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# Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Spaces using Contractive Condition

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## ABSTRACT

In this paper, employing the property (E.A), we prove a common fixed theorem for weakly compatible maps using contractive condition in intuitionistic fuzzy metric space.

**Key words:** *Intuitionistic Fuzzy metric space, weakly compatible maps, property (E.A.).*

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## 1. INTRODUCTION

In 1986, Jungck [8] introduced the notion of compatible maps for a pair of self mappings. However, the study of common fixed points of non-compatible maps is also very interesting (see [19]). Aamri and El. Moutawakil [1] generalized the concept of non-compatibility by defining the notion of property (E.A) and proved common fixed point theorems under strict contractive conditions. Jungck and Rhoades [9] initiated the study of weakly compatible maps in metric space and showed that every pair of compatible maps is weakly compatible but reverse is not true. In the literature, many results have been proved for contraction maps satisfying property (E.A.) in different settings such as probabilistic metric spaces [5, 7]; fuzzy metric spaces [12, 15, 18]; intuitionistic fuzzy metric spaces [11, 13, 14, 16].
Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [22] and later there has been much progress in the study of intuitionistic fuzzy sets [4]. In 2004, Park [20] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani [6]. Fixed point theory has important applications in diverse disciplines of mathematics, statistics, engineering, and economics in dealing with problems arising in: Approximation theory, potential theory, game theory, mathematical economics, etc.
In this paper, employing the property (E.A), we prove a common fixed theorem for weakly compatible maps using contractive condition in intuitionistic fuzzy metric spaces.

## 2. PRELIMINARIES

The concepts of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skelton that we use are characterization fuzzy intersections and union respectively. These concepts were originally introduced by Menger [17] in study of statistical metric spaces.

### 2.1 Definition [21]

A binary operation $*$ : $[0,1]\times[0,1] \to [0,1]$ is continuous $t$-norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0,1]$;
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in [0,1]$.

### 2.2 Definition [21]

A binary operation $\lozenge$: $[0,1]\times[0,1] \to [0,1]$ is continuous $t$-conorm if $\lozenge$ satisfies the following conditions:

(i) $\lozenge$ is commutative and associative;
(ii) $\lozenge$ is continuous;
(iii) $a \lozenge 0 = a$ for all $a \in [0,1]$;
(iv) $a \lozenge b \le c \lozenge d$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in [0,1]$.

Alaca et al. [2] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous $t$-norm and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [10] as :

### 2.3 Definition [2]

A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric space if $X$ is an arbitrary set, $*$ is a continuous *t*-norm, ◊ is a continuous *t*-conorm and *M*, *N* are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$ ;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(vi) for all $x, y \in X$ , $M(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is left continuous;

(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \in X$ ;

(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if x = y;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(xi) $N(x, y, t) \lozenge N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;

(xii) for all $x, y \in X$ , $N(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is right continuous;

(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$ .

Then *(M, N)* is called an intuitionistic fuzzy metric space on *X*. The functions *M(x, y, t)* and *N(x, y, t)* denote the degree of nearness and the degree of non-nearness between *x* and *y* w.r.t. *t* respectively.

## 2.4 Remark

Every fuzzy metric space *(X, M, *)* is an intuitionistic fuzzy metric space of the form *(X, M, 1-M, *, ◊)* such that *t*-norm * and *t*-conorm ◊ are associated as

$$x \lozenge y = 1-((1-x) * (1-y)) \text{ for all } x, y \in X .$$

## 2.5 Remark

In Intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$, *M(x, y, *)* is non-decreasing and *N(x, y, ◊)* is non-increasing for all $x, y \in X$ .

Alaca, Turkoglu and Yildiz [2] introduced the following notions:

## 2.6 Definition

Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric

space. Then

(a) a sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n\to\infty} M(x_n, x, t) = 1 \text{ and } \lim_{n\to\infty} N(x_n, x, t) = 0.$$

## 2.7 Definition [2]

An intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ is said to be complete if and only if every Cauchy sequence in *X* is convergent.

### 2.7.1 Example

Let $X = \{1/n : n \in N\} \cup \{0\}$ and let * be the continuous *t*-norm and ◊ be the continuous *t*-conorm defined by $a * b = ab$ and $a \lozenge b = \min\{1, a+b\}$ respectively, for all $a, b \in [0,1]$. For each $t \in (0, \infty)$ and $x, y \in X$ , define (M, N) by:

$$M(x, y, t) = \begin{cases} \dfrac{t}{t+|x-y|}, & t > 0, \\ 0 & t = 0 \end{cases}$$

and

$$N(x, y, t) = \begin{cases} \dfrac{|x-y|}{t+|x-y|}, & t > 0, \\ 1 & t = 0. \end{cases}$$

Clearly, $(X, M, N, *, \lozenge)$ is complete intuitionistic fuzzy metric space.

