

Mathematical Model to Predict Linear Phase Condition Transport of E.Coli in Pheratic Aquifer at Elele in Rivers State of Nigeria

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ABSTRACT

The developments of ground water that will be free from pollution have a lot of specification that must be applied. quality ground water in the deltaic environment is a problem in most part of Niger delta, Elele in deltaic environment is one of the victim in ground water pollution due to it types of geological formation that deposit in that environment, its stratum micropoles is high and has a fracture deposition that causes lost of lost of circulation observed in drilling explorations. These include fast migration of microbes confirmed through high rate of water related diseases in the study location. mathematical model was developed considering this condition and were applied including column experimental analysis both result were compared which explain the level of concentration in distance and duration, it shows that the concentration produced from both result are determine from the influence of environmental factors and deposition of the geological formation, the model from the figure will be applied as one of the design criteria to solve ground water pollution in deltaic environment.

Keywords: *mathematical model, prediction of linear phase, E. coli Transport*

1. INTRODUCTION

Linear phase condition in pheratic aquifer in deltaic environment at Elele Rivers State was discovered through lots of fracture in the geological deposition in the study area, this problem is through the level of transport of microbes, most ground water abstraction on those area in most location were confirmed to contain high concentration of the microbes within a short duration, the abstraction process of ground water were discovered to experience fast migration of pollution, lost of circulation from drilling fluid with a large volume were one of the confirmation of linear condition, whereby drilling to optimum depth of ground water aquifer will always consume great volume of water through reverse circulatory system on rotary or manual method of drilling, this process discovered confirmed the linear process of transport of this microbes in the study area, this has resulted to increase in concentration of this microbes at Elele in Niger Delta of Rivers State. Base on this ugly condition, it became imperative to carry out this study for eradication of this source of pollution, to avoid increase of death rate from water related diseases in the study location, other causes of linear transport of e.coli are base on manmade activities or natural origin, meanwhile Groundwater is water that is found in the subsurface in cracks and pore spaces in soil, sand and rocks. These materials form what is called the groundwater reservoir or aquifer. Groundwater is found in two different zones. The first is the unsaturated zone, which lies immediately

below the land surface; in this zone the spaces between the rock and soil particles are filled with a mixture of air and water. The second zone is the saturated zone in which all the pore spaces is completely filled with water. The top of the saturated zone is called water table, which may be found at widely varying depths. The water table may rise due to heavy rains or melting snow, it also may fall during periods of dry weather and because of over pumping, which takes place in areas depending mainly on groundwater for agriculture or as drinking water, like the area south-west of the Nile Delta in Egypt. (Hany, 2005). Groundwater is recharged generally by rainwater or snow melt, which infiltrate into the soil where some of it is evaporated, some is absorbed by plant roots, and some seeps down into the saturated zone. During long periods without rain the unsaturated zone may remain dry. In the saturated zone water infiltrates through the interconnected pore spaces, moving downward by the force of gravity, and upward toward zones of lower pressure. Where the water table intersects the surface, such as at a surface stream, lake, or swamp, the groundwater returns to the surface. Groundwater can also be extracted through a well drilled into the aquifer. A well is essentially a pipe in the ground that is perforated in some depth interval (called the well screen and usually located in the bottom part) through which it fills with groundwater. This water then can be brought to thin some areas of the world (e.g. south west Nile Delta, Egypt) people face serious water shortages because groundwater is used faster than it is naturally replenished. (Kamann,2004, Marsily1986) In other areas groundwater is polluted by different

