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Estimating CDMA Capacity and Performance in Mobile Network
(A Statistical Approach)

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ABSTRACT

This paper presents the modelling of Code Division Multiple Access (CDMA) capacity and performance in mobile networks. CDMA due to its capacity advantage serves as a tool for in ensuring quality of service (q_os) and proper network dimensioning. This is achieved by modelling telephone traffic using statistical approach to generate a CDMA blocking probability that is adapted into Erlang B formula for capacity calculations and matlab is used to model the blocking probability formula. The results showed that variation in network parameters affects CDMA capacity and performance and that CDMA has a huge capacity advantage over FDMA and TDMA.

Keywords: *FDMA, CDMA, TDMA, Traffic and network*

1. INTRODUCTION

The success of CDMA technology is due to the huge increase in capacity it offers. Its advantage over other multiple access schemes (like FDMA and TDMA) include higher spectral reuse efficiency, greater immunity to multipath fading, more robust handoff procedures, gradual overload capability and voice activity effects [Lee and Miller, 1995].

This paper is on teletraffic modelling of CDMA systems to enable capacity analysis of such systems leading to analytical tools aided by a software model to assist in performance analysis, capacity calculation, dimensioning and design of CDMA networks.

In CDMA cellular system, there is no fixed number of channels as we have in FDMA and TDMA system, because the capacity (allowable number of users) depends on the degree of interference that the network experience [Evans and Everrit, 1998]. Thus, the capacity is not hard limited but interference limited. Blocking of subscribers trying to make calls occurs when the reverse link multiple access interference power reaches a predetermined level that is set to maintain acceptable signal quality [Lee *et al*, 1998]. If the total user interference at a base station receiver exceeds some threshold ($Z > Z_0$), the system blocks (denies access) to the next user who attempts to place a call. The number of users for which the CDMA blocking probability equals a certain quality of service value is defined as Erlang capacity of the system and is related to an equivalent number of channels in an FDMA or TDMA cellular system [Evans and Everrit, 1995].

Because the number of users (call traffic) at a given time is random, and the interference power from a user is a random variable, the probability of blocking leads to an estimate of the average number of active users that is termed the Erlang capacity of the CDMA cell sector [Anyaegbu, *et al*, 2011]. The determination of Erlang capacity depends on the assumptions about the probability distributions of the call traffic and user interference [Evans and Everrit, 1998].

In CDMA however, the separation between traffic and transmission issues is not clear with capacity being determined by interference caused by all the transmitters in the network [Evans and Everrit, 1995].

The objective of this paper is to model the stochastic nature of call arrivals and departure in CDMA networks, evaluate and analyse the resulting compound sum using statistical methods.

2. MATERIALS AND METHODS

CDMA blocking model is developed as a tool for the capacity analysis of CDMA cellular networks. This blocking model is also modelled by software to allow for more flexibility in the analysis in this paper.

Whereas Erlang B model can be applied directly to the reverse links of FDMA and TDMA systems because the number of available channels for those systems is fixed, as have been pointed out in the introduction, the number of channels in a CDMA cellular system is not fixed, but fluctuates due to the fact that the reverse link (which the capacity depends on) is interference-limited [Viterbi and Viterbi, 1993]. Therefore, the mechanism for blocking in a CDMA cellular system needs to be examined before an application of the Erlang B theory can be made in its capacity analysis.

Blocking occurs when the reverse link multiple access interference power reaches a predetermined level that is set to maintain acceptable signal quality. If the total user interference at a base station receiver exceeds some threshold, the system blocks (denies access to) the next user who attempts to place a call [Evans and Everrit, 1995].

The number of users for which the CDMA blocking probability, denoted B_{CDMA}, equals a certain value (usually 1 or 2%) is defined to be the Erlang capacity of the system and is related to an equivalent number of channels in an FDMA or TDMA cellular system. Thus, the calculation of the CDMA blocking probability is based on an analysis of the other-user interference.

Because the number of users (call traffic) at a given time is random, and the interference power from a user is a Random Variable, the probability of blocking leads to an estimate of the average number of active users that is termed the Erlang capacity of the CDMA cell or sector. The determination of Erlang capacity depends on the assumptions about the probability distributions of the call traffic and user interference. In this paper, CDMA Erlang capacities are determined using two different assumed distributions for the total user interference power: Gaussian (main focus) and lognormal approximations.

