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Performance Evaluation of Two-Hop Wireless Link under Rayleigh and Nakagami-*m* Fading

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ABSTRACT

One of the major challenges in wireless communications, especially in urban areas, is to detect the digital information under different fading environments. In a two-hop wireless link, each hop is affected by fading and noise which degrades the overall performance of the communication system. In this paper, performance of a two-hop link is analyzed under Rayleigh and Nakagami-*m* fading environments taking maximal ratio combining (MRC) at the first hop and Alamuoti coding at the second hop. The objective of the paper is to observe the impact of fading effect on the bit error rate (BER) on the performance of the two-hop wireless link for 8-PSK and QPSK modulation schemes.

Key words: *Probability of symbol error, maximal ratio combining (MRC), Alamouti scheme, coherent demodulator, Nakagami-m fading channel.*

I. INTRODUCTION

The electromagnetic (EM) wave in wireless channel may be reflected and scattered by surrounding objects which results in multipath propagation of the signal. Therefore, multiple copies of the same signal arrive at the receiving end and they create delay spread. When the separation between the transmitter and the receiver is very large then the model is known as large scale propagation model but when separation between the transmitter and the receiver is small (less than 5km), usually in an urban or suburban area, then rapid variation of the signal strength within short distance or short duration is observed, this is known as small scale propagation model. Under a multipath propagation environment, the amplitude and phase of a composite modulated symbol usually vary widely and rapidly and this is known as fading effect. Two most important parameters of a fading channel are the coherence time and the coherence bandwidth [1]. A number of parameters like: symbol period, multipath delay spread, Doppler spread, coherence time/bandwidth, time variant or invariant property of the channel, channel gain etc. play vital role on the performance of a wireless link as summarized in [1] and [2].

A channel may be time-selective or frequency-selective depending on the time varying nature of the impulse response of the channel. From the time auto-correlation function of the channel impulse response, the channel may be classified as wide-sense stationary uncorrelated scattering (WSSUS) channel. The complex envelop of a modulated wave is a random variable (RV) which may follow Rayleigh, Ricean, or Nakagami-*m* distribution depending on the condition of the channel. For example, if there is no line-of-sight (LOS) between the transmitter and the receiver along with multipath fading, the distribution follows Rayleigh probability density function (PDF) instead of Ricean PDF.

Multi-hop transmission has drawn much attention in recent years both in the industry and academia. Relaying is a convenient solution to satisfy the requirements of the next generation wireless communication systems, such as: high data rates and large coverage areas [3]. Dual-hop relaying communication has a number of advantages over direct-link transmission in terms of connectivity, power saving and channel capacity for the high data-rate coverage required for future cellular and ad-hoc networks [4]. Cooperative relay in this multi-hop transmission network was introduced earlier to make use of the broadcast nature of the wireless channel and create virtual antenna arrays with communication nodes having single antennas [5]. In the open literature, works considering beamforming in fixed-gain relaying systems are not as rich as for variable-gain relaying or amplify-and-forward (AF) relaying techniques [6].

In [7], the authors derived asymptotic, average symbol error probability for AF cooperative diversity networks. The resulting expressions derived in [7] (using the bounding approach) are general for any type of fading distributions provided that the PDF of the instantaneous signal to noise ratio (SNR) is not zero. In both [8] and [9], multi-hop relaying over Nakagami-*m* fading channels have been studied. In [10], Karagiannidis presented the performance bounds for AF multi-hop systems over Rician, Nakagami-*m* and Nakagami-*q* (Hoyt) generalized fading channels.

This paper presents the performance of a cooperative relay network having a multi-antenna relay with identically distributed branches. It is to be mentioned here that in [5], the authors used a two-hop regenerative relay network under correlated Nakagami-*m* fading environment. But in this channel model they used only maximal ratio combining (MRC) for both the relay parts. However, in our present system model, we consider the link from source to relay performs MRC scheme and the link from relay to destination performs Alamouti coding scheme. Link performance has been evaluated for both Rayleigh fading channel and Nakagami-*m* fading channel.

The rest of the paper is organized as follows. Section II describes the system model while results from the system model are presented in Sec. III. Finally, Sec. IV concludes the paper.

