

Optimum Target Value for Two Processes in Series: Case of Hybrid Tubular Expansion

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ABSTRACT

In this paper, a manufacturing system with two machines in series was investigated. The optimum process mean and screening limits of a surrogate variable through two machines in series with 100% inspection are investigated. In this procedure, the variable is inspected first to decide whether to be accepted or rejected. After accepting the product from the first machine, it goes to the second machine which has three classifications, accept, or reject and rework on the second machine or reject then rework for both machines. Assuming that the performance of variables are jointed normally distributed, the optimum process mean are obtained by maximizing the expected profit or minimizing the production cost which includes selling price, production, reprocessing, inspection and penalty costs. This method is applied to joining tube-tubesheet expansion process in heat exchangers using hybrid method which includes hydraulic then rolling processes. The target dimension chosen is the wall reduction percentage of the tube thickness. A numerical example is presented.

Keywords: *Optimum target, tube, tubesheet, rolling, hydraulic, hybrid, expansion.*

1. INTRODUCTION

Several researchers have studied the optimum targeting for a process considering recycling of rejected products and other factors. The optimum target value that minimizes the reprocessing cost and the material costs for overfilled and under filled items was obtained. *Golhar* [1] studied the case in which a rejected product would be recycled (emptied and recycled) so that it would be sold in the primary market. *Golhar and Pollock* [2] determined the optimal process mean and optimal upper limit for recycling products. Recently *Gupta and Golhar* [3] determined both the optimal process mean and optimal lot size for the rejected recycled products. In all the above cases, researchers cited examples from the pharmaceuticals industry *Golhar* [1] and glass industry *Gupta and Golhar* [2] showing only a single stage operation. *Boucher and Jafari* [4] and *Al-Sultan* [5] discussed situations in which the items are subjected to lot-by-lot acceptance sampling rather than complete inspections. *Elsayed and Chen* [6] determined optimum levels of process parameters for products with multiple characteristics, and *Arcelus and Rahim* [7] developed a model for simultaneously selecting optimum target means for both variable and attribute quality characteristics. *Tang* [8] considered an economic model for determining the most profitable target value along with the optimum inspection precision level for a production process. *Tang* [8] presented a methodology to maximize product robustness by the appropriate allocating assembly and machining tolerances. Dimensional accuracy and cost optimization of the finishing stage for target machining of

cold formed parts was studied by *Kopac* [9]. *Al-Sultan* [10] presented the theory and methodology of optimum target values for two machines in series with 100% inspection. *Teeravaraprug* [11] presented a method of designing the optimal process target levels for multiple quality characteristics. The paper first studied a multivariate quality loss function to capture customer dissatisfaction with product quality, and then proposed an optimization scheme to determine the most economical process target levels for multiple quality characteristics. The optimization procedures were demonstrated in a numerical example, and the effects of process parameters were examined by conducting a sensitivity analysis. The study of *Jeang* [12] optimized the components parameters and components tolerances simultaneously via a computer simulation and response surface methodology RSM. *Duffuaa* [13] developed a model of a process targeting with multi-class screening and measurement error.

Joining tubes inside the tubesheet holes in a heat exchanger can be done with different methods such as rolling, hydraulic or explosive. Each process has advantages and disadvantages. To get benefit from the advantages while preventing disadvantages of these processes, hybrid process was implemented. The most common hybrid expansion is using hydraulic and rolling tube-to-tubesheet expansion. This is called hybrid expanding that takes advantage of fixing the tubes firmly in place and stiffening the tubesheet in the hydraulic expanding stage, and in the rolling stage, the inner shells of the tube end is strain hardened sufficiently to overcome spring back. This results in tube-to-tubesheet joints less prone to fatigue failure with adequate strength for the

service. The goal of this hybrid method is to reduce the wall thickness to gain maximum profit or minimize the overall cost.

2. METHODOLOGY

In this work, a hybrid expansion with two stages, hydraulic and then rolling of tube-to-tubesheet expansion are simultaneously used. The work concentrate on of determining the optimum process mean and screening limits of a surrogate variable associated with product quality under two machines in series with 100% inspection. The process targeting will be applied to the processes of tube-to-tubesheet hydraulic expansion followed by roller expansion. New or reworked tube enters the hydraulic process to get a tube wall reduction of percentage X_1 . If the percentage is less than the lower specified percentage by designer L_1 , the tube will be returned back to rework and expanded hydraulically again. When the hydraulically expanded tube wall reduction percentage is larger than L_1 , the tube will go to the rolling stage.