2.1 Formulation of the Blocking Probability

Considering a single, isolated CDMA cell with M active users. The total reverse link signal-plus-noise power received at the base station can be written as

$$\underbrace{\alpha_{r1}P_1+\alpha_{r2}P_2+\ldots\ldots+\alpha_{rM}P_M}_{M\ \text{reverse link signals}} + \underbrace{(N_0W)_c}_{\text{noise power}} \qquad (1)$$

Where $\{\alpha_{ri}\}$ are random variables representing the reverse link voice activity, which have the experimental values given as $E\{\alpha_{ri}\} = \overline{\alpha_r} = 0.4$ and $E\{\alpha_{ri}^2\} = \overline{\alpha_r^2} = 0.31$, $\{P_i\}$ are the random signal powers for the M active users. The number of signals M is itself an RV, assumed to have a Poisson distribution, so that $E\{M\}=\bar{M} = Var\{M\}$

To a potential $(M + 1)$st reverse link user, the total power for the M active users and the thermal noise is interference power. Thus, we may write

$$I' \equiv I'_0W = \alpha_{r1}P1 + \alpha_{r1}P_2 + \ldots + \alpha_{rM}P_M + (N_0W)_c \qquad (2)$$

Where

$$I'_0 = \frac{I'}{W} = \frac{\alpha_{r1}P1 + \alpha_{r1}P_2 + \ldots + \alpha_{rM}P_M + N_0}{W} \qquad (3)$$

is the power spectral density level for the total received interference power.

Normalized by I'_0R_b, where R_b is the data bit rate, the total interference is characterized by the quantity

$$\frac{I'}{I'_0R_b} = \frac{W}{R_b} = \alpha_{r1}\frac{E_{b1}}{I'_0} + \alpha_{r2}\frac{E_{b2}}{I'_0} + \cdots + \alpha_{rM}\frac{E_{bM}}{I'_0} + \frac{N_0}{I'_0}\cdot\frac{W}{R_b} \qquad (4)$$

Where $P=E_bR_b$

$$\frac{W}{R_b} = Z + \frac{N_0}{I'_0}\cdot\frac{W}{R_b} = Z + \eta\frac{W}{R_b}$$

$$Z \triangleq \sum_{i=1}^{M} \alpha_{r1}\,\rho_i = \frac{W}{R_b}(1-\eta) \qquad (5)$$

$$\rho_i \triangleq E_{bi}/I'_0 \qquad (6)$$

An`d

$$\eta \triangleq \frac{N_0}{I'_0} \text{ (thermal noise)} \qquad (7)$$

is a parameter indicating the loading of the CDMA system and W/R_b is the spread-spectrum processing gain. Given the value of η, the quality of the channel that is available to the $(M + 1)$ st mobile user is characterized by the value of the random variable *Z;* if *Z* exceeds some threshold value, then the channel is effectively unavailable (blocked) to the $(M + 1)$ st user. In terms of the distribution of the random variable *Z*, the probability that the $(M + 1)$ st mobile CDMA user will be blocked is the probability that *Z* exceeds some threshold value Z_0, as a function of a threshold value of the interference parameter η_0

If a probability density function $p_z(x)$ is known or assumed for Z, then the evaluation of B_{CDMA} is simply a matter of integrating that pdf over the region defined by $Z > Z_0$:

$$B_{CDMA} = \int_{Z_0}^{\infty} dx\,p_z(x) \qquad (8)$$

The exact pdf of *Z* is not known, however, so an approximation is needed to compute B_{CDMA}.