## 2.8 Definition [8]

A pair of self mappings *(S, T)* of a intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ is said to be compatible if $\lim_{n\to\infty} M(STx_n, TSx_n, t) = 1$ and $\lim_{n\to\infty} N(STx_n, TSx_n, t) = 0$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in *X* such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = u$ for some *u* in *X*.

## 2.9. Definition [9]

Let *S* and *T* be self maps of intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$. A point *x* in *X* is called a coincidence point of *S* and *T* iff *Sx* = *Tx*. In this case, *w* = *Sx* = *Tx* is called a point of coincidence of *S* and *T*.

### 2.10. Definition [9]

A pair of self mappings *(S, T)* of a intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ is said to be weakly compatible if they commute at the coincidence points i.e., if $Su = Tu$ for some $u \in X$, then $TSu = STu$.

It is easy to see that two compatible maps are weakly compatible but converse is not true.

### 2.11. Definition [1]

A pair of self mappings *(S, T)* of a intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ is said to satisfy the property (E.A) if there exist a sequence $\{x_n\}$ in $X$ such that $\lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sx_n = z$ for some $z \in X$.

### 2.11.1. Example

Let $X = [0,\infty)$.

Consider $(X,M,N,*,\Diamond)$ be an intuitionistic fuzzy metric space as in Example 2.7.1. Define $S, T : X \to X$ by

$Sx = \frac{x}{5}$ and $Tx = \frac{2x}{5}$ for all $x \in X$. Clearly, for sequence $\{x_n\} = \left\{ 1/n \right\}$, $S$ and $T$ satisfies property (E.A).

## 3. MAIN RESULTS

### 3.1. Theorem

Let $S$ and $T$ are weakly compatible self maps on an intuitionistic fuzzy metric space $(X,M,N,*,\Diamond)$ such that :

(3.1) the pair $(S,T)$ satisfies the property (E.A.);

(3.2) for any $u,v \in X$ and for all $t > 0$, there exists $k \in (0,1)$ such that,

$$M^2(Tu,Tv,kt) \geq \left[ \min \left\{ \begin{array}{l} M(Su,Sv,t), M(Su,Tu,t), \\ M(Sv,Tv,t), \\ M(Sv,Tu,t), M(Su,Tv,t) \end{array} \right\} \right]^2$$

and

$$N^2(Tu,Tv,kt) \leq \left[ \max \left\{ \begin{array}{l} N(Su,Sv,t), N(Su,Tu,t), \\ N(Sv,Tv,t), \\ N(Sv,Tu,t), N(Su,Tv,t) \end{array} \right\} \right]^2$$

If *S(X)* be a closed subset of *X*, then *S* and *T* have a unique common fixed point.

**Proof:** As the pair $(S,T)$ satisfies property (E.A.), then there exist a sequence $\{x_n\}$ in $X$ such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$. Since *S(X)* is closed subset of *X*, $z = Sa$ for some $a$ in *X*. Therefore, $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = z = Sa.$

By (3.2), take $u = x_n, v = a$, we get

$$M^2(Tx_n,Ta,kt) \geq \left[ \min \left\{ \begin{array}{l} M(Sx_n,Sa,t), M(Sx_n,Tx_n,t), \\ M(Sa,Ta,t), \\ M(Sa,Tx_n,t), M(Sx_n,Ta,t) \end{array} \right\} \right]^2$$

*and*

$$N^2(Tx_n,Ta,kt) \leq \left[ \max \left\{ \begin{array}{l} N(Sx_n,Sa,t), N(Sx_n,Tx_n,t), \\ N(Sa,Ta,t), \\ N(Sa,Tx_n,t), N(Sx_n,Ta,t) \end{array} \right\} \right]^2$$

As $n \to \infty$

$$M^2(Sa,Ta,kt) \geq \left[ \min \left\{ \begin{array}{l} M(Sa,Sa,t), M(z,z,t), \\ M(Sa,Ta,t), \\ M(Sa,Sa,t), M(Sa,Ta,t) \end{array} \right\} \right]^2$$

$$M^2(Sa,Ta,kt) \geq M^2(Sa,Ta,t) \geq M^2(Sa,Ta,kt)$$

and

$$N^2(Sa,Ta,kt) \leq \left[ \max \left\{ \begin{array}{l} N(Sa,Sa,t), N(z,z,t), \\ N(Sa,Ta,t), \\ N(Sa,Sa,t), N(Sa,Ta,t) \end{array} \right\} \right]^2$$

$$N^2(Sa,Ta,kt) \leq M^2(Sa,Ta,t) \leq N^2(Sa,Ta,kt)$$

Therefore,

$$M^2(Sa,Ta,kt) = M^2(Sa,Ta,t)$$

and

$N^2(Sa,Ta,kt) = N^2(Sa,Ta,t)$ for all $x \in [kt,t]$

$$M^2(Sa,Ta,t) = M^2(Sa,Ta,t/k)$$

and

$N^2(Sa,Ta,t) = N^2(Sa,Ta,t/k)$ for all $x \in [t,t/k]$.

