

UMTS Uplink Loading Probability Calculation Using Log-Normal Interferers Contributions

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ABSTRACT

In this paper we introduce the probabilistic notion of uplink loading in a UMTS network subject to uplink interferers from both the home cell and other neighbouring cells. Our study is based on the assumption that, for a given UMTS cell, all the interferers in the uplink are log-normally distributed and that the Gaussian noise is also taken into account. The latter is considered as a constant contribution to be added to the sum of all interferers that are summed up using Wilkinson's [1] calculation method. The uplink loading h is regarded as a random variable as it is directly dependent upon uplink log-normal interferers. In this paper we propose a new way of assessing this uplink loading and for a given User Equipments spatial distribution snapshot. Subsequently the probability that the uplink loading exceeds a given threshold, say h_0 , is thus derived and for different correlation figures and different noise levels as well.

Keywords: *Uplink Loading, Interference, Gaussian Noise, Log-Normal distribution.*

1. INTRODUCTION

The uplink interference loading in UMTS is defined as [2] the ratio of the sum of home cell plus other cell interference to the same amount of interference added to the Gaussian noise. This uplink loading is a critical parameter to the uplink capacity from one hand and to the uplink coverage from the other hand. The latter affects the dynamic sensitivity of the Node B and therefore leads to the well-known phenomenon of cell breathing that is characteristic to CDMA system in general. The maximum allowable path loss is markedly reduced and this leads to an obvious loss of uplink coverage footprint, creating a link unbalance with downlink and users see their communications dropped especially in the vicinity of the uplink cell border. Each UMTS signal can be assumed to be log-normally distributed when the received power is expressed in linear units (i.e. W or mW). However, when the power is expressed in dBm or dBW the distribution is simply a Gaussian one. It is important to outline that this loading changes during the day as the traffic is dynamically changing as well. In Busy hours, for example, the uplink loading can reach dramatic proportions leading to loss of uplink capacity and cell shrinking. New users cannot be serviced and this issue is penalizing to the mobileoperators. The latter are very much interested to control this uplink loading and there are several techniques to reduce the amount of interference, such as the use of smart antennas [4] or merely the use of advanced RRM algorithms to control the admission to the network [5].

2. PROBABILITY CALCULATION

Our starting point is the uplink loading expression [2] that is given by the following:

$$\eta = \frac{x+y}{x+y+n} \quad (1)$$

Where h is the uplink loading, x and y are respectively the uplink home cell and other cell interferences. Whereas n is the wideband noise at the Node B of the cell under study.

According to numerous investigations [6][7][8][9] x and y are log normally distributed and n is considered to be a constant value that we will detail its expression later on in this section.

In fact each interferer from the home cell is summed up and the resulting signal has a lognormal distribution. The same concept applies to the interferers from other cells. According to this statement one may write the following expressions for x and y :

$$x = 10^{\frac{x}{10}} = e^{\lambda X} \quad (2a)$$

and

$$y = 10^{\frac{y}{10}} = e^{\lambda Y} \quad (2b)$$

With $l=0.230$, X and Y are in dB (i.e. dBm or dBW)

The noise term n in eqn. (1) is given by

$$n = 10^{\frac{N}{10}} \quad (3)$$

N is the dBm value that is given by:

$$N = -174 \text{ dBm/Hz} + 10 \times \log_{10}(W) + NF \quad (4)$$

With W , the UMTS Bandwidth in Hz (i.e. 3.84×10^6 Hz) and NF is the Node B receiver noise figure in dB.

When we take a glance to eqn. (1) we notice that $x+y$ is also log-normally distributed and one may define a new random variable, say z , to account for $x+y$. As a result, eqn. (1) can be written in the following form:

$$\eta = \frac{z}{z+n} = \frac{e^{\lambda Z}}{e^{\lambda Z} + n} \quad (5)$$

At this point we have combined the home cell and the other cells lognormal interferers to have a unique lognormal variable that is z and whose dB value is Z .

