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Evaluating Mass Transfer Processes for Groundwater Contaminants Flow Models in Yenagoa, Nigeria

Awajiogak Anthony Ujile
Department of Chemical/Petrochemical Engineering,
Rivers State University of Science and Technology, P M B 5080, Port Harcourt, Nigeria.

ABSTRACT

Principles of mass transfer have been applied to evaluate groundwater contaminants flow model in Yenagoa, Nigeria. The groundwater flow and solute transport, to determine the concentration profile of contaminants in groundwater flow was carried out. Applications of the model in Yenagoa to determine the concentration profile of contaminants show that the solution is unconditionally convergent and stable. The sensitivity analysis in form of MATHCAD matrix printout has shown that bulk soil density, decay rates, aquifer characteristics affect the behavior of the contaminants. This work shows the choice of applicable treatment method for groundwater contaminants.

Keywords: *contaminants, spatial distribution, porous-media flow, reaction-diffusion*

1. INTRODUCTION

Mass transfer is the movement of any identifiable species from one spatial location to another to another. Chemical processes produce complex mixtures of compound from various sources. Since separation processes are based on the creation of composition differences within and between phases, a consideration of mass transfer principles is necessary to the analysis and design of such processes Donald, [1]. Contaminated water flows into the ground from the surface of the earth through pores in the soil and underlying geologic structures. Although these pores are very small and so account for only a small portion of the underground volume, contaminated water moves very slowly underground and can cover large distances in depth [2]. The quality of ground water in some parts of the world, particularly shallow ground water, is changing as a result of human activities. Ground water is less susceptible to bacterial pollution than surface water because the soil and rocks through which ground water flows screen out most of the bacteria. Bacteria, however, occasionally find their way into ground water, sometimes in dangerously high concentrations. But freedom from bacterial pollution alone does not mean that the water is fit to drink. Many unseen dissolved mineral and organic constituents are present in ground water in various concentrations. Most are harmless or even beneficial; though occurring infrequently, others are harmful, and a few may be highly toxic. The most common dissolved mineral substances are sodium, calcium, magnesium, potassium, chloride, bicarbonate, and sulphate. Ground water, especially if the water is acidic, in many places contains excessive amounts of iron. Iron causes reddish stains on plumbing fixtures and clothing. Like hardness, excessive iron content can be reduced by treatment USGS [3].

Many researchers have studied the problem of solute movement through saturated porous media both analytically and numerically. However, published literature related to a field problem on a regional scale such as the present one, (Niger Delta region) is scarce.

For example, Mercer,[4] modeled groundwater flow at Love canal; Bredehoeft and Pinder [5] modeled the distribution of contaminant in an aquifer of glacial sediments; solute transport in limestone aquifer by Schwartz [7]; and assessment of salt water encroachment phenomenon by Ashim, [8].

Of all the investigations carried out by other researchers there has not been any that considered regional assessment with the application of second order differential equations. This work involves not only solute transport and flow-model in a regional scale, but also multiple solute contaminants (Iron, chloride, hardness, salinity concentration levels) in the region with the application of second order differential equations. Abraham [9] used a single cell model to study the regional chloride and nitrate pollution pattern in part of the coastal aquifer in Israel. Spanoudaki [10] used the finite difference and orthogonal grids for Integrated Surface water-Groundwater modeling; Shahld and Rahman [11] applied a two-dimensional groundwater flow for the delineation of wellhead protection area around water well and Fatta [12] carried out numerical simulation of flow and contaminant migration at a municipal landfill. Park [13] applied transport modeling to the interpretation of groundwater tracer. Other researchers have recently applied analytical method for the mass transfer coefficient and concentration boundary layer thickness for dissolving non-aqueous phase liquid pool in porous media.

Donald, [1] established that mass transfer occurs due to a concentration gradient or difference within a phase. Mass transfer rate between two phases is proportional to their interfacial area and not the volumes of the phase present. Transfer owing to microscopic fluid motion or mixing is much more rapid than that due to molecular motion (diffusion).

From the standpoint of the above, it becomes imperative to establish solutions to second order partial differential equation that will be simple to all practitioners and researchers. This work has utilized the solutions obtained by Ujile [14] to evaluate the mass transfer processes in groundwater contaminant flow models.