pollutants. In general the pollution of air, water, and land has an effect on the pollution and contamination of groundwater. For example, when the air is polluted, rainfall will settle many pollutants on the ground, which can then seep into and contaminate the groundwater resource (Standish, and Leyshon 1981). In addition, the overuse of fertilizers and pesticides for agricultural purposes, industrial and municipal solid waste, and leakages from tanks containing all kinds of liquids have also polluted groundwater. If groundwater becomes polluted (Koltermann, and Gorelick. 1995.), it will no longer be safe to drink. Cleaning up the contaminated aquifers is usually extremely difficult and expensive, or even unfeasible. . (Hany 2005).The movement and the amount of storage of groundwater depend on soil properties such as porosity and permeability. Porosity is defined as the ratio of the volume of pore space in a unit of material to the total volume of material; porosity is affected by the shape, size and the arrangement of the soil particles, e.g., the smaller particles could fill in the void spaces between the larger particles, which would result in a lower porosity. Permeability is a measure of a soil's or rock's ability to transmit water so that the size of pore space and interconnectivity of the spaces help determine permeability. Often the term hydraulic conductivity is used when discussing groundwater and aquifer properties. Hydraulic conductivity has the units of velocity (Conrad, 2006, Conrad,2007) Groundwater flow is driven by differences in pressure and elevation between two points in the aquifer, which are described in terms of the gradient of the so-called hydraulic head. The basic principle linking the specific discharge q (volume flow of water per cross sectional area and time) to the hydraulic head h and the hydraulic conductivity k is Darcy's law, which for one-dimensional flow may be written as: $dhqkdx=-$ (1.1). The specific discharge, as the hydraulic conductivity, has the dimensions of a velocity, and hence is also called the Darcy velocity. e surface by a pump. The scarcity of water will be even more critical in the future, especially in arid and semi arid regions, which are characterized by receiving less than 200 to 250 mm/yr of precipitation, which may occur irregularly (Ingraham et al., 1998, Hubbert,1940.). Groundwater is considered as the main source of fresh water in arid and semi arid regions except in areas where large rivers are found such as the Colorado River in the American southwest and the Nile River in northeast Africa. Because of the steadily increasing demand for high quality fresh water due to population growth and rising standard of living, groundwater is frequently over-utilized. This results in a significant lowering of the groundwater table, allowing encroachment of saline water from the sea or from the deeper aquifers. For sustainable utilization of groundwater, the pumping has to be limited to the rate of groundwater recharge and the water quality has to be preserved. . (Hany, 2005). Environmental tracers can be used to provide information about groundwater systems

and their dynamics, in order to cope with the above mentioned guidelines for sustainable management. E.g., stable isotopes (δD and $\delta^{18}O$) give information about the origin of groundwater and the effect of evaporation, anthropogenic tracers such as 3H (possibly combined with 3He), CFCs and SF_6 give information about groundwater ages, and noble gases can be used to determine the recharge conditions (especially temperature) in the past (Kamann,et al 2007 Koltermann, and Gorelick. 1995). Environmental tracers have been used in this work to study groundwater in the area south-west of the Nile Delta, Egypt and in the Mountains of Oman, both of them belonging to arid regions. The main goal is to determine the origin of recharge and the recharge rate, which will help to define management guidelines for a sustainable use of groundwater in such areas.

2. MATERIALS AND METHOD

Sample Collection The method of sample collection was insitu method of sample collection from a point source discharge into a drain at. Elele in Rivers State from Niger Delta Environment

COLUMN EXPERIMENTS Column experiments were performed to monitor the level of transport of *E. coli* at different deposit of soil formation.

Experiment Set up The column was set up; the height is 1 metre of 10mm diameter steel pipe, positioned at vertical level, including a funnel of 30cm that contains 4 litres of waste water. Each sample level of average of 2000mg/l of waste water containing *E. coli* was poured inside the column. While the flow was passing through the column, a stop watch was used to monitor the speed level, to determine the level of transport of each sample of aquifer materials. The effluent 1000mg/l from the column were collected and subjected to thorough analysis to determine the level of transport of *E. coli* in each of the aquifer material, which determines the level of transport to aquiferious zone

3. DEVELOPED MODELS FOR LINEAR PHASE CONDITION

C	=	Concentration	[ML ⁻³]
V	=	Velocity	[LT ⁻¹]
D _A	=	Dispersion coefficient dimension less	
T	=	Time	[T]
X	=	Distance	L