The probability P that the uplink loading exceeds a given threshold is thus given by:

$$P\{\eta > \eta_0\} = P\left\{Z > \frac{1}{\lambda} \cdot \ln\left(n \cdot \frac{\eta_0}{1-\eta_0}\right)\right\} \quad (6)$$

Subsequently it is straightforward to derive the probability using the fact that Z is a Normal distribution whose statistical parameters are known. The mean and variance of Z are computed using one of the above mentioned summation techniques. For high accuracy purposes in the UMTS band we have chosen the Wilkinson's algorithm [1] that we supply in the annex of this paper. We define the dB resulting threshold L as:

$$\Lambda(\eta_0) = \frac{1}{\lambda} \cdot \ln\left(n \cdot \frac{\eta_0}{1-\eta_0}\right) \quad (7)$$

We find

$$P\{\eta > \eta_0\} = \frac{1}{2} \left(1 - \text{erf}\left[\frac{\Lambda(\eta_0) - m_z}{\sigma_z \sqrt{2}}\right]\right) \quad (8)$$

Where m_z and σ_z^2 are respectively the resulting uplink total interference mean and variance. These two parameters are computed by means Wilkinson's algorithm.

It is important to note that each uplink signal has its own mean and variance that is combined using Wilkinson's calculation method. Besides, these mean values are estimated by means of a propagation model. In our case we have made use of COST231-Hata [10] extended to UMTS band. The standard deviation of each signal, in a suburban-type environment, is about 7.5 dB. We also assume that all the signal have the same correlation coefficient r that we have used in our study. The noise n is

a constant value that is given by eqn. (3) combined with eqn. (4).

3. NUMERICAL RESULTS AND DISCUSSIONS

For our calculations we have chose the city of Taif and precisely near the University campus where we have chosen two UMTS sites and 15 User Equipments that are serviced by these two Node B's. The figure below illustrates the Node B's and the UE's.

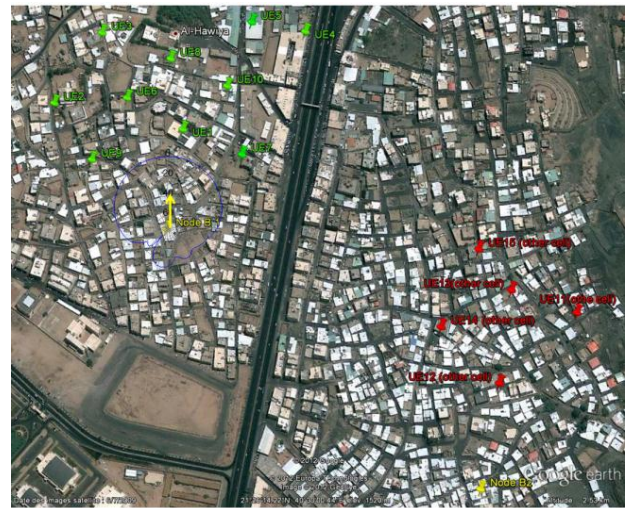


Figure 1: Illustration of the uplink load of Node B1(home cell) and the neighbouring Node B 2 whose UE's are the other cell interferers of Node B 1. The antenna pattern is marked in blue on Node B1.

The home cell interferes are marked in green colour, whereas the other cell interferers are in red colour in figure 1. The antenna that is used for the Node B1 is a commercial antenna of Kathrein manufacturer. The whole RF parameters are detailed below.

3.1 Input Parameters

For the sake of simplicity all the UE's are serviced with the same data throughput of speech 8 kbps. The traffic mix could also be studied but it is out of scope of this work.

Table 1. Node B RF Parameters and Antenna type

Antenna Height	Antenna Type	Horizontal Antenna Beamwidth	Antenna Electrical Tilt
25 m	Kathrein	65°	2°

The horizontal and vertical antenna patterns are given by the figure below

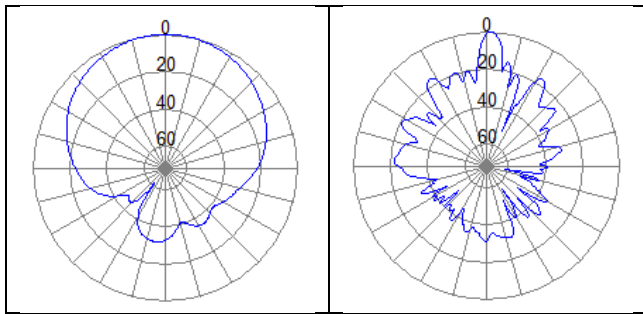


Figure 2: UMTS Kathrein Antenna Patterns

The calculation of m_z and s_z requires the knowledge of each individual mean and standard deviation of each uplink signal. In our case we need to compute 15 values. As mentioned above we have used COST231- Hata Model that is given in the Annex. To compute the probability in eqn. (8) we need to first calculate m_z and s_z . To do so we have to consider each uplink signal from each UE towards the Node B 1.