2. METHODOLOGY

We considered the movement of the contaminated fluid from a solid waste landfill into potable water aquifer located beneath it, which is an example of unwanted groundwater flow (Fig.1). The motion of fluid through porous rock induced by pressure and gravity forces is of great practical importance.

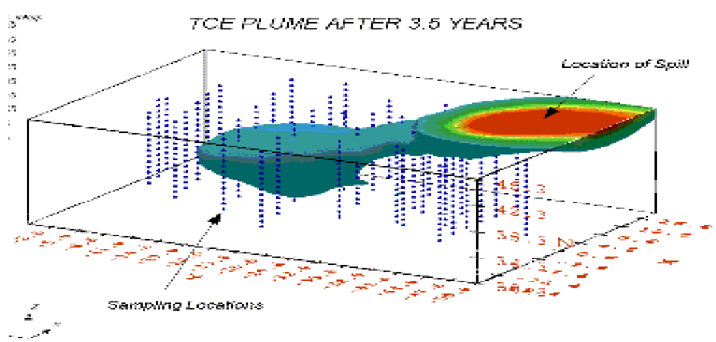


Fig.1. Plume behavior TCE (Trichloro ethylene) after 3.5 years NRC, [15].

Taking the material balance on the leaching process in Fig. 1 in the x, y and z directions Ujile [14] obtained the following expressions:

$$\frac{\partial}{\partial x}\left(D_x \frac{\partial C}{\partial x}\right) - V_x \frac{\partial C_i}{\partial x} + \frac{\partial}{\partial y}\left(Dy \frac{\partial C}{\partial y}\right) - Vy \frac{\partial C}{\partial y} + \frac{\partial}{\partial z}\left(D_z \frac{\partial C}{\partial z}\right) - V_z \frac{\partial C}{\partial z} = R \frac{\partial C}{\partial t} + KC \qquad (1)$$

Considering and correlating the plume of (trichloro-ethylene) TCE leachate (Fig. 1), the direction of flow in horizontal x and vertical y directions is larger than the transverse z direction. The field data obtained from the Niger Delta region shows variation in hydraulic conductivity in different directions

i.e $\frac{\partial c}{\partial x} > \frac{\partial c}{\partial y} >>> \frac{\partial c}{\partial z}$

hence $\frac{\partial c}{\partial z} = 0$, $V_x > V_y >>> V_z$; $V_z \simeq 0$

Then Eq. (1) becomes:

$$\frac{\partial c}{\partial x}\left(D_x \frac{\partial c}{\partial x}\right) - V_x \frac{\partial c}{\partial x} + \frac{\partial c}{\partial y}\left(D_y \frac{\partial c}{\partial y}\right) - V_y \frac{\partial c}{\partial y} = R \frac{\partial c}{\partial t} + KC \qquad (2)$$

Eq. (2) can be modified to the form.

$$\left(D_x \frac{\partial^2 C}{\partial x^2} + D_y \frac{\partial^2 C}{\partial y^2}\right) - \left(V_x \frac{\partial C}{\partial x} + V_y \frac{\partial C}{\partial y}\right) = R \frac{\partial C}{\partial t} + KC \qquad (3)$$

2.1 Basic Engineering Assumptions

Movement of the solute is assumed to be in the plane of the horizontal and vertical section of the aquifer and the porous medium is assumed to be anisotropic with respect to the dispersivity of the medium. The density and viscosity of groundwater is assumed to be constant. This is the case in most ground water flow system. The groundwater flow pattern is not altered by the presence of multiple contaminants in solution.

Aquifers with unconsolidated formation of sand and gravel are predominant in Niger Delta region, porosity is considered more or less uniform on a regional basis.

(Range 0.33-0.45) The first order decay rate in liquid phase is the same as in soil (solid) phase, (i.e. $K_L = K_S$)

The mass transfer coefficient of pollutant is considered on a macroscopic scale. This is because of hydraulic conductivity and porosity, which create irregularities in the seepage velocity and consequently additional mixing of the pollutant.