The rolling stage may lead to three cases. The first case occurs when the rolled tubes wall reduction percentage X_2 is less than the lower specified percentage by designer L_2 , hence the tube has to return back for retubing again. Second, the rolled tube percentage is larger than the upper specified limit U_2 , the tube is turned back to rolling again. Third, the acceptable tube joint has a tube wall reduction percentage on the range between the upper and lower limit. Figure 1 shows the flow chart of the whole process.

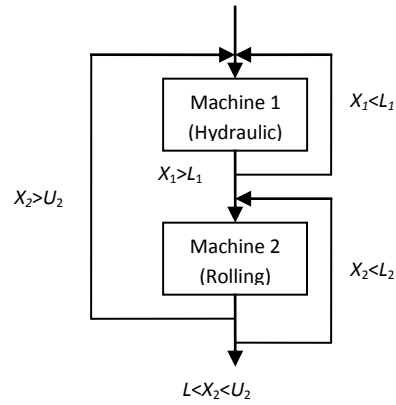


Fig.1. Flow Chart of the processes

3. MODELING

For a given μ_1 and μ_2 the expected value of the expansion cost per tube is $C_1\mu_1$ and $C_2\mu_2$ respectively. Let $[P(X_1, X_2, \mu_1, \mu_2)]$ be the profit for a single tube joining with tubesheet with wall reduction percentage X_2 and $E[P(X_1, X_2, \mu_1, \mu_2)]$ be the expected profit. If a joint failed to meet the specifications either after first process or second process, it is reworked at the cost of R_1 when rejected from process 1 and R_2 when rejected from process 2 with $X_2 > U_2$ and R_3 when rejected from process 2 with $X_2 < L_2$. This reworked tube will then realize the expected profit $E[P(X_1, X_2, \mu_1, \mu_2)]$. Given the above description of the problem, one can construct the following profit function per single tube joint:

Let $E = E[P(X_1, X_2, \mu_1, \mu_2)]$

$$P(x_1, x_2, \mu_1, \mu_2) = \begin{cases} A - C_1x_1 - C_2(x_1 - x_2) & \text{if } x_1 > L_1 \text{ and } L_2 < x_2 < U_2 \\ E(P(X_1, X_2, \mu_1, \mu_2)) - R_1 & \text{if } x_1 < L_1 \\ E(P(X_1, X_2, \mu_1, \mu_2)) - R_2 & \text{if } x_1 > L_1 \text{ } x_2 < L_2 \\ E(P(X_1, X_2, \mu_1, \mu_2)) - R_3 & \text{if } x_1 > L_1 \text{ and } x_2 > U_2 \end{cases} \quad (1)$$

where

$$C_1x_1 + C_2(x_1 - x_2) = (C_1 + C_2)x_1 - C_2x_2A$$

Hence the expected profit is given by:

$$E(P(x_1, x_2, \mu_1, \mu_2)) = \int_{L_1}^{\infty} \int_{L_2}^{U_2} (A - (C_1 + C_2)x_1 + C_2x_2) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 + \int_0^{L_1} (E(P(x_1, x_2, \mu_1, \mu_2)) - R_1) f(X_1; \mu_1, \sigma_1^2) dx_1 + \int_{L_1}^{\infty} \int_0^{L_2} (E(P(x_1, x_2, \mu_1, \mu_2)) - R_2) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 + \int_{L_1}^{\infty} \int_{U_2}^{\infty} (E(P(x_1, x_2, \mu_1, \mu_2)) - R_3) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 \quad (2)$$

4. ANALYSIS OF THE MODEL

The procedure of solving is by getting the optimum value of μ_1 and μ_2 . Then Equation (1) will be arranged as follow:

$$\begin{aligned}
 E(P(x_1, x_2, \mu_1, \mu_2)) = & \int_{L1}^{\infty} \int_0^{U2} (A - (C_1 + C_2)x_1 + C_2x_2) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 \\
 & - \int_{L1}^{\infty} \int_0^{L2} (A - (C_1 + C_2)x_1 + C_2x_2) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 \\
 & + (E(P(x_1, x_2, \mu_1, \mu_2)) - R_1) \int_0^{L1} f(X_1; \mu_1, \sigma_1^2) dx_1 \\
 & + (E(P(x_1, x_2, \mu_1, \mu_2)) - R_2) \int_{L1}^{\infty} \int_0^{L2} f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 \\
 & + (E(P(x_1, x_2, \mu_1, \mu_2)) - R_3) \int_{L1}^{\infty} \int_{U2}^{\infty} f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2
 \end{aligned} \tag{4}$$