Table II. Interferers coordinates and their distances to Node B 1

UE's	Longitude	Latitude	Distance to Node B
UE1	40°29'51.34"E	21°26'21.68"N	204 m
UE2	40°29'42.88"E	21°26'23.29"N	310 m
UE3	40°29'46.04"E	21°26'27.60"N	396 m
UE4	40°29'59.45"E	21°26'27.71"N	448 m
UE5	40°29'55.89"E	21°26'28.31"N	427 m
UE6	40°29'47.70"E	21°26'23.59"N	252 m
UE7	40°29'55.24"E	21°26'20.12"N	223 m
UE8	40°29'50.56"E	21°26'26.00"N	326 m
UE9	40°29'45.32"E	21°26'19.82"N	199 m
UE10	40°29'54.27"E	21°26'24.35"N	315 m
UE11	40°30'17.09"E	21°26'10.38"N	788 m
UE12	40°30'12.04"E	21°26'6.11"N	689 m
UE13	40°30'12.83"E	21°26'11.77"N	664 m
UE14	40°30'8.23"E	21°26'9.55"N	554 m
UE15	40°30'10.69"E	21°26'14.24"N	588 m

With these data and using the propagation model we obtain the following table for each uplink signal as perceived by Node B1. Node B and UE antenna heights are 25 and 1.5 m respectively.

The Wilkinson algorithm provides the following figures of m_z and s_z with different correlation coefficients.

It is important to note that figure 1 represents the network snapshot at a given time. We could imagine many cells interfering with each other and apply the same concept of our computation method. The following figures show the obtained results for three values of the noise figure NF of the Node B 1 receiver. The correlation coefficient is set to $r=0, 0.5$ and 0.9 .

Table II. Uplink received signals at Node B 1 premises (mean values)

UE's	m_i (dBm)
UE1	-94.27
UE2	-105.77
UE3	-105.27
UE4	-110.29
UE5	-107.14
UE6	-98.00
UE7	-101.95
UE8	-101.25
UE9	-100.14
UE10	-102.57
UE11	-144.00
UE12	-147.20
UE13	-140.30
UE14	-142.00
UE15	-135.41