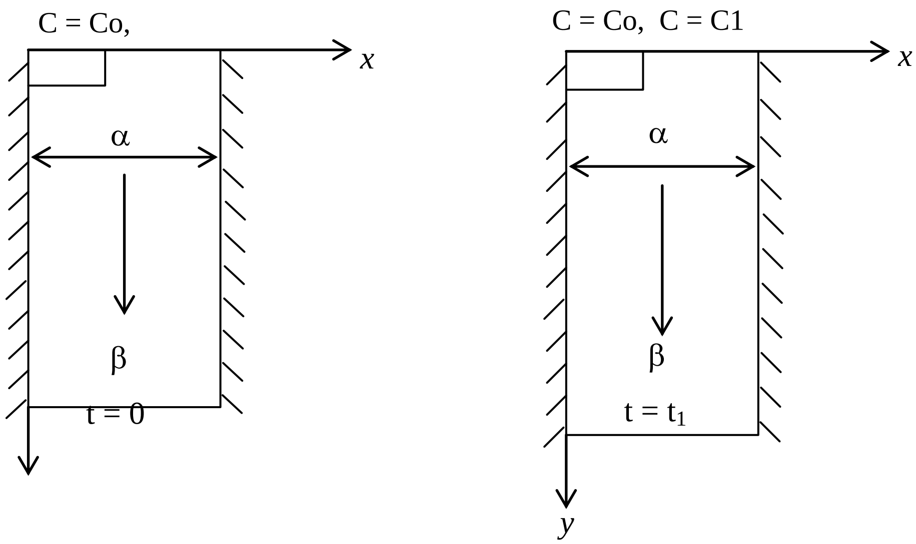


Fig. 2 Schematic sketch of Longitudinal and lateral dispersion of contaminant in groundwater flow in Niger Delta

$$C\,(x, y, o) = Co \quad 0 \le x \le \alpha;\ 0 \le y \le \beta \qquad (4)$$

$$\frac{\partial c}{\partial x}\,(o, y, x) = Co \qquad t > 0 \qquad (5)$$

If the boundary layer flow should satisfy the mass conservation, equation for incompressible flow:

$$\frac{\partial c}{\partial x} + \frac{\partial c}{\partial y} = 0 \qquad (6)$$

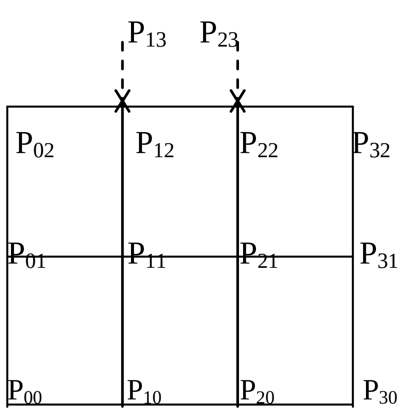


Fig. 3 Mixed boundary Value Problem grid, Kreyszig, [16]

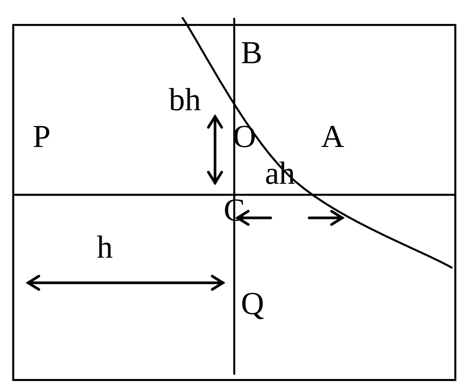


Fig. 4:Curved boundary C of a region R, a mesh point O near C and neighbours A, B, P, Q.

The solution obtained from Eq. (3)

Numerical models using approximation (e.g. finite differences or finite elements, Taylor theorem, Newmann methods, etc) yielded:

$$\text{In}\ \frac{C_{A'}}{C_o} = \text{In}\left[K_L\ (1+\rho_b\ Kd/\phi) + \frac{M}{h}\right] - \left\{\left[K_L\ (1+\rho_b\ Kd/\phi) + \frac{M}{h}\right]\Big/(1+\rho_b\ K_d/\phi)\right\}.t \qquad (7)$$

where M =

$$\left(\frac{2D_x}{h} - (1-a)V_x\right)\frac{1}{a} + \left(\frac{2D_y}{h} - (1-b)V_y\right)\frac{1}{b} \qquad (8)$$