II. SYSTEM MODEL

Let us consider a dual-hop wireless system where a source node S communicates with a destination node D through the help of a relay R, as shown in Fig. 1. Node S is equipped with a single transmitting antenna N1 and node D is equipped with a single receiving antenna D1, whereas the relay has two antennas R1 and R2. These two antennas R1 and R2 play the role of receiving antennas when they receive signals from the transmitting antenna N1. Therefore, from the path S to R the method of MRC is used as there are only one transmitter and two receivers. Now, from R to D, these two antennas R1 and R2 act as transmitting antennas and in this link, Alamouti Code [11] is used as there are two transmitters and one receiver and in this case a full rate transmission can be achieved. Let us consider that there is no direct link between S and D and the communication can be performed only through the relay R. This introduces fixed gain on the received signal regardless of the amplitude on the first hop, hence in an output signal with variable power, this type of fixed gain relay is cost effective to implement. Here, space diversity technique is used. Space diversity technique employs multiple transmit or receive antennas having some separation between the adjacent antennas. Usually the separation between two adjacent antenna elements are kept half of the wavelength or slightly greater. In this method, the antennas are separated vertically to increase the transmission and reception availability of LOS microwave links by reducing the duration and frequency of multipath fading events. During the period of multipath fading, deep fades of signals received on two vertically received antennas rarely overlap in time. Various techniques are available to combine the signals from multiple diversity branches. Maximal ratio combining (MRC) scheme is one of them. MRC represents a theoretically optimal combiner over fading channels as a diversity scheme in a communication system. Theoretically, multiple copies of same information signal are combined so as to maximize the instantaneous SNR at the output [2].

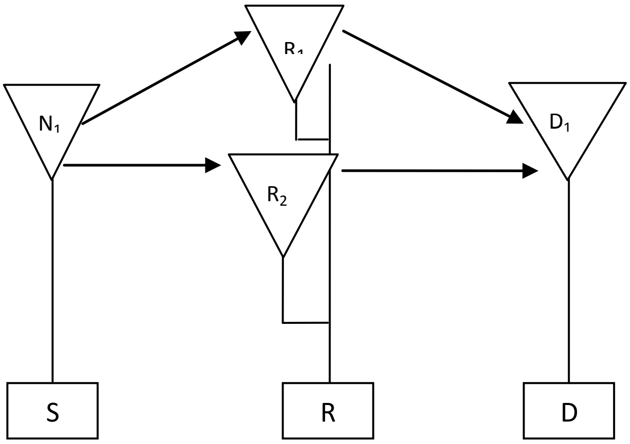

Fig. 1: System Model

The Rayleigh fading is a statistical model for the effect of a propagation environment on a radio signal, such as that used by wireless devices. This fading model assumes that the magnitude of a signal that has passed through such a transmission medium will vary randomly, or fade, according to a Rayleigh distribution. The Rayleigh fading is viewed as a reasonable model for the effect of heavily built-up urban environments or radio signals [12]. Rayleigh fading is most applicable when there is no dominant propagation path along a LOS between the transmitter and the receiver. If there is a dominant LOS, then Ricean fading model, about which we will talk later, is more applicable. Rayleigh fading is a reasonable model when there are many objects in the environment that scatter the radio signal before it arrives at the receiver. It can be shown that if there is sufficiently much scatter, the channel impulse response will be well-modeled as a Gaussian process irrespective of the distribution of the individual components. If there is no dominant component to the scatter, then such a process will have zero mean and phase evenly distributed between 0 and 2π. The envelope of the channel response will therefore be Rayleigh distributed. Calling this envelope R, it will have a probability density function.

$$p_{Rayleigh}(r) = \frac{2r}{\Omega} e^{-r^2/\Omega}, \qquad r \geq 0, \tag{1}$$

where $\Omega \equiv \mathbf{E}(R^2)$ is the average fading power, $\mathbf{E}(\cdot)$ is the statistical expectation operator. We note that Rayleigh fading is a small-scale effect. There will be bulk properties of the environment such as path loss and shadowing upon which the fading is superimposed.

Besides Rayleigh and Ricean fading, refined models for the PDF of signal amplitude exposed to mobile fading have been suggested. In the Nakagami-*m* fading, the distribution of the amplitude and signal power can be used to find probabilities on signal outages. If the envelope is Nakagami-*m* distributed, the corresponding instantaneous power is gamma distributed. With Nakagami-*m* distribution [13], sometimes denoted by *m*-Nakagami distribution, a wide class of fading channel conditions can be modeled. This fading distribution has gained a lot of attention lately, since the Nakagami-*m* distribution often gives the best fit to land-mobile and indoor mobile multipath propagation as well as in the ionospheric radio links [14]. Most recent studies also showed that Nakagami-*m* gives the best fit for satellite-to-indoor and satellite-to-outdoor radio wave propagation [15], [16]. The PDF for a Nakagami-*m* distribution channel can be expressed as [17]-[19]:

$$p_{Nakagami-m}(r) = \frac{2m^m r^{2m-1}}{\Omega^m \Gamma(m)} e^{-mr^2/\Omega}, \; r \geq 0, \qquad (2)$$