Table III. Uplink resulting mean and standard deviation

	$r=0$	$r=0.5$	$r=0.9$
m_z (dBm)	-87.40	-87.90	-89.00
σ_z (dB)	5.47	5.87	6.61

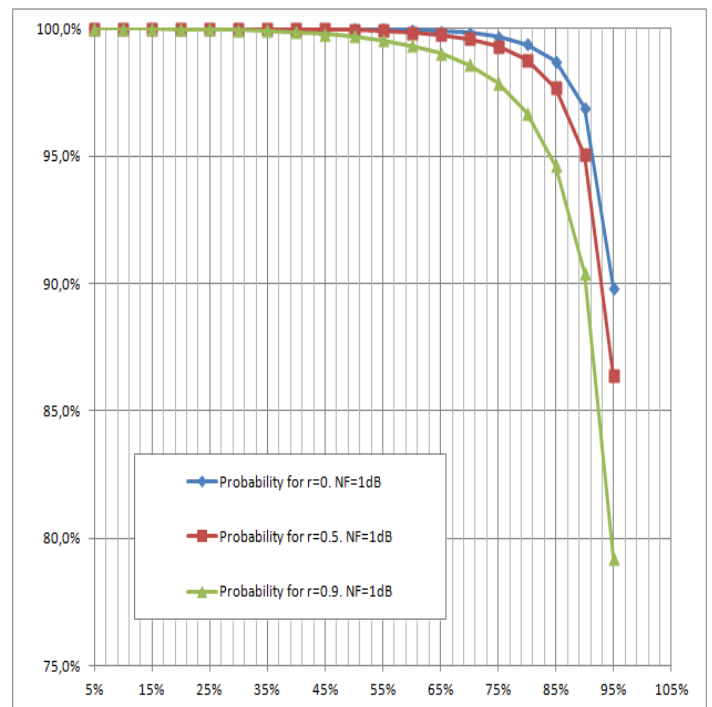


Figure 3: Probability versus the loading threshold for NF = 1 dB

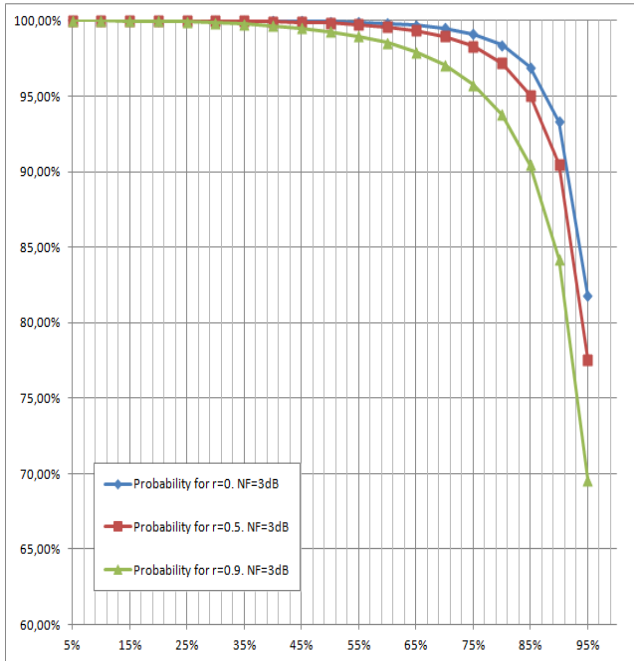


Figure 4: Probability versus the loading threshold for NF = 3 dB

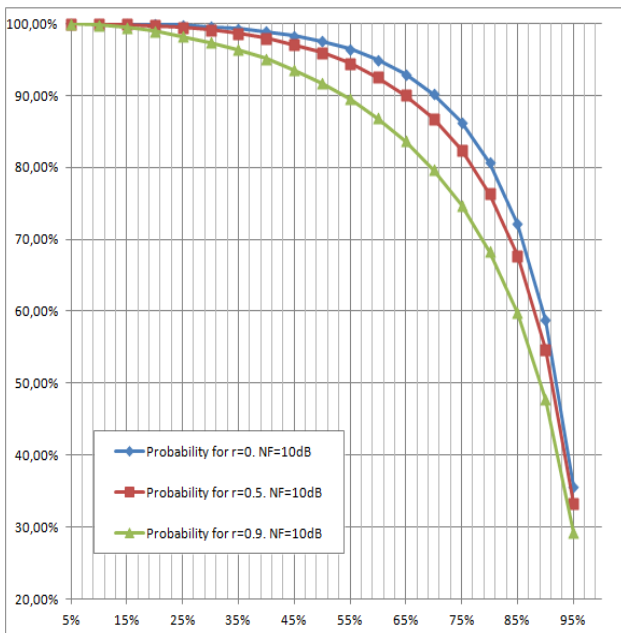


Figure 5: Probability versus the loading threshold for NF = 10 dB

The first thing we may notice is that when the interferers are highly correlated (i.e. for $r=0.9$) the probability that the total uplink loading does exceed a given threshold is the largest figure we can obtain (uppermost curve). Regarding the noise contribution, it is clear that the lower the noise figure the better is for the probability. The threshold h_0 we have used should not reach 100% otherwise a floating point error is returned according to

eqn. (7). Generally most of the operators assume that the maximum uplink loading does not exceed 75%, whereas the initial UMTS network planning assumes a default value of 50%. Therefore the used maximum value of 95% is an extreme figure approaching the figure for the pole capacity.

Furthermore, we noticed that when the noise level $N(\text{dBm})$ is below m_z the probability falls at about 55% loading and beyond, whereas when the noise level is higher ($NF = 10 \text{ dB}$) than m_z this probability starts falling much earlier at about 25% loading threshold. This clearly means that the choice of the receiver characteristics of the node B is crucial. Nowadays technology evolution has led the manufacturers to develop node B receivers with a maximum of 2.5 dB noise figure. In addition, the network operators should limit this uplink interference by using modern techniques such as smart antennas and highly directional antennas to reduce the other cell interference such as 6-sector sites for example.