$$C_A' = \frac{C_A}{h}\,(A + B + P + Q)$$

$$A = \left(\frac{2D_x}{h} - V_x\right)\frac{1}{a(a+1)}$$

$$B = \left(\frac{2D_y}{h} - V_y\right)\frac{1}{b(b+1)}$$

$$P = \left(\frac{2D_x}{h} + V_x a\right)\frac{1}{a(a+1)}$$

$$Q = \left(\frac{2D_{xy}}{h} + V_y b\right)\frac{1}{(b+1)}$$

3. APPLICATION OF THE MODEL EQUATIONS

3.1 Area Description: Yenagoa in Bayelsa State

Niger Delta is one of the largest wetlands in the world; and produces crude oil, which accounts for about 85% of the total Nigerian Government revenue. The states within the region are: Edo, Delta, Bayelsa, Rivers, Akwa Ibom, Ondo and Cross River State. Yenagoa is the headquarters of one of these States, was taken as the study area. The groundwater iron distribution map of part of the region is shown in Fig. 5. The figure shows that Yenagoa has very high iron concentration of 1 to 8 mg/l. The River Niger deposits all loadings from the upper Niger at the Deltaic zones. It bifurcates at some 130 km south of the apex into the rivers Nun and Forcados. Yenagoa is located east of the confluence of River Nun and Ekole River. Niger Delta Environmental Survey Report NDES report [17] asserted that the River Niger carries iron loadings from the deposits of Itakpe Iron Ore and perhaps through the processes of dispersion, advection, inter aquifer exchange move the pollutants to the groundwater aquifer.

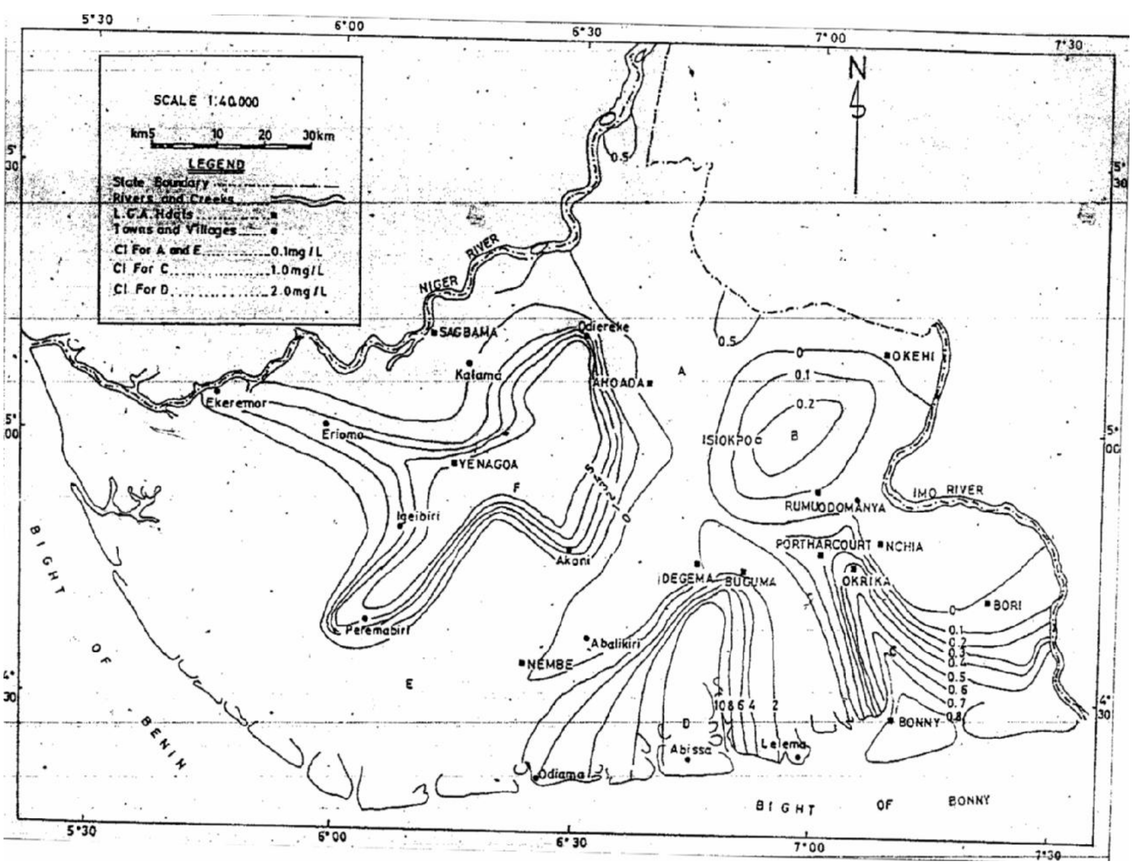


Fig.5. Groundwater Iron distribution map of Bayelsa and Rivers State in Niger Delta, (Ujile [18].

