



Modeling Tractive Force Requirements of Wheel Tractors for Disc Harrowing in Loamy Sand Soil

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ABSTRACT

In this research study, disc harrowing operations in a loamy sand soil, on an experimental plot of twenty different soil moisture levels at tillage speeds of 1.94m/s, 2.22m/s and 2.5m/s were conducted, using trace tractor techniques. The independent variables: drawbar pull force, rolling (motion) resistance, wheel slip, moisture content, cone index, wheel numeric, contact pressure, speed, width of harrow, depth of harrow, and dependent variable (Tractive force) were measured. Mathematical models using dimensional analysis, describing the tractor tyre-soil interaction were developed and validated. Regression analysis, was used to depict the relationships between independent variables and dependent variable. Analysis of variance using Randomized Complete Block Design in two way analysis was also used to study the effects and interactions of variables on tractive forces.

Validation results of the developed tractive force models conducted, revealed that harrowing operations recorded the highest coefficient of determination, $R^2 = 0.995$ at 2.5m/s tillage speed, while $R^2 = 0.990$ and 0.9 were obtained at tillage speeds of 1.94m/s and 2.22m/s respectively. Analysis of variance between measured and predicted tractive forces showed correlation coefficients, $R^2 = 0.9308, 0.8999, 0.9958$ and standard errors of 0.5844, 0.8628 and 0.78476 for harrowing at tillage speeds of 1.94m/s, 2.22m/s, and 2.5m/s respectively. The residuals analysis ranged from between -138.95 and $48.7117, -98.6106$ and $451.474, -33.3709$ and 32.5384 , and percentage (%) errors from -0.83458466 and $0.27430385, -0.396874637$ and 2.546385 and -0.191731686 and 0.189232 respectively. These indicate that there are no significant difference ($P \geq 0.05$) between the measured and predicted tractive forces, which are clear evidences of the test of goodness of fits of the models. Tillage speed of 2.5m/s illustrated the highest correlation coefficient of 0.9958 in this tillage operation. The tractive force models developed showed a good agreement between the measured and predicted results. It is therefore, recommended that the models be used for predicting tractive force in disc harrowing operations.

Keywords: *Tractive force, disc harrowing, loamy, sand, soil, modelling*

1. INTRODUCTION

In the early years of the twentieth century, agricultural engineers were concerned with efficiency in tractive force agricultural production especially in tillage operations. Tractive force in tillage operations has been a long term problem in farming activities. With advancement in improved agricultural production the use of tractor in tillage becomes unavoidable. Agricultural tractor specially for tillage operation is to perform drawbar pull work. Tillage is the manipulation or alteration of the soil to provide a condition favourable for crop growth and development (Adetayo, et al.,2004). Over the years, different kinds of tillage operations and implements have evolved in order to give the soil the best tilt for crop production. Tillage is an inevitable part of soil management and crop production. Harrowing is a secondary tillage operation intended to create a refined soil condition. This tillage operation is accomplished with

the aid of secondary tillage implement called disc harrow (Onwualu et al.,2006). Until recently, one of the challenges in design of agricultural field tractor, had been to equip it with a tractive device that can develop effective traction with minimum soil damage (Hassan and Broughton, 1975). Over time, the choice of tractive elements used for off-road vehicles to generate tractive force was mainly restricted to pneumatic tyres (Yu,2006). A tractor pulling a tillage implement through the soil must overcome the draught force which is generated by the traction between the tyres and the soil surface. Tractive force plays a vital role in tillage operations, therefore the need to determine the appropriate tractive force model becomes necessary in order to balance the variables in operations. However, establishing the required tractive force is a complex phenomenon as evident from numerous efforts made to develop appropriate model that characterized the operations. Different modeling tools such as empirical, analytical and numerical analysis have been used over the

years to model traction. However, there is still room for improvement. Hence, there is the need to improve on the efficiency of tillage operations to save time, money and increase productivity in mechanized agriculture. Researchers have shown that poor tractive performance in tillage operations is not cost effective. Optimizing the performance of agricultural tractors could, therefore help to minimize energy waste (Sefa and Kazim, 2004). Energy loss by the pneumatic tyre is therefore of concern to search for improved traction efficiency (Wong, 1993). This model determines the appropriate tractive force for tillage operations, which in turn gives the tractor operators proper condition for their fieldwork.

