IJET EXPLORE International Journal of Engineering & Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-3444

International Journal of Engineering and Technology Volume 2 No. 10, October, 2012

Roos-Extension of Default Logic in Artificial Intelligence

Fu Li
Department of Mathematics,
Qinghai Nationalities University, Xining 810007

ABSTRACT

One important defect of Reiter's default logic is inability to reason by cases. To overcome the defect definition of Reiter's extension. Roos presents a modified definition of a default extension that solves the problem. In this paper, we will discuss the properties of Roos-extension that similar to Reiter-extension, and will find some properties of Reiter-extension cannot still correct to Roos-extension. We point out the difference of them, and Roos-extension offers a new idea for inference of artificial intelligent, it can achieve a method to classify the information.

Keywords: *Reiter-extensions; Roos-extensions; semi-monotonicity; consequents, algorithm.*

1. INTRODUCTION

Reiter's default logic[1] is one of the most popular formalism for describing non-monotonic reasoning in Artificial Intelligence. One important defect of Reiter's default logic is, however, inability to reason by cases. A solution to this problem should enable reasoning by cases but may not result in introducing a contraposition for some of default rules. Several proposals have been made to extend default logic with reasoning by case [2-4]. As Moinard shows [3] these approaches all introduce in one way or another some forms of a contraposition. Moinard analyzes the problems and presents a modified definition of a default extension that solves the problem. He also shows that a simple transformation of the default rules, make it possible to realize reasoning by cases using Reiter's original definition of an extension. Independently, Voorbraak [5] has proposed a similar transformation, however, one aspect that has been ignored by all solutions presented so far, are the consequences for the default rules that are applicable if reasoning by cases is not by cases in clausal default logic. To overcome the defect, Roos [6] modified the definition of Reiter-extension, and discussed the relationship between Reiter-extensions and Roos-extensions. Fu [7] has investigated and discussed how to compute Roos-extensions of a default theory by means of clausal default theory, and attained some similar results. Xu [8] discussed on Reasoning by cases in causal default logic, they presented a kind of tree method to investigate the reasoning by cases in default logic and algorithms to compute the smallest set of literals from a set of clause, and applied this algorithms to computation of Roos-extensions, and presented some similar results to Reiter-extension.

In this paper, we will discuss the properties of Roos-extensions that are similar to Reiter-extension. We will find some properties of Reiter-extension can't still correct to Roos-extension ,and we give the algorithm of Roos-extension, point Roos-extension offers a new idea for inference of artificial intelligent, it can achieve a method to classify the information.

2. PRELIMINARIES

L is the set of wffs on the first language .

Definition 1.1[1] (Reiter-extension) Let Δ = (D, W) be a default theory. For any set of formulae S , let Γ(S) be a smallest set satisfying the following three conditions:

(i) W$\subseteq$ E,

(ii) E is deductively closed, i.e. Th(E)=E

(iii) If $\delta = \dfrac{\alpha : \beta_{1,\ldots}\beta_n}{\gamma} \in$ D, $\alpha \in$ E, $\neg\beta_1 \notin$ E, …, $\neg\beta_n \notin$ E,then $\gamma \in$ E .

E is called the extension of Δ ,if Γ (E)=E, then E is fixed point of.

Theorem 1.2[6,12] Let E be a set of formulas, and let Δ = (D,W) be a default theory. Define E_0 = W, and for i $\geq$ 0,

E_{i+1} =Th(E_i)$\cup$ {β| $\dfrac{\alpha : \beta_{1,\ldots}\beta_n}{\gamma}$ $\in$ D, α$\in$ E_i , $\neg\beta_1 \notin$ E, …, $\neg\beta_n \notin$ E }, then E is a Reiter-extension if and only if E= $\bigcup_{i=0}^{n} E_i$.

Definition 1.3[6] (Roos-extension) Let Δ=(D,W) be a default theory. For any set of formulas S, let Γ(S) = { T_1 ,…, T_n },T∈Γ(S) if and only if T is the smallest set satisfying the following three conditions:

(i) W⊆T;

(ii) T is equal to the deductive closure of the set of literals that T contains E , i.e.α∈T if and only if there exist a subset of literals T', T'⊆ T, such that T'├α;

(iii) if $\frac{\alpha:\beta_{1,\dots}\beta_n}{\gamma}$ ∈D,α∈Γ(S), ¬ β_1 ∉ E, …, ¬ β_n ∉ S, then γ∈Γ(S).

A set of formulas is a Roos-extension of theory = (D,W) if and only if E∈ Γ(E).