The geology of the region is dominated by the Quaternary Benin Formation (Coastal Plain Sands). Others are Agbada and Akata formations. A generalized stratigraphic succession can be found in Ujile's work [15]. The main body of groundwater in major parts of the region is contained in very thick and extensive sand and gravel aquifers of the Benin Formation

3.2 Application of the Model

The application of the model was carried out with the MATHCAD software systems. The expression in the parenthesis – $(1 + \rho_b K_d / \phi)$ is called the retardation factor. This expression determines the resistance of the contaminant to move through the groundwater aquifer. The analysis of the parameter effective porosity was based on the transient characterization and its effect on the groundwater flow systems. MATHCAD program was developed from the solution of the model equation, Appendix A and the results are shown in Figs. 6, 7 and 8. The input data for Yenagoa based on the hydrogeological features are:

Boundary Conditions

$0 \le x \le 500 \quad 0 \le y \le 49.5$

For h = 500m; a = (0.1 – 1.0); b = (0.009 – 0.099); Co = 4.5 mg/l (iron content). Other input data are shown in Appendix A and the output indicated in Appendix B, Figs. 6, 7 and 8.

4. RESULTS AND DISCUSSION

The results obtained in the form of matrix of function values (Appendix B), further produced the bar, surface, and contour plots over specified horizontal and vertical ranges as shown in Figs. 6, 7 and 8 respectively. They show rapid divergent when effective porosity is increased.

For conservation transport model, the following observations were made:

> A reduction in contaminant concentrations with increase in porosity values provides primary lines of evidence for natural attenuation of each contaminant.

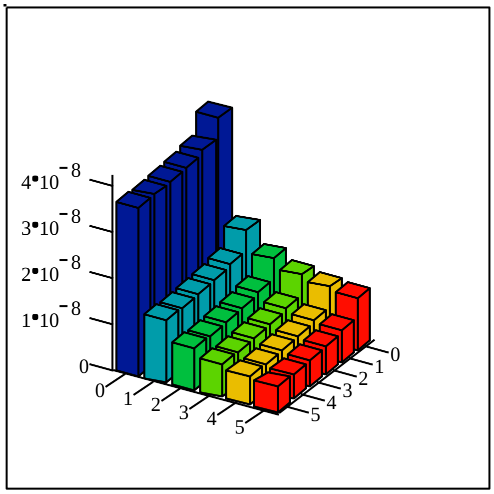
The contour plots provide secondary evidence for possible biodegradation of contaminant in a spatial trend analysis as shown in Fig. 7.

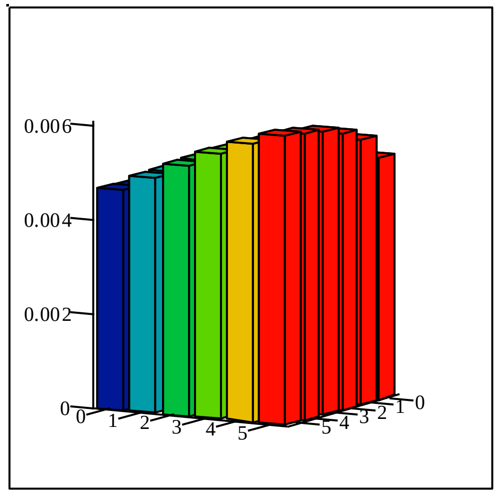
These observations may be correct for conditions where there are spillages of chemicals, dumpsites of municipal and hazardous wastes. However, the *high iron content and hardness levels* detected in most groundwater in the region especially in Yenagoa has been traced to the upcoming deposits from the Niger Delta river system distributions which affect the groundwater by processes of diffusion, advection, convection via interaquifer exchange, direct migration, infiltration and groundwater/ surface water interaction. Therefore the source of the contamination seems to be natural. Then attenuation and biodegradation phenomenon may not apply in the circumstance. For non conservative transport model, the following observations are made:

> An increase in contaminant concentrations with increase in porosity values shows that natural attenuation may not be tenable.
> The alternative solution to the groundwater problem will be to analyze borehole samples and the results determine the treatment method(s) to be applied to obtain potable water. For the non-conservative transport model there is increase in the contaminant concentration, as it moves from source to sink.