Simplify Equation (4) to get

$$\begin{aligned}
 E(P(x_1, x_2, \mu_1, \mu_2)) = & A(1 - \Phi[-z_1])(\Phi[-z_3]) - AA(1 - \Phi[-z_1])(\Phi[-z_2]) \\
 & - I \\
 & + (E(P(x_1, x_2, \mu_1, \mu_2)) - R_1)\Phi[-z_1] \\
 & + (E(P(x_1, x_2, \mu_1, \mu_2)) - R_2)(1 - \Phi[-z_1])(\Phi[-z_2]) \\
 & + (E(P(x_1, x_2, \mu_1, \mu_2)) - R_3)(1 - \Phi[-z_1])(1 - \Phi[-z_3])
 \end{aligned} \tag{5}$$

$$\Phi(z_1) = \Phi\left(\frac{\mu_1 - L_1}{\sigma_1}\right) \tag{6}$$

$$\Phi(z_2) = \Phi\left(\frac{\mu_2 - L_2}{\sigma_2}\right) \tag{6}$$

$$\Phi(z_3) = 1 - \Phi\left(\frac{\mu_2 - U_2}{\sigma_2}\right) A$$

$$\begin{aligned}
 I = & \int_{L1}^{\infty} \int_0^{U2} ((C_1 + C_2)x_1 - C_2x_2) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 \\
 & - \int_{L1}^{\infty} \int_0^{L2} ((C_1 + C_2)x_1 - C_2x_2) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2
 \end{aligned} \tag{7}$$

$$I = \int_{L1L2}^{\infty} \int_0^{\infty} ((C_1 + C_2)x_1 - C_2x_2) f(X_1; \mu_1, \sigma_1^2) f(XA_2; \mu_2, \sigma_2^2) dx_1 dx_2 \tag{8}$$

$$- \int_{L1U2}^{\infty} \int_0^{\infty} (C_1 + C_2)x_1 - C_2x_2) f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2$$

from Golhar [1] and Al-Sultan [10] we have

$$\int_L^{\infty} xf(x, y)dy = \mu\Phi(z) + \sigma\phi(-z) \tag{9}$$

$$\int_{L1L2}^{\infty} \int_{L1L2}^{\infty} x_1 f(X_1; \mu_1, \sigma_1^2) f(X_2; \mu_2, \sigma_2^2) dx_1 dx_2 = \mu\Phi(z_1)\Phi(z_2) + \sigma\Phi(z_1)\phi(z_1) \tag{10}$$

Then

$$I = (C_1 + C_2)\mu_1\Phi(z_1)\Phi(z_2) + (C_1 + C_2)\sigma_1\Phi(z_1)\phi(z_2) - C_2\mu_2\Phi(z_1)\Phi(z_2) - C_2\sigma_1\Phi(z_1)\phi(z_2) + (C_1 + C_2)\mu_1\Phi(z_1)\Phi(z_3) + A(C_1 + C_2)\sigma_1\Phi(z_1)\phi(z_3) - C_2\mu_2\Phi(z_1)\Phi(z_3) - C_2\sigma_1\Phi(z_1)\phi(z_3) \tag{11}$$

Simplify (11)

$$I = \Phi(z_1)[((C_1 + C_2)\mu_1 - C_2\mu_2)(\Phi(z_2) + \Phi(z_3)) + ((C_1 + C_2)\sigma_1 - C_2\sigma_2)(\phi(z_2) + \phi(z_3))] \tag{12}$$

Let $E = E[P(X_1, X_2, \mu_1, \mu_2)]$ and from equation (5) we have

$$E = A\Phi(z_1) - I + E + E\Phi(z_1)A[\Phi(z_3) - \Phi(z_2)] - R_1\Phi(z_1) - R_2\Phi(z_1)\Phi(-z_2) + R_3\Phi(-z_3) \tag{13}$$

From 12 and 13 after simplification we get

$$E = \left(\frac{1}{\Phi(z_2) + \Phi(z_3)} \right) * A[A - ((C_1 + C_2)\mu_1 - C_2\mu_2)(\Phi(z_2) + \Phi(z_3)) - ((C_1 + C_2)\sigma_1 - C_2\sigma_2)(\phi(z_2) + \phi(z_3)) - R_1 - R_2\Phi(-zA_2) + R_3\Phi(z_3)] \tag{14}$$

5. SOLUTION AND RESULTS

Equation (14) is the final equation of the profit. The task now is to maximize the profit. Equation (14) can be maximized by differentiating it with respect to μ_1 and μ_2 , then equating the result by zero. This method needs lengthy work. However the numerical methods or search methods may be easier and give roughly the same result. In this work the solver macro of MS Excel program is used. Data appear in table 1. Figures 2, 3 show the solver window and its option. Figure 4 shows the function 3D graph generated using MathCAD.