where Ω is defined earlier and $\Gamma(.)$ is gamma function. In Eq. (2), *m* is known as the Nakagami fading parameter or the shape factor of the Nakagami-*m* distribution. This parameter determines the severity of the fading [20]. The value of *m* ranges between ½ and ∞. When $m \to \infty$, the channel converges to a static channel. As special cases, Nakagami-*m* includes Rayleigh distribution when $m=1$, and one-sided Gaussian distribution for $m = 1/2$. This basically means that, if $m < 1$, the Nakagami-*m* distributed fading is more severe than Rayleigh fading, and for values of *m* > 1, the fading circumstances are less severe. For the values of $m > 1$, the Nakagami-*m* distribution closely approximates the following Ricean distribution [13]:

$$p_{Ricean}(r) = \frac{2(K+1)r}{\Omega} e^{-[K+(K+1)r^2/\Omega]} \times I_0\left(2\sqrt{\frac{K(K+1)}{\Omega}}r\right), \; r \geq 0. \qquad (3)$$

The parameter *m* and the Ricean factor *K* (which determines the severity of the Ricean fading) can be mapped via the following equation:

$$m = (1+K)^2/(1+2K) \text{ for } K \geq 0.$$

In the present investigation, we have been particularly interested to investigate the performance of two-hop wireless link by means of calculating the probability of error under Rayleigh and Nakagami-*m* fading processes.
The probability of error for a particular modulation scheme with symbol error probability $\text{Prob}(error, \gamma_{mrc})$ is [2]:

$$P_e = \int_0^\infty \text{Prob}(error, \gamma_{mrc})\, f_\Gamma(\gamma_{mrc})\, d\gamma_{mrc}, \qquad (4)$$

where, γ_{mrc} is the instantaneous output SNR of the MRC combiner, which can be written as [2]:

$$\gamma_{mrc} = (E/N_0)\sum_{k=1}^{N_r} \alpha_k^2$$

.

$$(5)$$

In Eq. (5), the quantity α_k is the signal multiplication factor, E/N_0 is the symbol energy-to-noise spectral density ratio and N_r is the total number of diversity branches at the Relay site.
In the following, we now consider two special cases of interest: Rayleigh fading case and Nakagami-*m* fading case.

Case A: Rayleigh Fading Environment

In this case, α_k is assumed to follow the Rayleigh distribution, then it can be shown that the PDF of γ_{mrc}, i.e., the sum of squares of α_k (*k* = 1, 2, …, N_r), (see Eq. (5)), follows the Chi-square distribution [2]:

$$f_{\Gamma\,\text{Rayleigh}}(\gamma_{mrc}) = \frac{1}{(N_r-1)!}\frac{\gamma_{mrc}^{N_r-1}}{\gamma_{av}^{N_r}}\exp\left(-\frac{\gamma_{mrc}}{\gamma_{av}}\right), \qquad (6)$$

where γ_{av} is the average SNR.

Adapting the formula for the probability of symbol error for QPSK over an additive white Gaussian-noise (AWGN) channel, we may write [2]:

$$\text{Prob}(error, \gamma_{mrc}) = \frac{1}{2}\,\text{erfc}\left(\sqrt{\frac{1}{2}\gamma_{mrc}}\right), \qquad (7)$$

where erfc (*x*) denotes the complementary error function with argument *x*.

Thus, the probability of error for the QPSK modulation scheme under the Rayleigh fading environment can be written as (by substituting Eqs. (6) and (7) into Eq. (4))

$$P_{e(QPSK)} = \frac{1}{2(N_r-1)!}\int_0^\infty \text{erfc}\left(\sqrt{\frac{1}{2}\gamma_{mrc}}\right) \times \frac{\gamma_{mrc}^{N_r-1}}{\gamma_{av}^{N_r}}\exp\left(-\frac{\gamma_{mrc}}{\gamma_{av}}\right) d\gamma_{mrc}, \qquad (8)$$

For 8PSK, it can be shown that the probability of symbol error is [2]:

$$\text{Prob}(error, \gamma_{mrc}) = \frac{1}{2}\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{8}\right). \qquad (9)$$

Thus, the probability of error for the 8PSK modulation scheme under the Rayleigh fading can then be written as (by substituting Eqs. (6) and (9) into Eq. (4))

$$P_{e(8PSK)} = \frac{1}{2(N_r-1)!}\int_0^{\infty}\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{8}\right)\right)\times\frac{\gamma_{mrc}^{N_r-1}}{\gamma_{av}^{N_r}}\exp\left(-\frac{\gamma_{mrc}}{\gamma_{av}}\right)d\gamma_{mrc} \qquad (10)$$