The dimensional analysis used provides advantages in reducing the number of variables which must be investigated and in formulating advantageous dimensionless variables (Murph, 1950, Langhaar, 1951, Kasprzak et al., 1990).

The objective of this study is to develop mathematical model for predicting the tractive force requirements of disc harrowing using dimensional analysis and to validate the models by comparison of the computed results from model with field results.

2. MATERIALS AND METHODS

2.1 Description

Experiments were conducted in the field (in-situ) using trace tractor technique. It was conducted at National Root Crops Research Institute (NRCRI) experimental farm, in Umudike, Umuahia, Abia State of Nigeria. Umudike is under the derived tropical humid ecological zone of Nigeria, and is 122m above sea level and lies on latitude $05^{\circ} 29'N$ and longitude $07^{\circ} 33'E$. The loamy sand soil where the experiment was conducted consists of clay-11.04% silt -4% and sand 84.96%.

2.2 Experimental Procedure

The instruments and implements used in measuring the tractive forces and other parameters are two tractors of the Massey Ferguson 435 model of 72 horse power, cone penetrometer, dynamometer, measuring tape, disc harrow, auger, stop watch and instrument for measuring weight of tractor (static hydraulic press). All field tests were conducted in a loamy sand soil for which the physical properties were determined. Before starting the field experiment, experimental layout area of 90m by 90m was designed with three different blocks of 90m x 27m each. Each block was divided into 9(nine) strips of 90m x 2m wide with a space of 3m between each strip. Disc harrowing operation was carried out on each of the blocks. Each day of the experiment were three replications of harrowing operations. This continued until the last day of minimal moisture levels. Hence, the total treatments were 9 x 20 days of the operations. Day 1, the

tillage operations started with harrowing on Block 1, Strip 1, Block 2, Strip 1 and Blocks 3, Strip 1. Day 2, harrowing on Block 1, Strip 2, Block 2 Strip 2, Block 3 Strip 2. Day 3, harrowing on Block 1 Strip 3, Block 2 Strip 3, Block 3, Strip 3. This pattern was followed for the remaining days up to the last day when minimum moisture content was achieved. Soil data were collected to the depths of 50mm, 150mm and 200mm respectively using soil auger, core sampler and a hand-operated soil cone penetrometer having an enclosed angle of 30° , with a base area of (323mm^2) mounted on a shaft of 2.03mm for the determination of moisture content, bulk density and soil resistance before tillage operation. In the sampling process, cone penetrometer was positioned between the operator's two legs and with his two hands on the handle pushed into the soil until the marked point on the shaft was reached, before readings were taken. The bulk density was measured using core sampler. Soil moisture content was determined using gravimetric method (oven dry method). The towing force and drawbar-pull forces were determined using trace- tractor technique. Two Massey Ferguson 435 model tractors of 72-horse power (hp) were used. The tractor carrying the implement with its engine disengaged (neutral gear) was coupled to another tractor which towed it with the dynamometer in between them. The first tractor pulled the second tractor coupled to the implement (disc harrow). The dynamometer reading was taken to determine the towing force. The drawbar-pull force was the difference between the towing force in neutral gear without implements in tillage operation and towing force when the implement was engaged in tillage operations. Depths and widths of harrowing were measured with rule (steel tape). The speed of operations was obtained by setting the tractor at a suitable gears of a gear reduction unit for targeted speed of 1.94m/s, 2.22m/s, 2.5m/s respectively. Simultaneously, the time taken to cover a fixed distance of 90m was recorded using a stopwatch to calculate the operating speed of the tractor and implement combination. Plate 1, shows the tractors –dynamometer – tractor-implement combination in action.