The data of table 1 are for a joined tube-to-tubesheet using hydraulic expansion until certain wall reduction percentage. The process continues with the rolling stage to get more accurate tolerances and harder joint. The cost of work and rework are listed. In addition, the standard deviation and the upper and lower specified limit are listed. The final total expected profit ET was achieved by changing the value of μ_1 and μ_2 using solver macro to get the maximum profit.

Table 1. Constant, variable and Results

	A	B	C	D	E	F
1	Constant		Variable		Result	
2	L1	2	μ_1	2.0000	E1	10.4132
3	L2	6	μ_2	6.5157	E2	24.5228
4	U2	7	$\Phi(z_1)$	0.5000	E3	-0.3806
5	A	25	$\Phi(z_2)$	0.0428	E4	4.3731
6	C1	7	$1 - \Phi(z_3)$	0.0532	ET	304.8624
7	C2	2	$\phi(z_1)$	0.4987		
8	R1	6	$\phi(z_2)$	0.3036		
9	R2	4	$(-1)\phi(z_3)$	-0.3612		
10	R3	15				
11	σ_1	0.8				
12	σ_2	0.3				

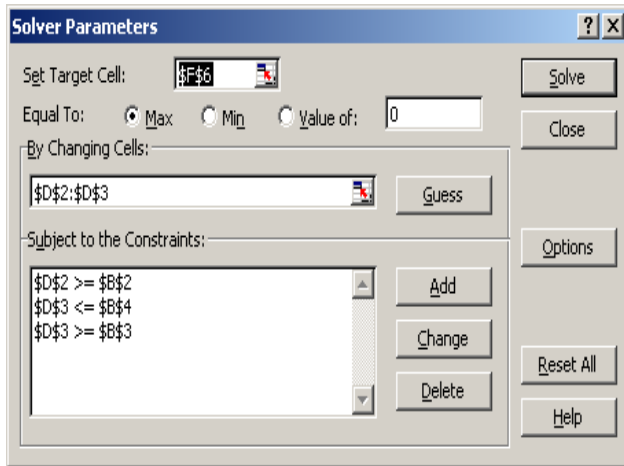


Fig. 2. Optimization macro (Solver)

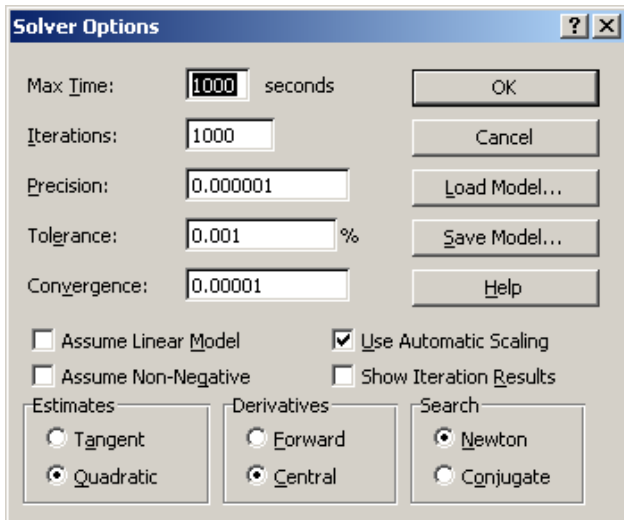


Fig. 3. Options of solver macro

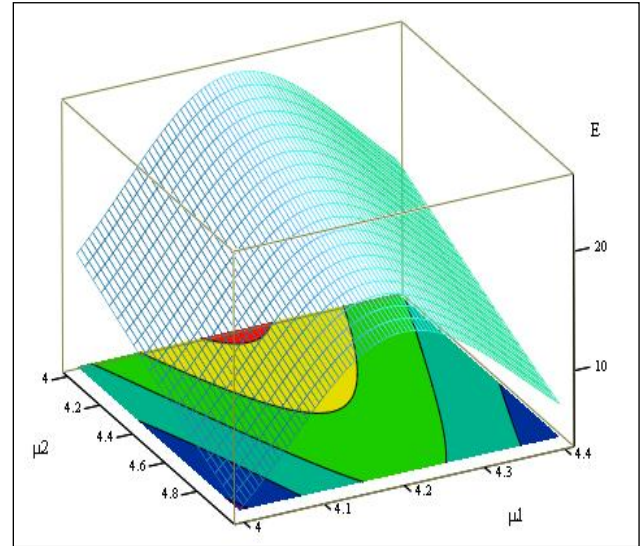


Fig. 4. 3D graph of the profit function

In a similar manner, we can minimize the function of the rework as

$$P[X_1 < L1] + (R1 + R2)P[X2 < L2] + R2P[X2 > U2] = R1\Phi(z1) + (R1 + R2)\Phi(z2) + R2(1 - \Phi(z3))$$

Plot of the function appears on Figure 5, and the table of solver optimization appears in table 2. In Figure 6, the disruptions of x_1 and x_2 are plotted by means of the data shown in table 1.