Case B: Nakagami-*m* Fading Environment

For Nakagami-*m* fading, the probability density function of the signal envelope is given by Eq. (2), whereas the PDF γ_{mrc} is gamma distributed. This PDF can be written as [14]:

$$f_{\Gamma\,\text{Nakagami}}(\gamma_{mrc}) = \frac{m^m}{\gamma_{av}^m\,\Gamma(m)}\,\gamma_{mrc}^{\,m-1}\exp\left(-\frac{m\gamma_{mrc}}{\gamma_{av}}\right). \qquad (11)$$

Therefore, for QPSK, the probability of error for Nakagami-*m* fading is (by substituting Eqs. (7) and (11) into Eq. (4))

$$P_e(QPSK) = \frac{1}{2(N_r-1)!}\int_0^{\infty}\sqrt{\frac{1}{2}\gamma_{mrc}}\times\frac{m^m}{\gamma_{av}^m\,\Gamma(m)}\,\gamma_{mrc}^{\,m-1}\exp\left(-\frac{m\gamma_{mrc}}{\gamma_{av}}\right). \qquad (12)$$

Similarly, for 8PSK, the probability of symbol error for Nakagami-*m* fading is derived to be (by substituting Eq. (9) and (11) into Eq. (4))

$$P_e(8PSK) = \frac{1}{2(N_r-1)!}\int_0^{\infty}\left(\sqrt{\frac{E}{N_0}}\sin\left(\frac{\pi}{8}\right)\right)\times\frac{m^m}{\gamma_{av}^m\,\Gamma(m)}\,\gamma_{mrc}^{\,m-1}\exp\left(-\frac{m\gamma_{mrc}}{\gamma_{av}}\right). \qquad (13)$$

To compensate the fading effect, the modulated signal is applied in orthogonal MISO (multiple-input single-output) system called Alamouti scheme. The Alamouti scheme is applied in the second part (relay to destination part) of our system.

It is to be mentioned here that the Alamouti code is a two-by-one orthogonal space-time block code (STBC). That is, it uses two transmit antennas and a single receive antenna. Space-time block code of Alamouti scheme of two antennas provides full diversity at full rate.

III. RESULTS

Figure 2 compares the performance of Alamouti scheme of single-hop, MRC of single-hop, combination of Alamouti and MRC scheme for two-hop links for both 8PSK and QPSK modulation schemes under Rayleigh fading condition. The probability of symbol error is found to be maximum for the two-hop 8PSK link (MRC and Alamouti) and Alamouti scheme of 8PSK (for single-hop) is very close to the previous one. The other scheme of QPSK modulation shows much better performance compared to 8PSK modulation scheme. MRC scheme of QPSK shows the best performance for the SNR of zero to 11.5dB but beyond that point, performance of Alamouti scheme is much better than MRC.

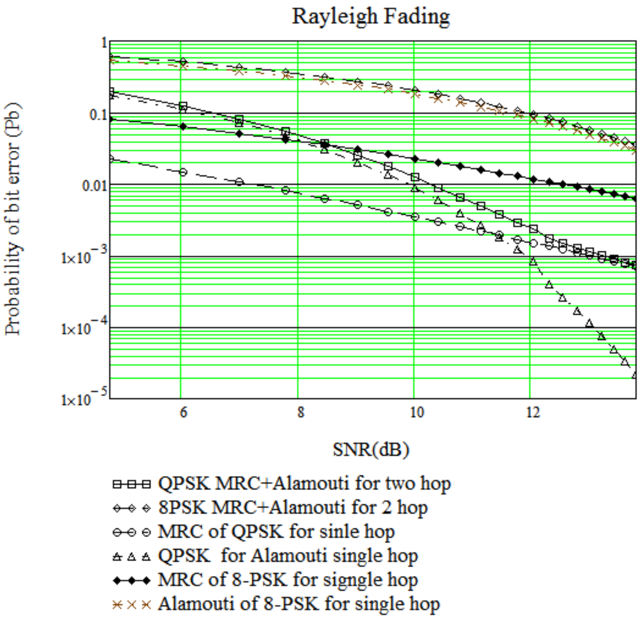

Fig. 2: Performance of single- and two-hop links of MRC and Alamouti schemes under Rayleigh fading.