Plate 1.A photograph depicting tractor-dynamometer, implement mounted position during disc harrowing operations

3. MODEL DERIVATION

Dimensional analysis was used to develop the prediction models for traction forces of wheel tractor in tillage operation. Soil-wheel interaction was considered in developing the predictive models. Based on the Buckingham Pi Theorem (Murph, 1950, Langhaar, 1951, Kasprzak et al., 1990), the number of dimensionless and independent quantities (Pi terms) required to fully express the relationship between the variables were determined as follows:

$$S = n - b \tag{1}$$

where, S is the number of Pi terms; n is the total number of variables; and b is the number of basic dimensions. Basic dimensions are mass (M), length (L) and time (T)

The pertinent variables that affect the traction performance of the wheel tractor in loamy sand soil during tillage are eleven variables, (Table 1) with their corresponding dimensions. Wheel slip(s), moisture content (μ), and wheel numeric (Cn) are dimensionless parameters

Table 1: Variables affecting traction requirement

Variables	Symbol	Unit	Derivation	Dimension
Dependent variable				
Traction force	F	N	Mass x Acceleration $\frac{m}{s^2}$ Mass x $\frac{m}{s^2}$	MLT^{-2}
Independent variables				
Drawbar pull	F_p	N	Mass x Acceleration $\frac{m}{s^2}$ Mass x $\frac{m}{s^2}$	MLT^{-2}
Rolling resistance	F_R	N	Mass x Acceleration $\frac{m}{s^2}$ Mass x $\frac{m}{s^2}$	MLT^{-2}
Wheel slip	S	%	(No derivation)	$M^0L^0T^0$
Moisture content	μ	%	(No derivation)	$M^0L^0T^0$
Cone index	CI	N/m^2	$\frac{stress}{strain} \Rightarrow \frac{Force}{Area/Number} = \frac{MLT^{-2}}{L^2/1}$	$ML^{-1}T^{-2}$
Wheel numeric	C_n	-	$\left(\frac{Cibd}{W} \right)$	$M^0L^0T^0$
Contact pressure	P_c	kPa	$\frac{Force}{Area} = \frac{MLT^{-2}}{L^2}$ kN/m^2	$ML^{-1}T^{-2}$
Speed	V	m/s	$\frac{Length}{Time} = \frac{L}{T}$	LT^{-1}
Width of harrow	a	m	-	L
Depth of harrow	b	m	-	L

Thus, tractive force F, may be expressed as a function of the other seven variables, after the dimensionless parameters are being taken away.

$$F = f(F_p, F_R, CI, P_c, V, a, b) \tag{2}$$

where, f is functional notation for tractive force

This equation may be re-written as

$$f(F, F_p, F_R, CI, P_c, V, a, b) = 0 \tag{3}$$

Therefore, the number of variables are, n = 8, but since the basic dimensions are MLT , = 3. Then, number of dimensionless π - terms,

$$S = n - b, \quad S = 8 - 3 = 5$$

Hence, five ρ_i - terms say, $\pi_1, \pi_2, \pi_3, \pi_4$ and π_5 were generated.

Thus,

$$F = f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) \tag{4}$$

∴ The equation (4) may be written as

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5) = 0 \quad (5)$$

where, f_1 is functional notation for π -terms

From equation (5) above, five π -terms ($\pi_1, \pi_2, \pi_3, \pi_4$ and π_5) are required to develop a general prediction equation for tractive force of wheel tractor tillage operation (ploughing) on loamy sand soil.

In the dimensionless terms determination exercise, the three dimensionless terms; wheel slip, (s), moisture content, (μ), and wheel numeric, (Cn) are excluded, but has to be added when the other dimensionless terms had been formed. Also, the chosen repeating variables: cone index (CI), tractor speed (V) and width of plough (a) should not form dimensionless term (Barenblat, 1987).

The repeating variables are:

$$CI = ML^{-1}T^2, \quad V = LT^{-1}, \quad a = L$$

When π -terms are formed based on the principle of dimensional homogeneity, the dimensionless π -terms obtained are:

$$\pi_1 = \frac{F}{CIa^2}, \quad \pi_2 = \frac{F_p}{CIa^2}, \quad \pi_3 = \frac{F_R}{CIa^2}, \quad \pi_4 = \frac{P_c}{CI}, \quad \pi_5 = \frac{b}{a}$$

and already determined dimensionless π -terms are :

$$\pi_6 = \mu, \quad \pi_7 = S, \quad \pi_8 = Cn$$

A new set of Pi terms are generated by multiplying and/or dividing present Pi terms with each other. In addition, present Pi terms can be reversed to make new Pi terms, while maintaining independency condition. Some new Pi terms groups were obtained after transformations:

$$\begin{aligned} \pi_1^1 &= \frac{\pi_1}{\pi_2} x \frac{1}{\pi_3} = \frac{F}{CIa^2} x \frac{CIa^2}{F_p} = \frac{F}{F_p} x \frac{1}{\mu} = \frac{F}{F_p \mu} \\ \pi_1 &= \frac{F}{F_p \mu} \\ \pi_2^1 &= \frac{\pi_3}{\pi_4} = \frac{F_R}{CIa^2} x \frac{CI}{P_c} = \frac{F_R}{P_c a^2} \\ \pi_2 &= \pi_2^1 x \pi_5 x \pi_3 = \frac{F_R}{P_c a^2} x \frac{b}{a} x \mu = \frac{F_R b \mu}{P_c a^3} \end{aligned} \quad (6)$$

$$\pi_2 = \frac{F_R b \mu}{P_c a^3} \quad (7)$$

$$\pi_3 = \mu \quad (8)$$

Through this, some existing π - terms were eliminated, then dimensionless Pi terms obtained are as follows:

$$\pi_1 = \frac{F}{F_{p\mu}}, \quad \pi_2 = \frac{F_R b \mu}{P_c a^3}, \quad \pi_3 = \mu \quad (\text{harrowing})$$

The traction force required is contained only in π_1 - term. Thus, π_1 is related to the other two π -terms as follows:

$$\pi_1 = \theta(\pi_2, \pi_3) \quad (9)$$

Thus,

$$\frac{F}{F_{p\mu}} = \theta \left(\frac{F_R b \mu}{P_c a^3}, \mu \right) \quad (10)$$

for harrowing at tillage speeds of 1.94m/s, 2.22m/s, 2.5m/s. Substituting the dimensionless parameters (Pi terms) gives:

$$\pi_1 = \text{constant } \pi_2 \pm \text{constant}_3 \pi_3 \pm \text{constant}_4 \pm \text{constant } (C)$$

$$\frac{F}{F_{p\mu}} = C_2 \frac{F_R b \mu}{P_c a^3} \pm C_3 \mu \pm C_4 \quad \text{for harrowing} \quad (11)$$

The Least-square method was used to determine the constants of the polynomials, expressed as

$$Y = a + bx + cx^2 + dx^3 \quad (12)$$

Thus, the least-square formulation for the polynomials is:

$$\begin{bmatrix} n & \sum x_i & \sum x_i^2 & \sum x_i^3 \\ \sum x_i & \sum x_i^2 & \sum x_i^3 & \sum x_i^4 \\ \sum x_i^2 & \sum x_i^3 & \sum x_i^4 & \sum x_i^5 \\ \sum x_i^3 & \sum x_i^4 & \sum x_i^5 & \sum x_i^6 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \\ \sum x_i^2 y_i \\ \sum x_i^3 y_i \end{bmatrix}$$

The constants a, b, c, and d are determined using the EView software (statistical package). The input variables (independent variables) are π_2 and π_3 and contain drawbar-pull, constant pressure, depth of plough, wheel

numeric, wheel slip, cone index, width of plough, moisture content and rolling resistance.

$$F = C_1 + C_2 F_{p\mu} + C_3 F_{p\mu} \pi_2 + C_4 F_{p\mu} \pi_3 \quad (13)$$

Estimation Equation:

The developed models for ploughing with different tillage speeds after substituted coefficients are:

$$F = 12674.26 + 1.0154 F_{p\mu} + 0.00005 F_p \frac{F_R b \mu^2}{P_c} - 0.077580 F_{p\mu}^2 \quad \text{for harrowing at 1.94m/s} \quad (14a)$$

$$F = 13241.76 + 0.889 F_{p\mu} + 0.783 F_p \frac{F_R b \mu^2}{P_c} - 0.0000774 F_{p\mu}^2 \quad \text{for harrowing at 2.22 m/s} \quad (15b)$$

$$F = 12795.46 + 1.010067 F_{p\mu} + 0.18420 F_p \frac{F_R b \mu^2}{P_c} - 0.039298 F_{p\mu}^2 \quad (15c)$$

4. RESULTS AND DISCUSSION

The values of coefficients for constants a, b, c and d as required to determine the predicted equations of 14a, 14b and 14c are shown in Table 2.