The CDMA blocking probability can be manipulated to the form

$$B_{CDMA} = \Pr\{ Z > Z_0 = \frac{W}{R_b}(1-\eta_0)\}$$

$$B_{CDMA} = \Pr\{Z > Z_0\} = \Pr\left\{\frac{Z - E\{Z\}}{\sqrt{Var\{Z\}}} > \frac{Z_0 - E\{Z\}}{\sqrt{Var\{Z\}}}\right\} \quad (9)$$

$$= \Pr\{\sum_{i=1}^{M} \alpha_{ri}\,\rho_i > \frac{W}{Rb}(1-\eta_0)\}$$

The approximation methods to be considered in what follows are

- Gaussian approximation: Based on the fact that Z is a sum (Central Limit Theorem) we can write

$$Qz(x) \approx Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} e^{-t^2/2}\, dt \quad (10)$$

That is, the blocking probability can be calculated using

$$B_{CDMA} = Q\left(\frac{Z0 - E\{Z\}}{\sqrt{Var\{Z\}}}\right) \quad (11)$$

- Lognormal approximation: Based on the fact that the SNRs in the sum are lognormal, Z itself can be approximately characterized as a lognormal variable.

These approximation methods are based on identifying the actual mean and variance of Z with the mean and variance of a Gaussian RV and the mean and variance of a lognormal RV, respectively. Next, we find the mean and variance of Z and specify its relations to Gaussian and lognormal RVs.

2.2 Mean and Variance of *Z*

The form of the interference statistics Z is the weighted sum of the M *RVs* $\{\rho i, i = 1, 2, \ldots, M\}$:

$$Z = \propto_{r1} \frac{E_{b1}}{I_0'} + \propto_{r2} \frac{E_{b2}}{I_0'} + \ldots + \alpha_{rM} \frac{E_{bM}}{I_0'}$$

$$= \propto_{r1} \rho_1 + \propto_{r2} \rho_2 + \cdots + \propto_{rM} \rho_M \quad (12)$$

Propagation measurements in general indicate that received signal powers, when expressed in dB units, are nearly Gaussian. Therefore, the $\{\rho i\}$ above, when measured in dB units, are close to having a Gaussian probability distribution with median m_{dB} and standard deviation σ_{dB}:

$$\rho_i\,(\mathrm{dB}) = 10\,log_{10}\,\rho i = m_{dB} + \sigma_{dB} G_i, \quad G_i = G(0, 1) \quad (13)$$

Therefore, the RV ρ_i is lognormal and can be written

$$\rho_i = 10^{(m_{dB}+\rho_{dB}Gi)/10} = (e^{\ln 10})^{(m_{dB}+\rho_{dB}Gi)/10}$$

$$= e^{\beta(mdB+\sigma dBGi)}, \text{ using } \beta = (In\ 10)/10 \quad (14)$$

The median, mean, and mean square of ρ_i are assumed to be same for all i and are obtained as follows:

- Median

$$\frac{1}{2} \triangleq \Pr\{\rho i \le \rho med\} = \Pr\{e^{\beta\,(mdB+\sigma dBG)} \le \rho med\}$$

$$Pr\left\{G \le \frac{\frac{1}{\beta}\ln \rho_{med} - m_{dB}}{\sigma_{dB}} = G_{med} = 0\right\} \quad (15)$$

Thus $\qquad \rho_{med=e^{\beta m_{dB}}}$ (16)

$$E\{\rho i\} = E\{e^{\beta(mdB+\sigma dBG)}\} = e^{\beta mdB} E\{e^{\beta\sigma dBG}\}$$

- Mean

$$= \rho med\ E\{e^{uG}\}|u = \beta\sigma dB \quad (17)$$

Where

$$E\{e^{uG}\} = M_G(u) = e^{u^2/2} \qquad (MGF)$$

Thus

$$E\{\rho i\} = \rho med\ M_G\,(\beta\sigma dB) = \rho med e^{\frac{(\beta\sigma dB)^2}{2}} \quad (18)$$

- Means square

$$E\{\{\rho_i^2\} = E\left\{[e^{\beta(mdB+\beta\sigma dBG}\ ]^2\right\} = E\left\{e^{2\beta(mdB+\beta\sigma dBG)}\right\}$$

$$= e^{2\beta mdB}\ E\{e^{2\beta\sigma_{dB}G}\} = \rho_{med}^2 M_G\,(2\beta\sigma dB)$$

$$= \rho_{med}^2\ e^{1/2(2\beta\sigma dB)^2} = \rho^2 med\ e^{2(\beta\sigma dB)^2} \quad (19)$$