Linear Condition

$$C_{(x)} \frac{\partial v(x)}{\partial t} - \frac{V}{t} \left[C(x) - \frac{x \partial c(x)}{\partial t} \right] - KC(x) \frac{V}{t} (x) = \frac{V \partial c(x)}{\partial t} \quad (1)$$

If $\frac{\partial v}{\partial t} + \frac{V\partial c(x)}{\partial t}$ and (2)

$$C_{(x)} \frac{-\partial v(x)}{\partial t} = \beta \quad (3)$$

Such that

$$\frac{V\partial c(x)}{\partial t} + \frac{V}{t}C(x) \frac{-x\partial c(x)}{\partial x} - KC(x)\frac{V}{t}(x) = \beta \quad (4)$$

We have

$$\frac{V\partial c(x)}{\partial t} = \frac{-V}{t}C(x) \frac{-x\partial c(x)}{\partial x} - KC(x)\frac{V}{t}(x) - \beta \quad (5)$$

By transformation of eqn. (5) we have

$$C_{(x)} = T_x$$

It implies that $\frac{\partial c(x)}{\partial t} = T^1x$

Obtaining it from separation of variables we have

$$\frac{\partial c(x)}{\partial t} = T_x^1$$

Substituting in eqn. (5) we have

$$V(T^1x) = \frac{-V}{t}T_x^1 - T_x \frac{\partial v(x)}{\partial t} \quad (6)$$

Expanding on equation (5) we have

$$VT^1X \frac{-V}{t}T_x^1 - T_x \frac{\partial v(x)}{\partial t} \quad (7)$$

Dividing equation (7) by T_x we have

$$TX \frac{\partial v(x)}{\partial t} \frac{-V}{t} \frac{TX - XTX^1}{T_x} - KTX \frac{VC(x)}{TX} \frac{-VT^1X}{TX} \quad (8)$$

This implies that

$$\frac{\partial v(x)}{\partial t} \frac{-V}{t} \left[1 - \frac{X^1}{X} \right] \frac{-KV(x)}{t} \frac{-VT^1}{T} \quad (9)$$

But if $\frac{V\partial c(x)}{\partial t} = \lambda^2$

We have

$$\frac{\partial v(x)}{\partial t} \frac{-V}{t} \left[1 - \frac{X^1}{X} \right] \frac{-KV(x)}{t} \frac{-VT^1}{T} = \lambda^2 \quad (10)$$

If $\frac{V\partial c(x)}{\partial t} = \lambda^2$ (11)