The model equation has proved that the concentration levels of iron pollutant in groundwater depend on distance from source of pollution, time, processes of transfer and the regime of flow.

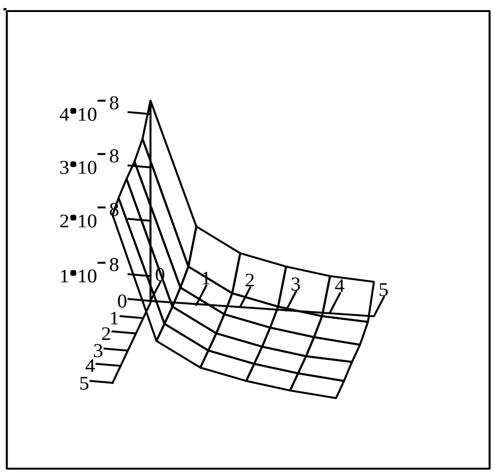
The distribution map of iron concentration was considered for comparison of the simulated model output. The MATHCAD graphical representations of C and C1 for Yenagoa compare the distribution of the contaminants. C on the printout shows the gradual penetration of the dissolved molecules of the contaminant into groundwater when there is no reaction. Comparison with C1 shows the change owing to the reaction with first order decay rate. As time elapses the concentration profiles with reaction, take on a different shape and approach a limiting exponential profile for which reaction and diffusion are in step at each value of boundary conditions. The profile remains steady because the rate of diffusion at each value of h, a, b (which determine the y and x positions) is exactly equal to the total rate of reaction throughout the aquifer length, x and depth, y. When these occur, there is no further decrease in the mass transfer rate of contaminants into the groundwater bodies. The print out result (Appendix B) shows that at x direction concentration of contaminant increases on the surface and decreases in the y-direction perhaps owing to adsorption processes in the rock matrix. The surface, bar and contour plots of C and C1 illustrate the spatial distribution of contaminants without reaction and with first order reaction respectively.



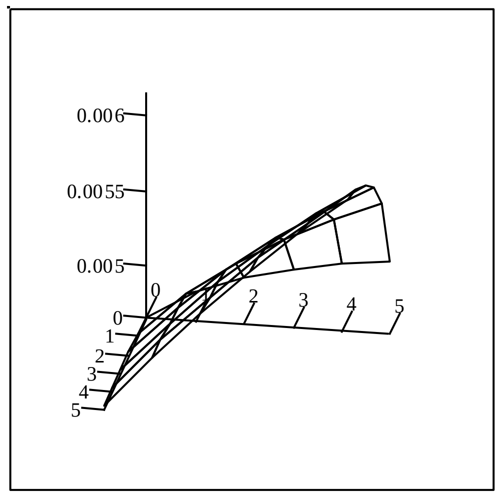


C CI

Fig.6 Bar plots for C and C1from matrices solution of Appendix B

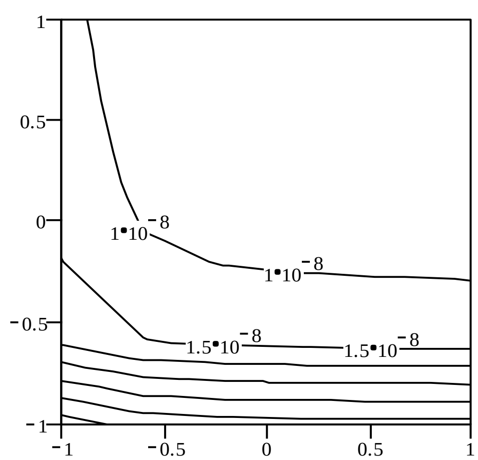


C

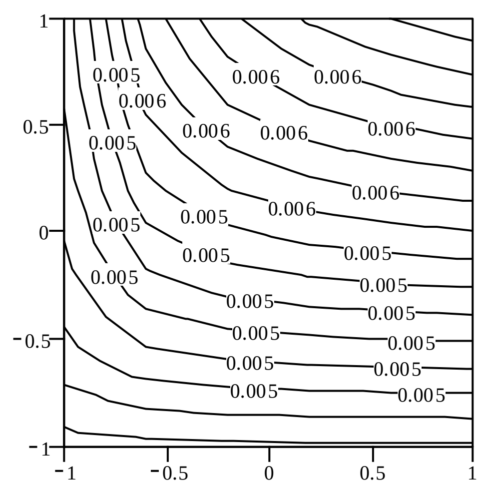


CI

Fig. 7. Surface plots for C and C1 from matrices solution of Appendix B



C



CI

Fig. 8. Contour plots for C and C1 from matrices solution of Appendix B

5. CONCLUSIONS

Mass transfer processes with the application of the second order partial differential equations for groundwater flow to determine the concentration profile of iron has been carried out. The application of the model equation has provided simple results that are similar to works carried out by other researchers with complex results.