Because M is analogous to the number of calls in progress through a switch, which has a Poisson distribution, it is reasonable to postulate that the mean and variance of M are equal, as in the case of Poisson RV. The mean, mean square and variance of Z therefore are

$$E\{Z\} = E_M\{E\{Z|M\}\} = E_M\{\sum_{i=1}^{m} E\{\alpha_{ri}\rho_i\}\}$$

$$= E_M\{ M\,E\{\alpha_{ri}\rho_i\}\} = E\{M\}\,E\{\alpha_{ri}\rho_i\}$$

$$= \overline{M}E\{\alpha_{ri}\rho_i\} = \overline{M}\overline{\alpha_r}e^{\beta dmB+\frac{1}{2}\beta^2\sigma^2 dB} \quad (20)$$

$$E\{Z^2\} = E_M\{ E\{Z^2 \mid M\}\} = E_M\{\sum_{i=1}^{M}\sum_{j=1}^{M} E\{\alpha_{ri}\alpha_{rj}\rho_i\rho_j\}$$

$$= E_M\left\{ME\{\alpha_{ri}^2\rho_i^2\} + M(M-1)[E\{\alpha_{ri}\rho_i\}]^2\right\}$$

$$= E_M\ \{M\{E\{\alpha_{ri}^2\rho_i^2\} + [E\{\alpha_{ri}\rho_i\}]^2\}\,\overline{M^2}\,E\{\alpha_{ri}\rho_i\}]^2\}$$

$$= \overline{M}Var\{\alpha_{ri}\rho_i\} + \overline{M^2}\,[E\{\alpha_{ri}\rho_i\}]^2 \quad (21)$$

$$Var\{Z\} = E\{Z^2\} - [E\{Z\}]^2$$

$$= \bar{M}\, Var\{\alpha_{ri}\rho_i\} + \left(\overline{M^2} - \bar{M}^2\right)[E\{\alpha_{ri}\rho_i\}]^2$$

$$= \bar{M}\, Var\{\alpha_{ri}\rho_i\} + Var\{M\}\, [E\{\alpha_{ri}\rho_i\}]^2$$

$$= \bar{M}\, Var\{\alpha_{ri}\rho_i\} + [E\{\alpha_{ri}\rho_i\}]^2\},$$
$$because\ Var\{M\} = \bar{M}$$

$$= \bar{M}\, E\{\alpha_{ri}^2\rho_i^2 = \bar{M}\, \overline{\alpha_r^2} e^{2BmdB + 2\beta^2\sigma_{dB}^2} \quad (22)$$

The interference due to mobiles in other cells can be accounted for by using first- and second-order *frequency reuse factors* $F = 1 + \xi$ and $F^1 = 1 + \xi^1$, respectively, where

$$\xi = \frac{Total\ other\ cell\ received\ (median) power}{Total\ same - cell\ received\ (median) power}$$

And (23)

$$\xi' = \frac{Total\ other\ cell\ mean\ square\ received\ power}{Total\ other\ cell\ mean\ square\ received\ power}$$

A typical analytical value of $\xi^1 = 0.086$ and $\xi = \xi^1 = 0.55$.

With this method of accounting for interference from other cells, the mean and variance for Z becomes

$$E\{Z\} = = \bar{M} \propto r\, \rho_{med} e^{\left(\frac{1}{2}\right)\beta^2\sigma_{dB}^2} \,.(1 + \xi) \quad (24)$$

$$\text{and } Var\{Z\} = \bar{M}\, \overline{\propto_r^2}\, \rho_{med}^2 e^{2\beta^2\sigma_{dB}^2} \,.(1 + \xi') \quad (25)$$