Solving it term by term we have

$$SV_{(s)} - V_{(o)} = \lambda^2 \quad (12)$$

$$V_{(o)} = C_{y_1}$$

$$SV_{(s)} - C_{y_1} = \lambda^2 \quad (13)$$

$$V_{(s)} = \frac{\lambda^2 + C_{y_1}}{S} \quad (14)$$

Since Laplace inverse of $1/S = 1$ we obtain

$$V_t = \lambda^2 + C_{y_1}$$

$$\lambda^2 = \frac{V_t}{C_{y_1}}$$

$$T_{(t)} = VC_{y_1} \ell^{\frac{\lambda^2}{V}t} \quad (15)$$

$$\frac{V}{t} 1 - \frac{X^1}{X} \quad (16)$$

$$X_t = \frac{V}{t} C_{y_2} \ell^{\frac{\lambda^2}{V}t} \quad (17)$$

$$K \frac{V(x)}{t} = \lambda^2 \quad (18)$$

$$V_{(x)} = K \frac{V}{t} C_{y_3} \ell^{\frac{\lambda^2}{V}t} \quad (19)$$

$$\frac{VT^1}{T} = \lambda^2 \quad (20)$$

$$VT^1 = \lambda^2 T \quad (21)$$

Let $T_{(o)} = 0$

$$VT^1 - \lambda^2 T = 0 \quad (22)$$

$$V(ST_{(s)} - T_{(o)} - \lambda^2 T(s)) = 0 \quad (23)$$

Considering the boundary condition we have

$$T_{(o)} = C_{y_4}$$

Where C_{y_1} is the initial concentration

$$V(ST_{(s)} - C_{y_4}) - \lambda^2 T_{(s)} = 0 \quad (24)$$

$$VST_{(s)} - VC_{y_4} - \lambda^2 T_{(s)} = 0 \quad (25)$$

$$(VS - \lambda^2) T_{(s)} = VC_{y_4} \quad (26)$$

$$T_{(s)} = \frac{VC_{y_4}}{VS - \lambda^2} \quad (27)$$

$$VS - \lambda^2 = 0$$

$$VS = \lambda^2$$

$$S = \frac{\lambda^2}{V} \quad (28)$$

$$C_{(x)} = VC_{y_4} \ell^{\frac{\lambda^2 t}{V}} \quad (29)$$

Since we have

$$\frac{\partial v(x)}{\partial t} - \frac{V}{t} \left[1 - \frac{X^1}{X} \right] - \frac{KV(x)}{t} - \frac{VT^1}{T} = \lambda^2 \quad (30)$$

If we let $C_{(x)} = T_{(x)}$ we have

$$\frac{\partial v(x)}{\partial t} - \frac{V}{t} \left[1 - \frac{X^1}{X} \right] - \frac{KV(x)}{t} - \frac{VT^1}{T} \quad (31)$$

Integrating both sides gives

$$VC_{y_1} = VC_{y_4} \ell^{\frac{\lambda^2 t}{V}} = \frac{K}{V} C_{y_2} \ell^{\frac{\lambda^2 t}{V}} \quad (32)$$

Therefore

$$C_{(x)} = VC_{y_1} \ell^{\frac{\lambda^2 t}{V}} = \frac{KV}{V} C_{y_3} \ell^{\frac{\lambda^2 t}{V}} \quad (33)$$

$$\text{But if } \lambda^2 = \frac{Vt}{Cy_1}$$

$$C_{(x)} = VC_{y_1} \ell^{\frac{Vt}{Cy_1}} = \frac{V}{t} Cy_2 + K \frac{V}{t} Cy_3 + Cy_4 \ell^{\frac{Vt}{Cy_1}} \quad (34a)$$

Or

$$C_{(x)} = VC_{y_1} \ell^{\frac{Vt}{Cy_1}} = \frac{V}{t} Cy_2 + K \frac{V}{t} Cy_3 + Cy_4 \ell^{\frac{\lambda^2 t}{V}} \quad (34b)$$

$$C_{(x)} = T_x = T_x X_t$$

$$C_{(x)} = \left(VC_{y_1} \ell^{\frac{V^2 t}{V}} \right) \left(\frac{V}{t} Cy_2 \ell^{\frac{V^2 t}{V}} \right) \quad (35)$$

$$C_{(x)} = \left(VC_{y_1} \ell^{\frac{V^2 t}{V}} \right) \left(\frac{KV}{t} Cy_3 + Cy_4 \ell^{\frac{V^2 t}{V}} \right) \quad (36)$$

Given the constrain below

$$\text{Since } t = 0, X = 0, C_{(x)} = C_m$$

$$\text{We have } C_m = Cy_1 Cy_2 Cy_3$$

$$\text{Such that } Cy_2 = \frac{C_m}{Cy_3}$$

Integrating through we have

$$C_{(x)} = \left(\frac{VC_m}{Cy_3} \ell^{\frac{Vt^2}{Cy_4}} \right) \left(\frac{KV}{t} Cy_3 \ell^{\frac{Vt^2}{Cy_4}} \right) \quad (37)$$

$$C_{(x)} = \left(\frac{VC_m}{Cy_3} \ell^{\frac{\lambda^2 t}{V}} \right) \left(\frac{V}{t} \ell^{\frac{\lambda^2 t}{V}} \right) \quad (38)$$

$$C_{(x)} = \left(\frac{VC_m}{Cy_3} \ell^{\frac{\lambda^2 t}{V}} \right) \left(\frac{KV}{t} Cy_3 \ell^{\frac{\lambda^2 t}{V}} \right) \quad (39)$$

$$\therefore C_{(x)} = \left(\frac{VC_m}{Cy_3} \ell^{\frac{\lambda^2 t}{V}} \right) \left(\frac{KV}{t} \ell^{\frac{\lambda^2 t}{KV}} \right) \quad (40)$$