The variation of porosity values has caused significant changes in the concentration of groundwater contaminant.

This analysis can be applied for the design of natural attenuation landfill for both municipal and industrial wastes. Materials of construction at the bottom of the landfill can be based on the principle of attenuation of groundwater contaminants. The determination of concentration profile gives a guide to the treatment method(s) that is/are applicable.

The reaction that is simulated is non-linear equilibrium adsorption of a single dissolved species. However several different zones with different sorptive and reactive properties are required: e.g distribution coefficient, decay coefficient, yield coefficient. It is therefore recommended that further analysis/ research on the model equations developed by the author be carried out as to determine the impact of soil bulk density, chemical decay rates and other hydro geological parameters on groundwater contamination.

Federal and State Ministries of Environment should consider groundwater protection an issue of paramount importance. Enabling Laws should be put in place for waste disposal systems and these Laws should be enforced.

Nomenclature

C = Concentration of the contaminant at time t, (for conservative transport model),mg/Ls

C1 = Concentration of the contaminant at time t, with 1st order reaction (for non conservative transport model), mg/L

C_o = Initial concentration of the contaminant mg/Ls

Dx, Dy, = Directional hydrodynamic dispersion coefficients, m^2 / s

exp = exponent

ln = Natural logarithm

h = Distance of flow moved by pollutant; m

K_d = Distribution coefficient.

K_l ,K_s = First order decay rate in the liquid phase and soil respectively, 1/s

K = Overall first order decay rate = $K_l + K_s \rho_b K_d / \phi$, 1/s..

l = Characteristic linear dimension, m.

M = Bulk mass transfer coefficient of pollutant in groundwater flow in x-y direction, m/s flow in x-y direction, m/s.

R = Retardation factor = $1 + \rho_b K_d / \phi$.

t = Process time, or time since the start of the simulation, s.

Vx, Vy = Directional seepage velocity components, m/s

Greek letter

ϕ = Effective porosity; porosity of aquifer

ρ_b = bulk density of soil, kg/m^3

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Appendix A

Concentration Profile of Iron contaminant in groundwater-MATHCAD PRINT OUT Yenagoa, Nigeria

$h := 500\,m$

$Dx := 2.1 \cdot 10^{-2}\,\frac{m^2}{s}$

$Dy := 8.6 \cdot 10^{-3}\,\frac{m^2}{s}$

$Vx := 8.8 \cdot 10^{-6}\,\frac{m}{s}$

$Vy := 1.9 \cdot 10^{-7}\,\frac{m}{s}$

$\rho b := 1.9\,\frac{g}{L}$

$\phi := 0.5355$

$Co := 4.5\,\frac{mg}{L}$

$Kd := 2.6\,\frac{L}{g}$

$R := 1 + \rho b \cdot \frac{Kd}{\phi}$

$R = 10.225$

$A(a) := \left(2\cdot\frac{Dx}{h} - Vx\right)\cdot\frac{1}{a\cdot(a+1)}$

$B(b) := \left(2\cdot\frac{Dy}{h} - Vy\right)\cdot\frac{1}{b\cdot(b+1)}$

$P(a) := \left(2\cdot\frac{Dx}{h} + a\cdot Vx\right)\cdot\frac{1}{(a+1)}$

$Q(b) := \left(2\cdot\frac{Dy}{h} + b\cdot Vy\right)\cdot\frac{1}{(b+1)}$

Mathcad program for $\phi = 0.5355$

$M(a,b) := 2\cdot\frac{Dx}{h\cdot a} + \left(2\cdot\frac{Dy}{h\cdot b}\right) - \left[\frac{(1-a)}{a}\cdot Vx + \frac{1-b}{b}\cdot Vy\right]$