The analysis in Table 3, shows the summaries of the residuals and percentage errors of predicted and measured tractive force values at tillage speeds of 1.94m/s, 2.22m/s and 2.5m/s. The residuals analysis depicted the difference

between measured experimental results and predicted models values. In harrowing operations the residuals analysis range from between - 3.10259 and 48.7117, - 7.3666 and 451.474, -8.88218 and 32.5384, percentage (%) errors from -0.0182237 and 0.27430385, -0.08605 and 2.546385 and -0.0709 and 0.189232 respectively. The general results of residual analyses and percentage (%) errors for harrowing operations at all the tillage speeds showed no significance differences.

Table 3: Shows the residuals and percentage errors for harrowing at various tillage speed

Harrowing 1.94m/s		Harrowing 2.22m/s		Harrowing 2.5m/s	
Residual	% error	Residual	% error	Residual	% error
-4.5741	-0.027315072	-98.6106	-0.572485341	17.0198	0.096320317
42.3949	0.25279845	-28.7996	-0.166279446	4.27077	0.024176451
-9.46654	-0.056601136	21.0527	0.121516306	29.6566	0.16783588
-138.95	-0.83458466	-26.4129	-0.152014066	7.45303	0.042059989
-15.0812	-0.090250923	-7.3666	-0.042653743	8.88218	-0.050423957
48.7117	0.274303847	3.26145	0.017804618	22.9219	0.122820018
-3.30725	-0.019383718	-41.8705	-0.243122169	-16.6161	-0.09458701
-3.10259	-0.01822373	-20.4423	-0.118781522	-11.946	-0.068048989
17.4344	0.101068986	-42.9474	-0.247345839	13.5658	0.076608313
19.1869	0.111876968	-41.0984	-0.237124394	2.54129	0.014380319
-10.4227	-0.061022834	-68.3815	-0.396874637	-30.1367	-0.171474822
-4.59125	-0.026881327	451.474	2.546384659	-19.4575	-0.110711238
1.58887	0.00953017	-42.052	-0.244034355	-18.2461	-0.103806679
-30.2986	-0.182796983	-11.029	-0.064365334	-32.7444	-0.187324943
28.7787	0.174575068	43.6316	0.255978879	25.1829	0.144812536
17.7812	0.109288261	10.6608	0.063268843	32.5384	0.189231753
39.6241	0.244744287	-25.8525	-0.154573991	9.26512	0.054277211
-25.5764	-0.155008485	-35.1396	-0.205976553	-33.3709	-0.191731686
26.1507	0.162679316	-14.3152	-0.086054704	19.4349	0.114457597
3.72013	0.022336415	-25.7629	-0.14965379	-12.4507	-0.070903759

Figures 1,2 and 3 depicted graphical comparison between the measured and predicted values of tractive force for

disc harrowing at tillage speeds of 1.94m/s ,2.22m/s and 2.5m/s respectively. These were plotted to find out if there

are acceptable agreements between the measured and predicted tractive force values. The results showed positives linear relationships between the predicted and measured values of tractive force. These confirmed good

agreement between measured and predicted values of tractive force with coefficients of determination $R^2=$ 0.990, 0.9 and 0.995 for the disc harrowing speeds of 1.94m/s, 2.22m/s and 2.5m/s respectively.