By indices it simplifies

$$C_{(x)} = V^4 \frac{V}{t} \frac{KV}{t} C_m \ell^{\left(\frac{\lambda^2 t}{V} + \frac{\lambda^2 t}{V} + \frac{\lambda^2 t}{KV} \right)} \quad (41)$$

But if $V = \frac{d}{t}$

$$C(x)_t = \frac{d^3}{t^3} - \frac{V}{t} - \frac{KV}{t} C_m \ell^{\left(\frac{\lambda^2 t}{d} + \frac{\lambda^2 t}{V} + \frac{\lambda^2 t}{KV} \right)} \quad (42a)$$

Or

$$C(x)_d = \frac{d^3}{t^3} \frac{V}{t} \frac{KV}{t} C_m \ell^{\left(\frac{\lambda^2 t}{d} + \frac{\lambda^2 t}{V} + \frac{\lambda^2 t}{KV} \right)} \quad (43b)$$

$$C(x) = \frac{d^3}{t^3} \frac{V}{t} \frac{KV}{t} C_m \ell^{\left(\frac{\lambda^2 t}{d} + \frac{\lambda^2 t}{V} + \frac{\lambda^2 t^2}{KV} \right)} \quad (44)$$

4. RESULT AND DISCUSSION

Comparison theoretical and experimental values of linear phase condition of e.coli are presented

Table 1 Concentration of E.coli at various Times

Time	Theoretical model Result Conc. m/(CVCLCD/a)
10	2.00E-06
20	1.78E-15
30	5.10E-15
40	3.43E-15
50	8.59E-12
60	2.60E-07
70	1.11E-15
80	1.77E-15
90	8.00E-15
100	8.33E-15

Table 2 Concentration of E.coli at various Distances

Distance	Theoretical model Result Conc. m/(CVCLCD/a)
3	2.00E-06
6	1.78E-15
9	5.10E-15
12	3.43E-15
15	8.59E-12
18	2.60E-07
21	1.11E-15
24	1.77E-15
27	8.00E-15
30	8.33E-15

Table 3 Comparison of theoretical model value with column Experimental Result versus time

Distance	Exp Result Conc. M/L	Theoretical model Result Conc. M/L(CVCLCD/a)
3	1.80E-06	2.00E-06
6	1.75E-08	1.78E-15
9	5.09E-08	5.10E-15
12	3.41E-08	3.43E-15
15	8.54E-08	8.59E-12
18	2.55E-08	2.60E-07
21	1.10E-07	1.11E-15
24	1.75E-09	1.77E-15
27	7.50E-08	8.00E-15
30	8.31E-09	8.33E-15

Table 4 Comparison of theoretical model value with column Experimental Result versus time

Time	Exp Result Conc. M/L	Theoretical model Result Conc. M/L(CVCLCD/a)
10	1.80E-06	2.00E-06
20	1.75E-08	1.78E-15
30	5.09E-08	5.10E-15
40	3.41E-08	3.43E-15
50	8.54E-08	8.59E-12
60	2.55E-08	2.60E-07
70	1.10E-07	1.11E-15
80	1.75E-09	1.77E-15
90	7.50E-08	8.00E-15
100	8.31E-09	8.33E-15