Similarly,

$M^2(Sa,Ta,t) = M^2(Sa,Ta,t/k^2)$ and

$N^2(Sa,Ta,t) = N^2(Sa,Ta,t/k^2)$

for all $x \in [t/k, t/k^2]$.

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Continuing like this, we can prove that

$M^2(Sa,Ta,t) = M^2(Sa,Ta,t/k^n)$

and

$N^2(Sa,Ta,t) = N^2(Sa,Ta,t/k^n)$

for all $x \in [t/k^{n-1}, t/k^n]$.

Take $n \to \infty$

$M^2(Sa,Ta,t) = 1$ and $N^2(Sa,Ta,t) = 0,$

hence, *Sa* = *Ta*. As *S* and *T* are weakly compatible, *STa* = *TSa* = *TTa* = *SSa*. Now, we claim that *Ta* is common fixed point of *S* and *T*.

By (3.2), take $u = a, v = Ta$, we get

$$M^2(Ta,TTa,kt) \geq \left[\min\left\{\begin{matrix} M(Sa,STa,t), M(Sa,Ta,t), \\ M(STa,TTa,t), \\ M(STa,Ta,t), M(Sa,TTa,t) \end{matrix}\right\}\right]^2$$

$$M^2(Ta,TTa,kt) \geq \left[\min\left\{\begin{matrix} M(Ta,TTa,t),1,1, \\ M(TTa,Ta,t), M(Ta,TTa,t) \end{matrix}\right\}\right]^2$$

$$M^2(Ta,TTa,kt) \geq M^2(Ta,TTa,t) \geq M^2(Ta,TTa,kt)$$

and

$$N^2(Ta,TTa,kt) \leq \left[\max\left\{\begin{matrix} N(Sa,STa,t), N(Sa,Ta,t), \\ N(STa,TTa,t), \\ N(STa,Ta,t), N(Sa,TTa,t) \end{matrix}\right\}\right]^2$$

$$N^2(Ta,TTa,kt) \leq \left[\max\left\{\begin{matrix} N(Ta,TTa,t),0,0, N(TTa,Ta,t), \\ N(Ta,TTa,t) \end{matrix}\right\}\right]^2$$

$$N^2(Ta,TTa,kt) \leq N^2(Ta,TTa,t) \leq N^2(Ta,TTa,kt)$$

Therefore,

$M^2(Ta,TTa,kt) = M^2(Ta,TTa,t)$

and

$N^2(Ta,TTa,kt) = N^2(Ta,TTa,t)$

for all $x \in [kt, t]$.

By applying same steps as above, one can easily show that

$M^2(Ta,TTa,t) = 1$

and

$N^2(Ta,TTa,t) = 0$. This gives, *Ta* = *TTa*. Therefore, *STa* = *TTa* = *Ta*. Hence, *S* and *T* has a common fixed point.

For, uniqueness, let *w* be another fixed point of *S* and *T*, then by (3.2), take *u* = *w*, *v* = *z*, we get

$$M^2(Tw,Tz,kt) \geq \left[\min\left\{\begin{matrix} M(Sw,Sz,t), M(Sw,Tw,t), \\ M(Sz,Tz,t), \\ M(Sz,Tw,t), M(Sw,Tz,t) \end{matrix}\right\}\right]^2$$

$$M^2(w,z,kt) \geq \left[\min\left\{\begin{matrix} M(w,z,t), M(w,w,t), \\ M(z,z,t), \\ M(z,w,t), M(w,z,t) \end{matrix}\right\}\right]^2$$

$$M^2(w,z,kt) \geq M^2(w,z,t) \geq M^2(w,z,kt)$$

and

$$N^2(Tw,Tz,kt) \leq \left[\max\left\{\begin{matrix} N(Sw,Sz,t), N(Sw,Tw,t), \\ N(Sz,Tz,t), \\ N(Sz,Tw,t), N(Sw,Tz,t) \end{matrix}\right\}\right]^2$$

$$N^2(w,z,kt) \leq \left[\max\left\{\begin{matrix} N(w,z,t), N(w,w,t), \\ N(z,z,t), \\ N(z,w,t), N(w,z,t) \end{matrix}\right\}\right]^2$$

$$N^2(w,z,kt) \leq N^2(w,z,t) \leq N^2(w,z,kt)$$

this gives,

$M^2(w,z,kt) = M^2(w,z,t)$

and

$N^2(w,z,kt) = N^2(w,z,t)$ for all $x \in [kt, t]$.