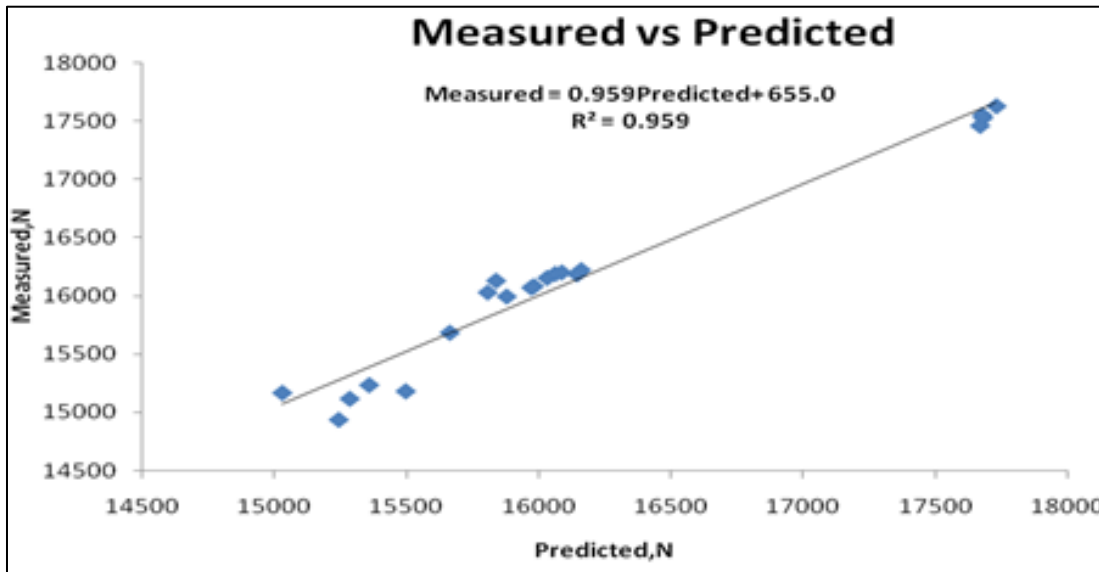


Figure 1: A plot of relationship between Measured and Predicted Tractive Force, at tillage speed of 1.94m/s, ploughing operation

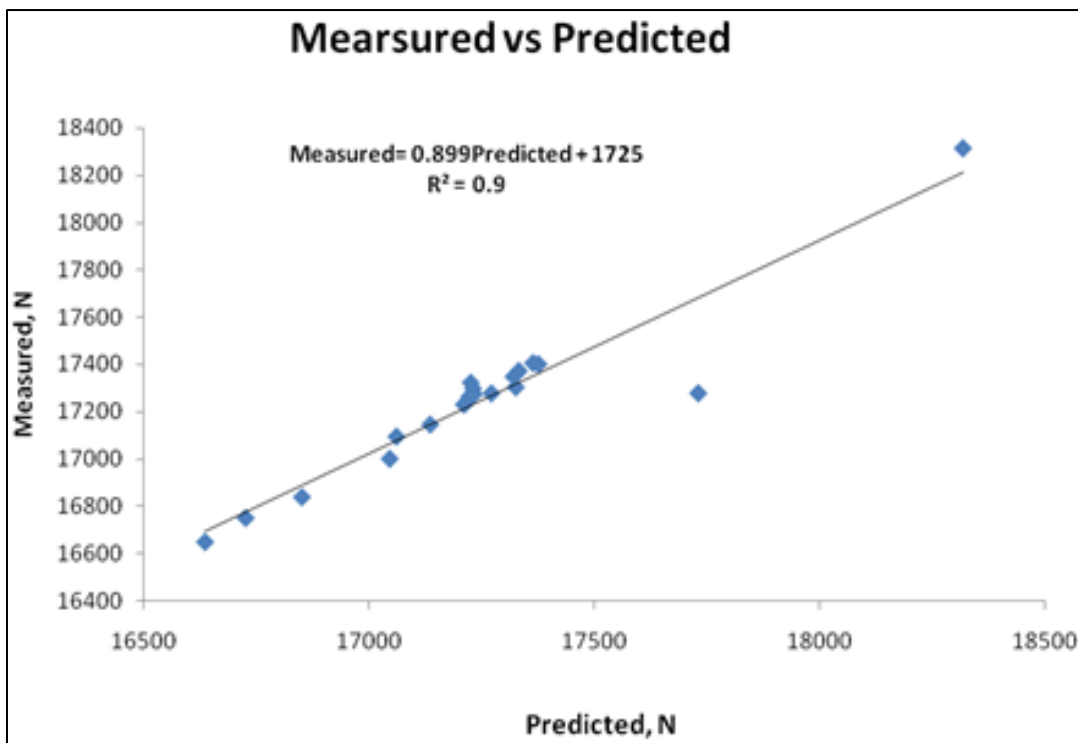


Figure 2 : A plot of relationship between Measure Predicted Tractive Force at tillage speed of 2.22m/s, harrowing operation