Notice that the difference of the two kinds of definitions about extension is the conditions (ii) and (ii).

Definition 1.4[6,12] $\delta = \frac{\alpha:\beta_{1,\dots}\beta_n}{\gamma}$ ∈D is applicable to a deductively closed set of formulae E iff α∈ E ,and $\exists\beta_i$ ∈ Just(δ),such that $\neg\beta_i$ ∈ E(i=1,…,n).

Definition 1.5 The application of a default rule $\delta = \frac{\alpha:\beta_{1,\dots}\beta_n}{\gamma}$ ∈D is blocked in an extension E , if α∈ E ,and $\exists\beta_i, \neg\beta_i \in E$.

We consider the following examples, we will find:

Example 1 Let W={α∨β},D={ $\frac{\alpha:\gamma}{\gamma}, \frac{\beta:\gamma}{\gamma}$ },then E=Th({α∨β}) is a Reiter-extension, and E_1=Th({α,γ}), E_2=Th({β,γ}) are all Roos-extensions. Clearly, E⊆ E_1 ,E⊆ E_2. Additionally, We note γ∈ E_1, E_2, γ∉E. E_1, E_2 are two different Roos-extensions for normal default theory (D,W),But $E_1 \cup E_2$ is consistent. That is,

(i) the orthogonality of Roos-extension is false in a normal default theory.

We know, a closed default theory (D,W) has an inconsistent Reiter-extension if W is inconsistent; and if a closed default has an inconsistent extension then this is its only extension. However, these propeties are flase in Roos-extension, for example:

Example 2 Let D = { $\frac{brid:exceptpenguin,canfly}{canfly}$, $\frac{penguin:exceptpenguin,canfly}{canfly}$, $\frac{penguin:exceptpenguin,exceptbird}{exceptbrid}$, $\frac{ostrich:exceptostrich,canfly}{canfly}$, $\frac{ostrich:exceptostrich,exceptbird}{exceptbird}$ },

W={bird, penguin ∨ ostrich},

then E=Th({bird,penguin ∨ ostrich, can fly})is a Reiter-extension, and E_1= Th({bird, penguin, can fly}), E_2 =Th({bird, penguin, can fly, except bird}), E_3 =Th({bird, ostrich, can fly}), E_4=Th({bird, ostrich, can fly, except bird}) are Roos-extensions .Having $E_1 \subseteq E_2$, $E_3 \subseteq E_4$,but $E_1 \neq E_2$, $E_3 \neq E_4$. So, minimality of Roos-extensions is false for closed default theory, and we find E_2, E_4 are inconsistent, and $E_1 \cup E_2$, $E_3 \cup E_4$ are inconsistent, but W is consistent. So,

(ii) Roos- inconsistent extension can have more than one for closed default theory.

(iii) Conversely, if W is inconsistent, (D,W) can have more than one consistent Roos-extension.

Example 3 Let W={α∨¬α },D={ $\frac{\alpha:\delta}{\delta}, \frac{\neg\alpha:\delta}{\delta}$ },then E_1=Th({α, δ}), E_2=Th({¬α,δ}) are Roos-extensions .Obviously, E_1, E_2 are consistent.

(iv) There is closed default theory no Roos extension.

Example 4 Let W=ϕ, D={ $\frac{:\neg\alpha,\neg\beta}{\alpha\vee\beta}$ },then Δ= (D,W) has Reiter-extension E=Th({α∨β}), but it has no Roos-extension.

In the later, we will discuss some properties of Roos-extension that are similar to Reiter-extension.

3. ROOS - EXTENSION FOR CLOSED DEFAULT-THEORY

Theorem 2.1 Let E be a set of closed wffs, and Δ=(D,W) be a closed default theory. Define E_0=W, and for i≥ 0, E_{i+1}=Th(E_i)⋃ { γ | $\frac{\alpha:\beta_1,\ldots,\beta_m}{\gamma} \in D$, α∈ E_i ,and ¬ β_1,…,¬ $\beta_m \notin$ E},then E is a Roos-extension for Δif and only if E∈ $\bigcup_{i=0}^{\infty} E_i$.

Definition 2.2 Suppose Δ =(D,W) is a closed default theory and E is a Roos-extension for Δ. The set of generating default for E with respect to Δ is defined to be GD(E,Δ) = { $\frac{\alpha:\beta_{1,\ldots,}\beta_m}{\gamma} \in D \mid \alpha \in E$, and $\neg\beta_1,\ldots,\neg\beta_m \notin E$ }. If D is any set of default (not necessarily closed) then Con(D) = { γ | $\frac{\alpha:\beta_{1,\ldots,}\beta_m}{\gamma}$ $\in D$ }, that is , it is the set of consequents of the defaults of D.