$t := 2\ yr$

$t = 6.311\cdot10^{7}\ \cdot s$

$KL := 3.1\cdot10^{-8}\ s^{-1}$

$K := KL\cdot R$

$K = 3.17\cdot10^{-7}\ \cdot s^{-1}$

$\tau(a,b) := K + \frac{M(a,b)}{h}$

$C(a,b) := Co\cdot\tau(a,b)\cdot\frac{e^{\frac{\tau(a,b)\cdot t}{R}}}{e^{\frac{\tau(a,b)\cdot t}{R}} - 1}$

$CI(a,b) := \frac{C(a,b)\cdot h}{A(a) + B(b) + P(a) + Q(b)}$

$alow := 0.1$

$ahigh := 1$

$an := 6$

$i := 0..\ an - 1$

$aind_i := alow + i\cdot\frac{ahigh - alow}{an - 1}$

$blow := 0.009$

$bhigh := 0.099$

$bn := 6$

$j := 0..\ bn - 1$

$bind_j := blow + j\cdot\frac{bhigh - blow}{bn - 1}$

$C_{i,j} := C\left(aind_i, bind_j\right)$

$CI_{i,j} := CI\left(aind_i, bind_j\right)$

Appendix B

Mathcad output for C and C1

$$C = \begin{bmatrix} 4.249\cdot 10^{-8} & 1.968\cdot 10^{-8} & 1.512\cdot 10^{-8} & 1.316\cdot 10^{-8} & 1.208\cdot 10^{-8} & 1.139\cdot 10^{-8} \\ 3.813\cdot 10^{-8} & 1.533\cdot 10^{-8} & 1.077\cdot 10^{-8} & 8.812\cdot 10^{-9} & 7.726\cdot 10^{-9} & 7.035\cdot 10^{-9} \\ 3.719\cdot 10^{-8} & 1.438\cdot 10^{-8} & 9.821\cdot 10^{-9} & 7.866\cdot 10^{-9} & 6.78\cdot 10^{-9} & 6.09\cdot 10^{-9} \\ 3.677\cdot 10^{-8} & 1.397\cdot 10^{-8} & 9.407\cdot 10^{-9} & 7.452\cdot 10^{-9} & 6.367\cdot 10^{-9} & 5.677\cdot 10^{-9} \\ 3.654\cdot 10^{-8} & 1.374\cdot 10^{-8} & 9.175\cdot 10^{-9} & 7.22\cdot 10^{-9} & 6.135\cdot 10^{-9} & 5.446\cdot 10^{-9} \\ 3.639\cdot 10^{-8} & 1.359\cdot 10^{-8} & 9.026\cdot 10^{-9} & 7.072\cdot 10^{-9} & 5.987\cdot 10^{-9} & 5.298\cdot 10^{-9} \end{bmatrix} \cdot kg \cdot m^{-3} \cdot s^{-1}$$

$$CI = \begin{bmatrix} 4.656\cdot 10^{-3} & 4.852\cdot 10^{-3} & 4.969\cdot 10^{-3} & 5.047\cdot 10^{-3} & 5.103\cdot 10^{-3} & 5.145\cdot 10^{-3} \\ 4.675\cdot 10^{-3} & 4.962\cdot 10^{-3} & 5.187\cdot 10^{-3} & 5.369\cdot 10^{-3} & 5.519\cdot 10^{-3} & 5.645\cdot 10^{-3} \\ 4.679\cdot 10^{-3} & 4.995\cdot 10^{-3} & 5.265\cdot 10^{-3} & 5.497\cdot 10^{-3} & 5.7\cdot 10^{-3} & 5.878\cdot 10^{-3} \\ 4.682\cdot 10^{-3} & 5.012\cdot 10^{-3} & 5.304\cdot 10^{-3} & 5.565\cdot 10^{-3} & 5.8\cdot 10^{-3} & 6.013\cdot 10^{-3} \\ 4.683\cdot 10^{-3} & 5.021\cdot 10^{-3} & 5.328\cdot 10^{-3} & 5.608\cdot 10^{-3} & 5.865\cdot 10^{-3} & 6.102\cdot 10^{-3} \\ 4.684\cdot 10^{-3} & 5.028\cdot 10^{-3} & 5.345\cdot 10^{-3} & 5.637\cdot 10^{-3} & 5.91\cdot 10^{-3} & 6.164\cdot 10^{-3} \end{bmatrix} \cdot kg \cdot m^{-3}$$