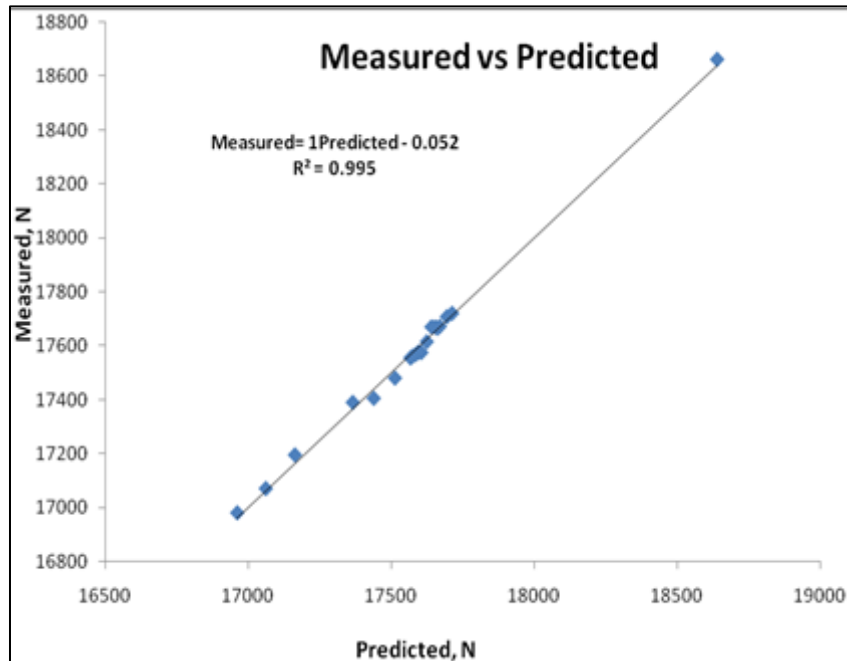


Figure 3: A plot of relationship between Measured and Predicted Tractive Force, at tillage speed of 2.5m/s, harrowing operation

Analysis of variance (ANOVA) was used to show significance differences between the predicted and measured tractive force values for disc harrowing operations at various tillage speeds (Tables 4, 5 and 6). It was obvious that there are no significance differences ($p \geq 0.05$) between the measured and predicted dependent variables (tractive forces). The correlation coefficients of $R^2 = 0.93089$, 0.89994 , 0.9958 for disc harrowing, at 1.94m/s , 2.22m/s and 2.5m/s respectively are clear evidence that variables have good relationship. These analyses revealed standard errors of 0.5844 ,

0.8628 , and 0.7847 for disc harrowing at 1.94m/s , 2.22m/s and 2.5m/s respectively. The values obtained are negligible and agree with the fact that the smaller the standard error values between measured and predicted results the better the models. These high correlations and no significance difference between measured and predicted data obtained, indicated that the developed models are good. These agree with the findings of Alder and Roessler, (1977) which states that the smaller the root mean square error (RMSE) values between measured and predicted results, the better the models.

Table 4: Analysis of variance between measured and predicted tractive force values at harrowing speed of 1.94m/s

Source	df	SS	MS	F	Significance F
Regression	1	2716592	2716592	-0.0580	1.6859 NS
Residual	18	201654.7	11203.04		
Total	19	2918246			

N.S: No significant different ($P \geq 0.05$) Regression Statistics: Multiple R 0.964831,

R Square 0.930899, Adjusted R Square 0.92706, Standard Error 0.58444, Observations 20

Table 5: Analysis of variance between measured and predicted tractive force values at harrowing speed of 2.22m/s

Source	df	SS	MS	F	Significance F
Regression	1	1883604	1883604	-9.3E-05	0.9999
Residual	18	209419.1	11634.39		
Total	19	2093023			

N.S: No significant different ($P \geq 0.05$)

Regression Statistics: Multiple R 0.948654, R Square 0.899944, Adjusted R Square 0.894386, Standard Error 0.8628, Observations 20