4. CONCLUSIONS

In this investigation we have derived the probability of the uplink loading using a stochastic approach based upon the combination of log-normally distributed interferers. The calculations are based on a single network snapshot and by means of a semi-empirical propagation model the mean values for the uplink signals have been estimated. It is therefore obvious that the accuracy of the probability calculation is highly dependent upon the accuracy of the propagation model. COST231-Hata model is known to have limited accuracy as the standard deviation of the error (difference between prediction and measurement) is about 5-6 dB. To assess more accurately this probability it would be more suitable to use ray tracing based propagation model (GTD/UTD) [11] with a high resolution Digital Terrain Model. Besides this our work is based on only one snapshot of the network and it would be more representative to use Monte Carlo random positions for the UE's and re-calculate this probability for several snapshots. An average probability for a complete set of snapshots with several mobiles (100 and more) could thus be derived. Finally the algorithms we have used along with the probability calculation method we have proposed could also be integrated in a UMTS planning and optimization tool to assess accurately the loading phenomenon from the stochastic standpoint. Further investigations could also consider other summation methods such as Schwartz & Yeh [8] or Chia-Lu Ho [9].

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ANNEX

A. Wilkinson Algorithm

In summary the values for m_z and σ_z are given by:

$$m_z = 2 \ln(u_1) - \frac{1}{2} \cdot \ln(u_2) \tag{A.1}$$

$$\sigma_z = \sqrt{\ln(u_2) - 2 \cdot \ln(u_1)} \tag{A.2}$$

With u_1 and u_2 are given by, for t signals, the formulas are as follows:

$$u_1 = \sum_{i=1}^t e^{m_{y_i} + \frac{\sigma_{y_i}^2}{2}} \tag{A.3}$$

$$u_2 = \sum_{i=1}^t e^{2m_{y_i} + 2\sigma_{y_i}^2} + 2 \sum_{i=1}^{t-1} \sum_{j=i+1}^t \left\{ e^{m_{y_i} + m_{y_j}} e^{\frac{1}{2}(\sigma_{y_i}^2 + \sigma_{y_j}^2 + 2r \cdot \sigma_{y_i} \sigma_{y_j})} \right\} \tag{A.4}$$

The correlation r is present in the calculation of u_2 according to eqn. A.4

m_{y_i} are the mean values of the uplink signals (interferers). These are calculated by means of COST231-Hata model extended to UMTS band (2100 MHz).

B. COST231-Hata Model

For Taif city we have assumed the environment to be a “small city” and the model used is given by the following equations:

$$PathLoss(dB) = A + a(H_m) + B \log_{10}(d_{km}) \tag{B.1}$$

With A, B and a(H_m) are

$$A = 46.3 + 33.9 \log_{10}(f_{MHz}) - 13.82 \log_{10}(H_b) \tag{B.2}$$

$$B = 44.9 - 6.55 \log_{10}(H_b) \tag{B.3}$$

$$a(H_m) = [1.1 \log_{10}(f_{MHz}) - 0.7] H_m - [1.56 \log_{10}(f_{MHz}) - 0.8] \tag{B.4}$$

With $H_b = 25m$ and $H_m = 1.5m$, and a frequency of 2100 MHz we obtain:

$$A = 139.6 \text{ dB}$$

$$B = 35.74 \text{ dB/m}$$

$$a(H_m) = 0.05 \text{ dB}$$

The uplink received power by Node B 1 is given by:

$$Pr(dBm) = EiRP_{max} - AntennaPatternLoss(q) - PathLoss(dB)$$

The $EiRP_{max}$ in UMTS for Speech is 21 dBm and the antenna gain of the UE is assumed to be 0 dBi. The

Antenna Pattern Loss(q) for figure 1 is given by the following table:

UE Interferers	Antenna Pattern Loss (dB)
UE1	0.30
UE2	5.30
UE3	1.00
UE4	4.10
UE5	1.70
UE6	0.75

UE7	6.60
UE8	0.00
UE9	6.55
UE10	1.85
UE11	29.10
UE12	34.30
UE13	28.00
UE14	32.50
UE15	25.00