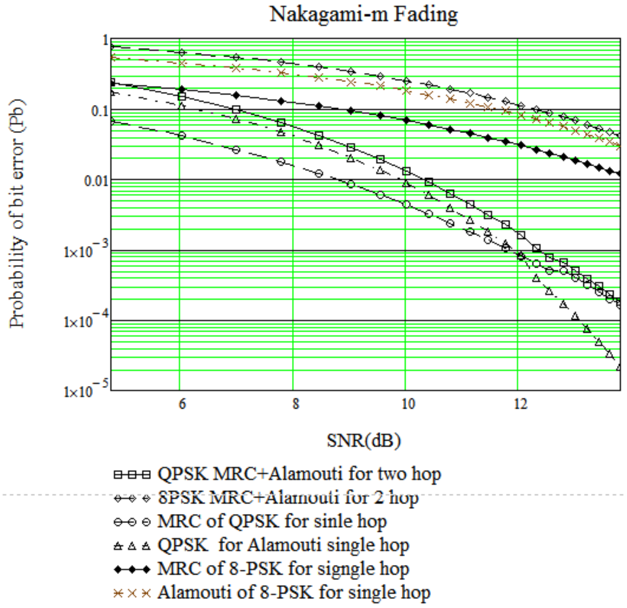

Fig. 3: Performance of single- and two-hop links of MRC and Alamouti schemes under Nakagami-*m* fading.

The performance of MRC of QPSK of single-hop merges with combination of MRC and Alamouti of the two-hop above 11 dB. The profiles of all the plots are found almost very close for Nakagami-*m* fading taking *m* = 7 as visualized from Fig. 3.

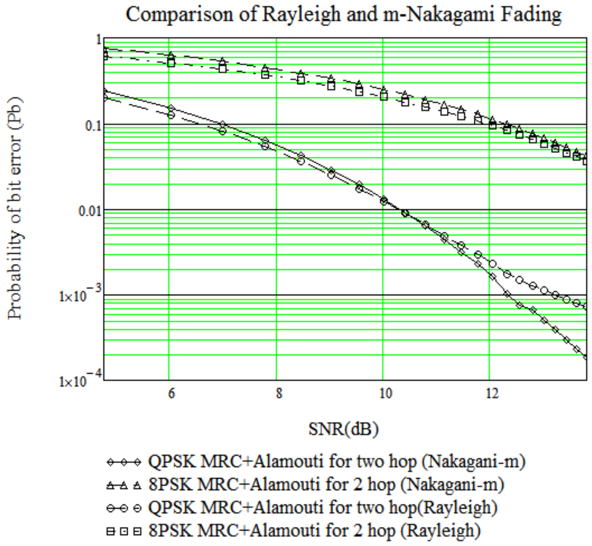

Fig. 4: Comparison of Rayleigh and Nakagami-*m* fading channels for the combination of MRC and Alamouti scheme.

The comparison of Rayleigh and Nakagami-*m* fading channels for two-hop case is shown in Fig. 4. Performances under both the fading cases are found to be much closer to each other. The behavior of the profiles of P_b in Fig. 2 can be explained from the signal space analysis of the constellation diagram. The signal space of the constellation point of QPSK scheme is very wide; therefore, the effect of multipath environment is less prominent on it. The incorporation of MRC and Alamouti schemes has combined the multipath signals better for the Rayleigh fading than that for the Nakagami-*m* fading at low SNR. It is important to note that situation would be different for the case of a single antenna link, where performance of Nakagami-*m* fading channel is better than that of Rayleigh fading channel at any SNR. When the SNR is above 11dB, the small scale fading of the channel becomes less prominent, and therefore the performance of the channel follows the basic conventional fading. Thus, the performance of Nakagami-*m* fading becomes better than that of Rayleigh fading case under MRC with the Alamouti scheme. For 8PSK case, the curves under both fading are almost parallel where performance of Rayleigh fading is found slightly better than the Nakagami case. The analytical results as discussed above are found by taking fading parameters of Alamouti model as $\alpha_1 = 0.051$ and $\alpha_2 = 0.26$. For the MRC case, only two antennas are considered and SNR of both the links are assumed to be very close to each other.

IV. CONCLUSION

In this paper, performance of a two-hop wireless link is evaluated under Rayleigh and Nakagami-*m* fading environments. It has been found that at higher SNR, the performance of the two-hop link is better under Nakagami-*m* fading case compared to that of Rayleigh fading for QPSK modulation scheme. But at lower SNR, performance under Rayleigh fading is slightly better. For 8PSK case, the relative impact of fading is found almost invariant to SNR. In this paper, two antenna space diversity scheme is used to obtain orthogonality of the STBC. The work can be extended for more than 2 antennas with applying quasi-orthogonal STBC [20]. Furthermore, the entire analysis can be repeated applying zero forcing adaptive equalizer at the repeater and the receiving terminals to improve the overall system performance. This work is under investigation and will be reported soon.

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