Table 6: Analysis of variance between measured and predicted tractive force values at Harrowing speed of 2.5m/s

	df	SS	MS	F	Significance F
Regress on	1	2066870	2066870	0.00028	0.4998 NS
Residual	18	8542.363	474.5757		
Total	19	2075412			

N.S: No significant different ($P \geq 0.05$) Regression Statistics: Multiple R 0.99794, R Square 0.995884, Adjusted Square 0.995655, Standard Error 0.78476, Observations 20

5. CONCLUSION

The study was conducted to investigate the tractive force efficiency of wheel tractors on agricultural soil during disc harrowing operations. To this effect experiments were designed to: develop mathematical models for wheel tractor tyre-soil interactions using dimensional analysis and the models were validated by comparison with experimental data from agricultural field tests and computational results. These equations produced sufficiently accurate results for predicting the tractive force requirements of disc harrowing in loamy sand soil at the reference tillage speeds. It was observed that the tillage speed of 2.5m/s illustrated the highest coefficient of determination $R^2=0.995$ in the operations. The residual analyses and percentage (%) errors at all the tillage speeds showed no significant difference ($P \geq 0.05$) and at minimum and maximum values were negligible values. The analysis of variance for measured and predicted tractive forces revealed standard error values of 0.3672552, 0.8628 and 0.8047 respectively. These are negligible and agree with the fact that the smaller the standard error values between measured and predicted results the better the models.

In conclusion, tractive force models developed showed good agreement between the predicted and experimental results, therefore are recommended for use to predict tractive force requirements for disc harrowing at chosen speeds and soil condition, with measurable accuracy.

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Table 2: Results of Least Square method on coefficients of multivariable regression equations for ploughing at different tillage speeds

Variables	Ploughing at 1.94m/s				Ploughing at 2.22m/s				Ploughing at 2.5m/s			
	Coefficient	Std. Errors	t-statistic	Prob.	Coefficient	Std. Errors	t-statistic	Prob.	Coefficient	Std. Errors	t-statistic	Prob.
C ₁	4489.715	1446.749	3.103313	0.0068	3322.100	1620.130	2.050514	0.0571	1151.419	1698.534	0.677890	0.5075
C ₂	0.001457	0.001358	1.073212	0.2991	2.309034	0.289724	7.969763	0.0000	2.433855	0.257868	9.438371	0.0000
C ₃	3.145007	3.799150	0.827819	0.4199	0.000543	0.000315	1.726404	0.1035	0.000297	0.000293	1.014968	0.3252
C ₄	2.334356	0.3160007	7.387050	0.0000	1.152594	2.759663	0.417657	0.6817	0.840530	0.711583	1.181212	0.2548
R ²		0.949056				0.955526				0.959391		

APPENDIX AH 1: Measured and Predicted tractive force values at various harrowing speeds

Ploughing at 1.94m/s		Ploughing at 2.2m/s		Ploughing at 2.5m/s	
Measured	Predicted	Measured	Predicted	Measured	Predicted
16745.7	16750.2	17225	17323.6	17670	17653
16770	16727.6	17320	17348.8	17665	17660.7
16725	16734.5	17325	17303.9	17670	17640.3
16649	16788	17375.3	17401.7	17720	17712.5
16710.3	16725.4	17270.7	17278	17615	17623.9
17758.3	17709.6	18318	18314.7	18663	18640.1
17062	17065.3	17222	17263.9	17567	17583.6
17025	17028.1	17210	17230.4	17555	17566.9
17250	17232.6	17363.3	17406.3	17708	17694.4
17150	17130.8	17332	17373.1	17672	17669.5
17080	17090.4	17230	17298.4	17575	17605.1
17079.7	17084.3	17730	17278.5	17575	17594.5
16672	16670.4	17232	17274.1	17577	17595.2
16575	16605.3	17135	17146	17480	17512.7
16485	16456.2	17045	17001.4	17390	17364.8
16270	16252.2	16850	16839.3	17195	17162.5
16190	16150.4	16725	16750.9	17070	17060.7
16500	16525.6	17060	17095.1	17405	17438.4
16075	16048.8	16635	16649.3	16980	16960.6
16655	16651.3	17215	17240.8	17560	17572.5