Theorem 2.3 Suppose E is a Roos-extension for a closed default theory Δ = (D,W) Then $E \in Th(W \cup Con(GD(E,\Delta)))$.

Theorem 2.4 Suppose that E is a Roos-estension for a closed default theory (D,W), and that $B \subseteq E$ .Then E is also a Roos-extension for (D,W∪B).

4. ROOS-EXTENSION FOR NORMAL DEFAULT THEORY

In this section we derive a variety of properties about closed Roos-extension normal default theories that will provide some difference from the Reiter-extension.

Theorem 3.1 Every closed normal default theory have Roos-extension, which are not necessary the only.
Example: Consider Δ=(D,W), in which W= ϕ, D= $\left\{\frac{:\alpha\vee\beta}{\alpha\vee\beta}\right\}$,having $\bigcup_{i=0}^{\infty} E_i$ E_1=Th({α}), E_2 =Th({β}) are Roos-extensions. so,的Roos-extension of Δ is not unique.

Theorem 3.2 (Semi-monotonicity) Suppose D and D' are sets of closed normal default with D' ⊆D. Let E' be a Roos-consistent extension for the closed normal default theory Δ'=(D',W) and let Δ=(D,W). Then Δ has a Roos-consistent extension E such that: (1) E'⊆ E (2) GD(E',Δ') ⊆ GD(E,Δ).

Corollary 3.3 Suppose Δ =(D,W) is closed normal default theory such that W∪ Con(D) is consistent . THen Δ can have more than one Roos-extension.
For example, we can consider example 1,W∪ Con(D) ={α∨β, δ } is consistent, E_1 =Th({ α , δ }), E_2 =Th({ β , δ }) are Roos-extensions.

Theorem 3.4 Suppose Δ =(D,W) is closed normal default theory, and that D'⊆ D. Suppose further that E_1', E_2' are distinct Roos- consistent extensions for (D',W), then Δ have distinct Roos- consistent extensions E_1 and E_2 such that $E_1' \subseteq E_1$,and $E_2' \subseteq E_2$.

Remark This theorem has the interesting interpretation that the addition of new closed normal defaults to a closed normal default theory (D,W) can never lead to a default theory (D',W) with fewer Roos-consistent extension than them original. The number of Roos- consistent extension of a closed normal default theory is monotone non-decreasing under the addition of closed normal defaults. In fact, to all Roos-extension this is correct, that is, so is Roos- inconsistent extension.

5. PROOF THEORY OF DEFAULT THEORY

If D is a finite set of closed normal default, define Pre(D)= $\wedge\alpha$, where the conjunction is taken over all wff α such that $\frac{\alpha:\gamma}{\gamma} \in D$, thus ,Pre(D)= ∧ Pre(δ), $\delta \in D$.

Definition 4.1 Let Δ =(D,W) be a closed normal default theory, and $\beta \in$ Ł a closed wff. A finite sequence $D_0,\ldots,D_k$ of finite subsets of D is a default proof with respest to Δ if

(P_1) W∪ Con (D_0) ├ β ;

(P_2) for $1 \le i \le k$,W $\cup$ Con (D_i) $\vdash$ Pre(D_{i-1});

(P_3) $D_k = \phi$;

(P_4) W $\cup$ $\bigcup_{i=0}^{k}$ Con (D_i is satisfiable.

We can prove the completeness of default proofs for closed normal default theories.

Lemma 4.2 Suppose Δ=(D,W) is a closed normal default theory with $\frac{\beta : \upsilon}{\upsilon} \in$ D ,if W $\vdash \beta$, and W $\cup$ {v} is consistent, then any Roos-consistent extension for(D,W $\cup$ {v}) is also Roos-consistent extension for (D,W).

Corollary 4.3 Suppose Δ = (D,W) is a closed normal default theory, and that D' $\subseteq$ D, such that W $\cup$ Con(D')is consistent and such that W $\vdash$ Pre(D'). Then any Roos-consistent extension for(D,W $\cup$ Con(D')) is also a Roos-consistent extension for (D,W).

Theorem 4.4 Suppose Δ=(D,W) is a closed normal default theory, and $\beta \in$ Ł be a closed wff. If β has a default proof $D_0 ,\ldots, D_k$ with respect to Δ, then has a Roos-consistent extension E such that $\beta \in$ E .