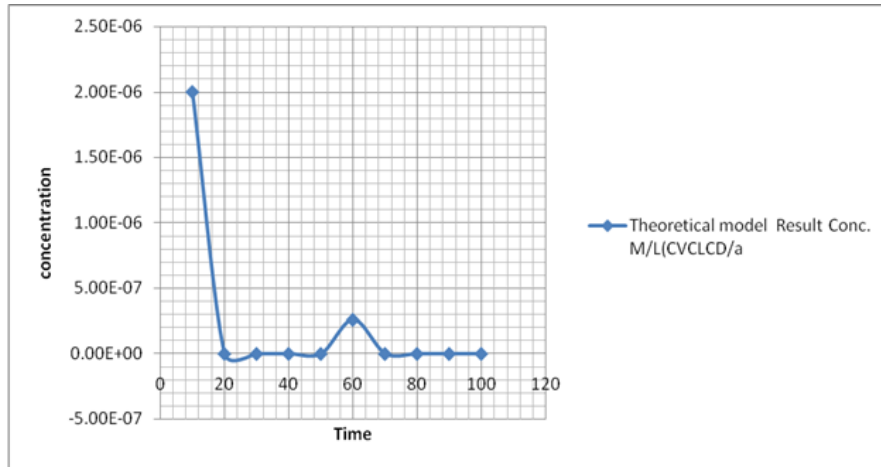


Figure 1 concentration of E.coli at various Times

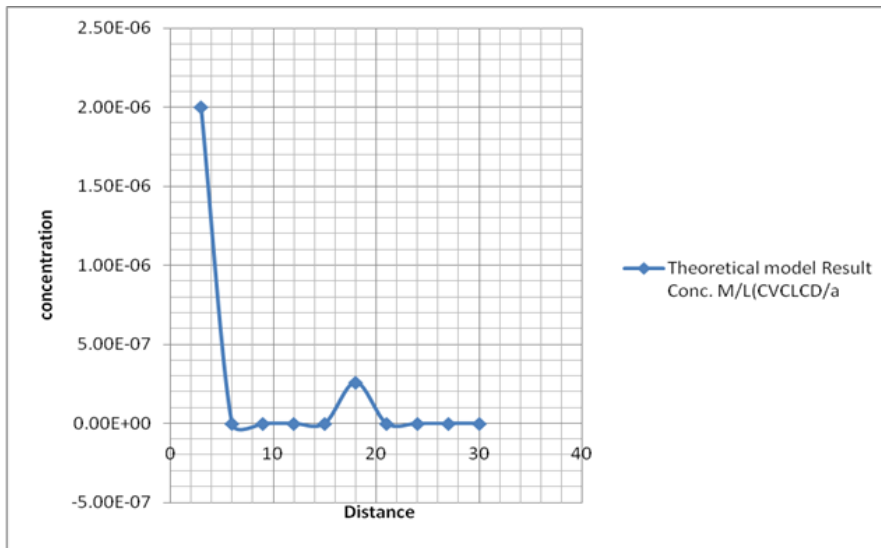


Figure 2 concentration of E.coli at various Distances

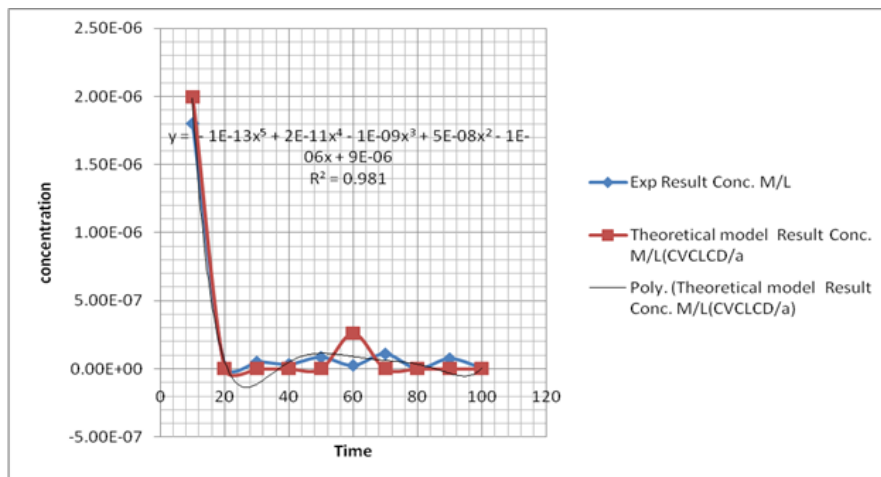


Figure 3 Comparison of theoretical model value with experimental result of column experiment linear phase condition versus Time