Continuing in a same manner as above, one can easily show that *w* = *z*. Hence, *z* is unique common fixed point of *S* and *T*.

## 4. APPLICATION

### 4.1. Definition [7]

Two families of self mappings $\{A_i\}_{i=1}^m$ and $\{B_j\}_{j=1}^n$ are said to be pairwise commuting if

(i) $A_i A_j = A_j A_i$, $i, j \in \{1,2,3,\ldots m\}$,

(ii) $B_i B_j = B_j B_i$, $i, j \in \{1,2,3,\ldots n\}$,

(iii) $A_i B_j = B_j A_i$, $i \in \{1,2,3,\ldots n\}$, $j \in \{1,2,3,\ldots n\}$.

As an application of Theorem 3.1, we prove a common fixed point theorem for two finite families of mappings on intuitionistic fuzzy metric spaces. While proving our result, we utilize Definition 4.1 which is a natural extension of commutativity condition to two finite families.

## 4.2. Theorem

Let $\{S_1, S_2, ..., S_p\}$ and $\{T_1, T_2, ..., T_q\}$ be four finite families of self mappings of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ such that $S = S_1.S_2.....S_p$ and $T = T_1.T_2.....T_q$ satisfying the conditions (3.1), (3.2) and $S(X)$ be a closed subset of $X$. If, the pairs of families an $(\{S_k\}, \{T_i\})$ commute pairwise then $S$ and $T$ have a unique common fixed point. Moreover, $\{S_k\}_{k=1}^{p}$ and $\{T_i\}_{i=1}^{q}$. have a unique common fixed point.

**Proof:** As the pairs of families an $(\{S_k\}, \{T_i\})$ commute pair wise, we first show that $TS = ST$ as

$TS = (T_1T_2 \dots T_q)(S_1S_2 \dots S_p) = (T_1T_2 \dots T_{q-1})(T_q S_1S_2 \dots S_p)$

$= (T_1T_2 \dots T_{q-1})( S_1S_2 \dots S_p T_q)$

$= (T_1T_2 \dots T_{q-2})(T_{q-1} S_1S_2 \dots S_p T_q)$

$= (T_1T_2 \dots T_{q-2})(S_1S_2 \dots S_p T_{q-1}T_q)$

$= \dots = T_1(S_1S_2 \dots S_p T_2 \dots T_q)$

$= (S_1S_2 \dots S_p)( T_1T_2 \dots T_q) = ST.$

and hence, obviously the pair $(S, T)$ is compatible and $(S, T)$ is weakly compatible. Now using Theorem 3.1, we conclude that $S$ and $T$ have a unique common fixed point in $X$, say $z$.

Now, one needs to prove that $z$ remains the fixed point of all the component mappings.

For this consider

$S(S_k z) = ((S_1S_2 \dots S_p)S_k )z = (S_1S_2 \dots S_{p-1})(S_p S_k )z$

$= (S_1S_2 \dots S_{p-1})(S_k S_p)z = (S_1S_2 \dots S_{p-2})(S_{p-1} S_k S_p)z$

$= (S_1S_2 \dots S_{p-2})(S_k S_{p-1} S_p)z = \dots = S_1(S_kS_2 \dots S_p)z$

$= (S_1S_k )(S_2 \dots S_p)z$

$= (S_k S_1)(S_2 \dots S_p)z = S_k (S_1S_2 \dots S_p)z = S_k Sz = S_kz.$

Similarly, one can prove that

$T(S_k z) = S_k (Tz) = S_kz$, $S(T_i z) = T_i(Sz) = T_iz$ and $T(T_i z) = T_i(Tz) = T_i z$.

which show that (for all $k$ and $i$ ) $T_iz$ and $S_kz$ are other fixed point of the pair $(S, T)$. As $S$ and $T$ have a unique common fixed point, so, we get

$z = S_k z = T_i z,$

for all $k = 1, 2, . . . , p$ and $i = 1, 2, . . . , q,$

which shows that z is a unique common fixed point of $\{S_k\}_{k=1}^{p}$ and $\{T_i\}_{i=1}^{q}$.

## 4.3. Remark

Theorem 4.2 is a slight but partial generalization of Theorem 3.1 as the commutativity requirements in this theorem are slightly stronger as compared to Theorem 3.1.

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