tag{25}$$

Correspondingly, the solution for the charge dipole line density satisfying Eq. (19) can be shown to be

$$f_4(\xi) \;=\; \frac{2}{b_4\varepsilon_{11}\kappa\mu_e}\left(e_{15}\boldsymbol{7}-\frac{\varepsilon_{11}\kappa\mu_e-e_{15}^2}{\varepsilon_{11}}\mathcal{D}\right)\frac{\sin\,(\pi\xi/2h)}{\left[\sin^2(\pi c/2h)-\sin^2(\pi\xi/2h)\right]^{1/2}}\,. \tag{26}$$

Having derived the above representations for the necessary densities $f(\xi)$ and $f_4(\xi)$, expressions for all the components of the stress and electric fields can be found explicitly from Eqs. (11) – (14), as desired.

Customarily from a practical viewpoint, it is the magnitudes of the components directly ahead of the crack tips which are usually of particular interest in developing fracture criteria for the cracks.

The shear stress component on the plane y = 0 is obtained by summing the effects of all the image cracks. The infinite integrals involved can be replaced by finite integrals, using Eqs. (20) and (21), to give

$$\sigma_{yz}(\xi,0) \;=\; \boldsymbol{7}+\frac{b(\varepsilon_{11}\kappa\mu_e-e_{15}^2)}{2\pi\varepsilon_{11}}\int_{-c_1}^{c_1}\frac{f_1(\xi_1')}{\xi_1-\xi_1'}\,d\xi_1'+\frac{b_4e_{15}}{2\pi}\int_{-c_1}^{c_1}\frac{f_{41}(\xi_1')}{\xi_1-\xi_1'}\,d\xi_1'\,, \tag{27}$$

with $f_{41}(\xi_1) = f_{41}(\sin(\pi\xi/2h)) \equiv f_4(\xi)$. However, the cumbersome expression that follows by substituting for f_1 and f_{41} from Eqs. (25) and (26) can be manipulated and rearranged to neatly yield

$$\sigma_{yz}(\xi,0) \;=\; \mathcal{T} + \frac{\mathcal{T}}{\pi}\int_{-c_1}^{c_1} \frac{\xi_1'}{\left(c_1^{\,2} - \xi_1'^{2}\right)^{1/2}}\,\frac{d\xi_1'}{\xi_1 - \xi_1'}\,. \tag{28}$$

With the integral involved evaluated by complex variable contour integration and recalling Eqs. (22), this finally leads to

$$\sigma_{yz}(\xi,0) \;=\; \mathcal{T}\;\frac{\sin(\pi\xi/2h)}{\left[\sin^2(\pi\xi/2h) - \sin^2(\pi c/2h)\right]^{1/2}} \tag{29}$$

Similarly the corresponding electric displacement component on the plane $y = 0$ can be shown to be

$$D_y(\xi,0) \;=\; \mathcal{D} + \frac{be_{15}}{2\pi}\int_{-c_1}^{c_1} \frac{f_1(\xi_1')}{\xi_1 - \xi_1'}\,d\xi_1' - \frac{b_4\varepsilon_{11}}{2\pi}\int_{-c_1}^{c_1} \frac{f_{41}(\xi_1')}{\xi_1 - \xi_1'}\,d\xi_1'$$

$$\;=\; \mathcal{D}\;\frac{\sin(\pi\xi/2h)}{\left[\sin^2(\pi\xi/2h) - \sin^2(\pi c/2h)\right]^{1/2}}\,. \tag{30}$$

The resulting stress and electric intensity factors, $K_{\mathcal{T}}$ and $K_{\mathcal{D}}$, respectively , are then given by

$$K_{\mathcal{T}} = \lim_{\xi\to c+}\;(\xi - c)^{1/2}\sigma_{yz}(\xi,0) \;=\; \mathcal{T}\left[\frac{h}{\pi}\tan\left(\frac{\pi c}{2h}\right)\right]^{\frac{1}{2}}, \tag{31}$$

$$K_{\mathcal{D}} = \lim_{\xi\to c+}\;(\xi - c)^{1/2}D_y(\xi,0) \;=\; \mathcal{D}\left[\frac{h}{\pi}\tan\left(\frac{\pi c}{2h}\right)\right]^{\frac{1}{2}}. \tag{32}$$

The non-dimensional variations of $\sigma_{yz}(\xi,0)/\mathcal{T}$ (and likewise those of $D_y(\xi,0)/\mathcal{D}$) with ξ/c for a range of values of c/h are presented graphically in Fig. 1. It can be seen that the curves have fundamentally the same shapes for all the values of the ratio c/h, but it is found that the intensity increases as the value of c/h increases.