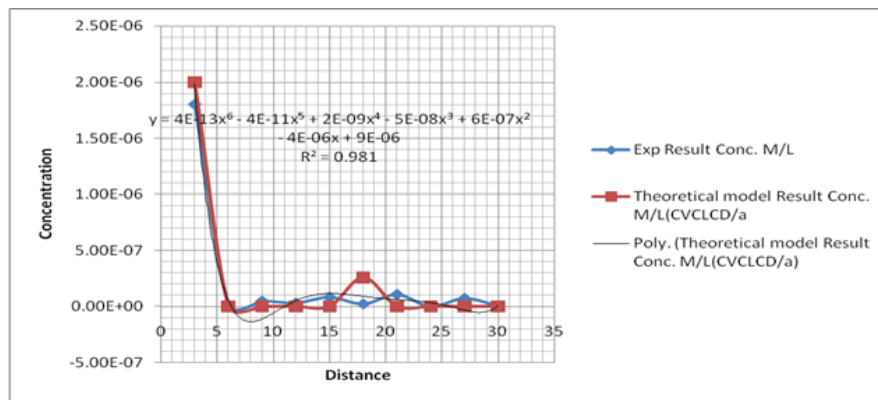


Figure 4 Comparison of theoretical model value with experimental result of column experiment linear phase condition versus Distance

The figure presented shows that the concentration increase with increase in a time at ten days where it obtained its optimum value, suddenly a decrease with increase in distance were observed in an vacillation form to hundred days, this figure explain these process with respect to linear phase condition, that in some instance concentration of the microbes may migrate from high level of concentration to low level of concentration with time, this implies that linear condition may have been influenced by the lithology of the soil, were the formation may have influence the degradation of the solute, these may also influence microbial degradation in some condition. figure 2 shows that the concentration increased distance to a point where an optimum value was observed at three metres, suddenly a decrease in concentration was observed in fluctuation form from nine metres to thirty metres, the figure presented explain that at linear phase the migration of the microbes are determined by the geological formation of the study area, the soil matrix and the deposition of the micropoles including the particle size distribution are part of the geological influence, this condition are deposited in Elele and that has resulted to fast linear transport of microbes generating more death trap to the settler in the study area. Figure 3 the concentration increases with time where an optimum value was recorded at ten days and it gradually decreased with increase in time (duration) down to hundred days, the same to the experimental result, it also recorded its optimum value at ten days with increase in concentration but the result was lower than that of the theoretical value and the experimental gradually decrease down to hundred days. The theoretical value fit in with column experimental result, this shows that the physical process transport of E.coli in linear phase condition, this condition can also be attributed to environmental factors It produces the best fit line equation of $Y = -1E-13x^5 + 2E-11x^4 - 1E-09x^3 + 5E-08x^2 - 1E-06x + 9E-06$ with Root of ($R^2 = 0.981$). figure 4 presented, shows that the concentration increased with distance, where an optimum

value was recorded at six metres and gradually decreased in a linear form down to thirty metres, while that of experimental result produced the same level of result, but the concentration were higher a little, the figure presented shows how the concentration are affected from other influence on the soil, the formation determined the level of concentration as for seen from the study, from the figure it also explain that despite the fast transport of microbes, the level of decrease in concentration are determine by geological formation deposited in the study location, Elele the study area has the type of deposition and the abstraction of ground water become difficult as most part of the area are confirmed through the volume water consumed during bore drilling were lost of circulation is experienced during drilling exploration, including high level of microbial transport. The figure produced the best fit line equation displayed on the graph.

5. CONCLUSION

Ground water pollution is a serious matter, because it involve human health, in every part of the world today, the issue of ground water pollution is of serious concern, this is because million of people are experiencing water scarcity including water related diseases, deltaic environment are experiencing serious death trap from water related diseases from every angle this ugly serge call for thorough research considering every dimensions of microbial transport, that include microbial behaviour, so that transport of microbes will be thoroughly monitored. The model of linear phase condition will be a benchmark to solve ground water pollution, on those area in the deltaic environment that are confirm to have such kind of geological deposition like the study area Elele,

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