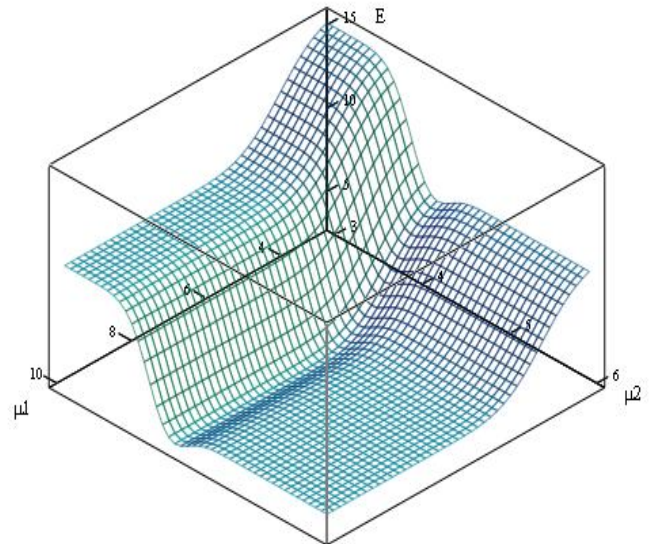


Fig. 5. Function of the rework

Table 2. Minimization of the cost function

	A	B	C	D	E	F
1	Constant		Variable		Result	
2	L1	2	μ_1	3.0000	$R1\Phi(z_1)$	6.0000
3	L2	6	μ_2	6.5862	$(R1+R2)(\Phi(z_2))$	1.1828
4	U2	7	$\Phi(z_1)$	0.1056	$R2(1-\Phi(z_3))$	1.2583
5	A	25	$\Phi(z_2)$	0.0254	Min(Total)	8.4411
6	C1	7	$1-\Phi(z_3)$	0.0839		
7	C2	2	$\phi(z_1)$	0.2283		
8	R1	6	$\phi(z_2)$	0.1971		
9	R2	4	$(-1)\phi(z_3)$	-0.5136		
10	R3	15				
11	σ_1	0.8				
12	σ_2	0.3				

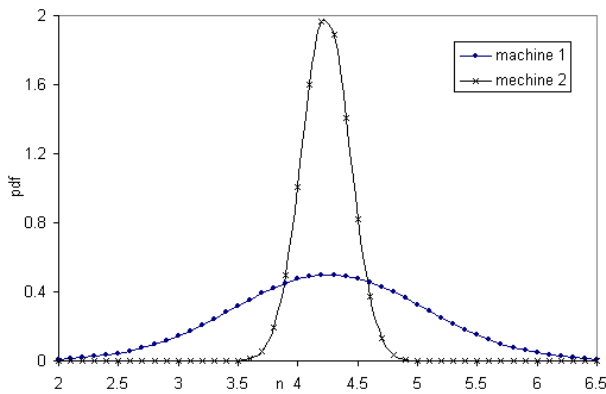


Fig. 6. Distributions of x1 and x2

6. CONCLUSION

In this paper, two machines in series were studied to find the optimum profit by controlling of the two dependent means. A practical example was introduced from industry, the hydraulic expansion of tube-to-tubesheet then the rolling expansion. The tube wall reduction after hydraulic expansion and after roller expansion is our controlling parameter to increase the profit. Results of the solver program is reasonable which give the profit of SR 5.8654 for the sailing price of SR 25.0 per process and with $\mu_1=4.26$ and $\mu_2=4.2227$ However when we try to minimize the rework cost, by the same technique, we got the cost SR 4.4454 with $\mu_1=4.5$ and $\mu_2=4$. The two results contradict each other but the first one with maximizing the profit is more generalized and takes on consideration the other parameters such as selling price and cost of work.

Contribution could be made by considering other parameters of hydraulic expansion such as expanding pressure, expansion length, or tube-tubesheet clearance and rolling expansion such as roller speed and lubrication. Furthermore, the use of other method joining the tube-to-tubesheet such as welding and explosive tube expansion could be used.

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