If has a default proof $D_0 ,\ldots, D_k$,then define its default support as DS(P_β)= $\bigcup_{i=0}^{k}$ D_i

Lemma 4.5 Suppose (D,W) is a closed normal default theory,and β', $\beta'' \in$Ł are closed wffs. Then $\beta' \wedge \beta''$ has a default proof with respect to Δ, such that W $\cup$ Con(DS($P_{\beta'}$) $\cup$ DS($P_{\beta''}$)) is satisfiable.

Corollary 4.6 Suppose (D,W) is a closed normal default theory, and $\theta_1 ,\ldots ,\theta_r$ have default proofs $P_{\theta_1} ,\ldots,$ P_{θ_r} respectively with respect to Δ. Furthermore, suppose that W $\cup$ Con($\bigcup_{i=0}^{r}$ (DS(θ_i)))is satisfiable. Then $\theta_1 \wedge \ldots \wedge \theta_r$ has a default proof with respect to, such that DS(P)= $\bigcup_{i=0}^{r}$ (DS(θ_i)). (θ_i)).

Theorem 4.7 Suppose α has a default proof P_α with respect to a closed normal default theory Δ =(D,W) suppose further that the wff $\alpha \supset \beta$ is valid . Then P_α is also a default proof of β with respect to Δ

Theorem 4.8 Suppose that E is a Roos-extension for a consistent closed normal default theory Δ=(D,W) ,and $\beta \in$ E,then β has a default proof with respect to Δ.

Theorem 4.9 Let $\beta \in$Ł be a closed wff, a consistent closed normal default theory Δ has a Roos-consistent extension E such that $\beta \in$ E if and only if β has a default proof with respect to Δ.

The proof about these theorem, lemma are easy by means of the theorem 2.1 and the definitio , reader can see[1],in here, we omit its.

We find that an intuitively plausible way of dealing with propositions that hold no longer be derivable as a result of reasoning by case, can have far reaching consequence, One of the consequences is that disjunctions must be viewed as describing possible extension, that is Roos-extension. Roos-extensions have some different properties from Reiter's.

6. THE ALGORITHM OF ROOS-EXTENSION

All Roos-extensions of (D,W)

input: A finite defaut theory (D,W)

output: The list of Roos-extensions of (D,W)
for E Th(E) do

$$E_{i+1}\text{=Th(}E_i\text{)} \cup \{ \gamma \mid \frac{\alpha : \beta_1,\ldots,\beta_m}{\gamma} \in D,\ \alpha \in E_i$$

,and $\neg\beta_1,\ldots,\neg\beta_m \notin$ E}

then output E_{i+1}

Algorithm for computing Roos-extensions of default theories

We discuss the properties of Roos-extensions so that using Roos-extensions can achieve to classify data, information in the field of artificial intelligence.

REFERENCES

[1] R.Reiter,A Logic for Default Logic[J],AI 1980, 13: 81--132

[2] K.Konolige, On the Relation Between Default and Autoepistemic Logic[J], AI 1988,35:343-382

[3] Y.Moinard,Reasoning by Cases without Contraposition in Default Logic,in: Proceedings ECAI-94, Ansterdam,1994:381-385

[4] J.P.Delgrande,W.K.Jackson.Default Logic Revisisted, in: Proceedings KR-91 , Cambrige,MA,1991:118-127

[5] F.Voorbraak,Preference-based Semantics for Non-monotonic Logics .in: Proceedings IJCAI-93,Washington DC.1993:584-589.

[6] Nico Roos,Reasoning by Cases in Default Logic[J],AI 1998,99:165-183

[7] Fuli the extension of default logic[M], China Scienceand technology press,2006

[8] Xu Dao-Yun,Ding De-Cheng,Zhang Ming-Yi,On Reasoning by Cases in Default Logic,Juurnal of Software,2001,12(8):1140-1146

[9] Georg Gottlob,Zhang Ming Yi,Cumulative Default Logic:finite characterization[J],algorithms and complexity 1994 69:329--345

[10] G.Brewka,Cumulative Default Logic:in defense of nonomontic inference rules[J],AI 50(1991): 183--205

[11] Grigoris Antonion,Nonmonotic Reasoning [M],Massachusatts Institue of Technology 1997

[12] V.W.Marek,M.Tuszczy\'nski,Nonmonotic Logic (Context—Default Reasoning)[M],Springer--Verlag Berlin Heidelberg 1993

[13] Lawra Girordano,Alberto Martelli:on Cumulative Default Logics[J], AI 66(1994): 161—179