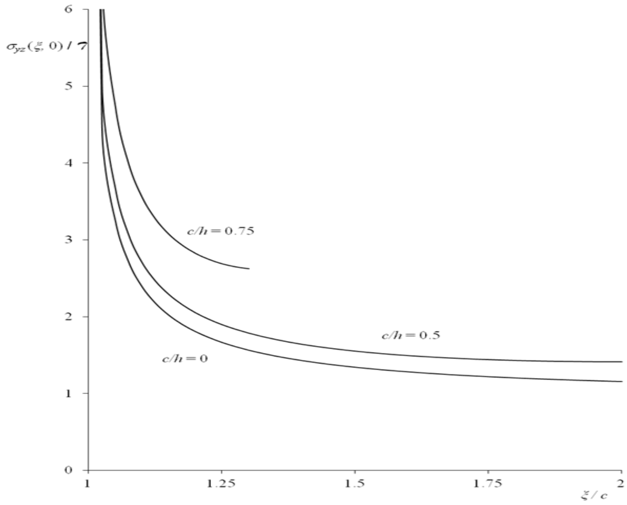

Figure 1. The variation of the scaled stress component $\sigma_{yz}(\xi,0)/\mathcal{T}$ with ξ/c for a range of values of c/h

The intensity factors can be regarded as the extensions to cracks moving within piezoelectric crystals of the fundamental stress intensity factor of a stationary crack in a purely elastic material. The density functions given by Eqs. (25) and (26) depend upon both $\mathcal{T}$ and $\mathcal{D}$, as well as the piezoelectric constants, the speed v and the parameters c and h. But it is interesting to observe that the stress intensity factor, $K_{\mathcal{T}}$ given by Eq. (31), depends upon $\mathcal{T}$ alone, and likewise the electric intensity factor $K_{\mathcal{D}}$ given by Eq. (32) upon $\mathcal{D}$ alone, together with c and h.

Clearly, it can be noted from Eqs. (25) and (26) that the above analysis is invalid when $\kappa = 0$ at $\delta_e = 1$. Recalling Eqs. (9), this occurs when the speed of the crack, v, reaches the piezoelectrically stiffened bulk shear wave speed, v_e, which is given by

$$v_e \quad = \quad \sqrt{\mu_e/\rho}\,. \qquad (33)$$

The material constants of crystals may vary with the manufacturer and year of production, but typical values for zinc oxide (ZnO) and a piezoelectric ceramic (PZT-4), respectively, are $c_{44} = -4.247\times 10^{10}\,\mathrm{Nm}^{-2}$, $e_{15} = -0.59\,\mathrm{Cm}^{-2}$, $\varepsilon_{11} = 0.738\times 10^{-10}\,\mathrm{Fm}^{-1}$, $\rho = 5.676\times 10^{3}\,\mathrm{kg\ m}^{-3}$ (from Shintani and Minagawa [31]) and $c_{44} = 2.56\times 10^{10}\,\mathrm{Nm}^{-2}$, $e_{15} = 12.7\,\mathrm{Cm}^{-2}$, $\varepsilon_{11} = 0.646\times 10^{-10}\,\mathrm{Fm}^{-1}$, $\rho = 7.5\times 10^{3}\,\mathrm{kg\ m}^{-3}$ (from Li and Mataga [16]). The corresponding values of v_e are therefore about 2883 ms^{-1} and 18334 ms^{-1}.

5. CONCLUSIONS

The deformation created around an infinite row of constant-width, Griffith-type, piezoelectric, moving cracks has been studied using an extension of the powerful method of dislocation layers.

But furthermore, since the distributions of the elastic dislocations and charge dipole lines are antisymmetrical about the planes $x = \pm\, h$, it is clear from an image construction that the stress component σ_{xz} and electric displacement component D_x both vanish on these two planes. Additionally therefore the elastic and electric fields around a single moving crack which initially occupies the region $-c < x < c$ in a finite piezoelectric plate $-h < x < h$, whose surfaces $x = \pm\, h$ are stress and charge free, are given by the above analysis.

Moreover, since the distributions are odd about $x = 0$ also, it further provides the solution for an edge crack $0 < x < c$ in a finite plate $0 < x < h$.

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