2.3 Approximations for the Probability Distribution of *Z*

Because the *M* RVs {ρi, i=1, 2,…, M} are lognormal RV, the interference statistic Z is the weighted sum of lognormal RVs. One approximation for the distribution of *Z i*s based on assuming that the summing of variables to produce *Z* causes its distribution to converge to a Gaussian distribution according to the CLT. Another approach is to assume that the lognormal character of the {ρi} makes *Z* have an approximately lognormal distribution. Thus:

$$Z = \propto_{r1} \rho_i + \propto_{r2} \rho_2 + \cdots + \propto_{rM} \rho_M$$

$$= \propto_{r1} e^{\beta(mdB + \sigma dBG_1)} + \propto_{r2} e^{\beta(mdB + \sigma dBG_1) +} + \propto_r{}^{Me^{\beta(mdB + \sigma dBG_M)}}$$

$$\approx \begin{cases} m_M + \sigma_M G & \text{Gaussian approximation} \\ e^{m_M + \sigma_M G}, & \text{Lognormal approximation} \end{cases} \quad (26)$$

2.4 CDMA Blocking Probability Formula for Gaussian Assumptions

Under the Gaussian assumption, the mean and variance of *Z* are identified as the mean and variance of a Gaussian RV, $m_M + \sigma_M G$:

$$B_{CDMA} = \Pr\{Z > Z_0\}$$

$$\approx \Pr\{m_M + \sigma_M G > Z_0\}$$

$$= \Pr\{ G > \frac{Z_0 - M_M}{\sigma M} \} = \sigma \left(\frac{Z_0 - E\{Z\}}{\sqrt{Var\{Z\}}}\right) \quad (27)$$

Substituting the expressions for the mean and variance of *Z*, we obtain a general expression for the CDMA blocking probability under the Gaussian approximation for the interference statistic, given by

$$B_{CMDA} = Q\left(\frac{\frac{W}{R_b}(1-\eta_0) - \bar{M}\overline{\propto_r}\rho_{med} e^{1/2\beta\sigma_{dB}^2(1+\xi)}}{\sqrt{\bar{M}\overline{\alpha_r^2}\rho_{med}^2 e^{2\beta^2\sigma_{dB}^2(1+\xi')}}}\right) \quad (28)$$

In which the the Erlang capacity is $\bar{M}$.

The interference parameter η is related to cell loading parameter by the relation

$$\eta = 1 - X \quad (29)$$

Thus the threshold η_0 can be converted to loading threshold as $X_0 = 1 - \eta_0$.

Using typical numerical values for the parameters with the following set of values [Jhong S. L. *et al*, 1998]:

σ_{dB}=2.5dB; m_{dB}=7dB; W=1.2288MH;

R_b=9.6kbps; X_0= 0.9; $\overline{\propto_r}$ =0.4; $\overline{\alpha_r^2}$ =0.31;

$\beta = (In\ 10)/10$, $\rho_{med} = e^{\beta mdB}$

The B_{CDMA} can be reduced to

$$B_{CDMA} = Q\left(\frac{115.2 - 2.37(1+\xi)\bar{M}}{3.89\sqrt{(1+\xi')\bar{M}}}\right) \quad (30)$$

This is the blocking probability of CDMA network as a function of Erlang capacity $\bar{M}$. ξ, ξ' can assume typical experimental values such as $\xi = \xi'$ =0.55.

2.5 CDMA Blocking Probability Formula for Lognormal Assumptions

Under lognormal assumption, the mean and variance of *Z* are identified as the mean and variance of lognormal RV, ζ, where

$$\zeta = e^{m_M + \sigma m_G} \quad (31)$$

The mean, mean square, and variance of ζ are given by

$$E\{\zeta\} = e^{m_M} E\{e^{\sigma_M G}\} = e^{m_M + \frac{1}{2}\sigma^2 M} \quad (32)$$

$$E\{\zeta^2\} = e^{2m_M} E\{e^{2\sigma_M G}\} = e^{2m_M + 2\sigma_M^2}$$

$$Var\{\zeta\} = E\{\zeta^2\} - [E\{\zeta\}]^2 = e^{2m_M + \sigma_M^2}[e^{\sigma_M^2} - 1] \quad (33)$$

By setting E{*Z*} = E{ ζ } and Var{*Z*} = Var{ ζ }, where the mean and variance of *Z* are given in (23) and (24). We solve for m_M and σ^2_M:

$$\bar{M}\,\overline{\alpha_r}\, e^{\beta m_{dB} + \frac{1}{2}\beta^2\sigma_{dB}^2}(1+\xi) = e^{m_M + \frac{1}{2}\sigma_M^2}$$

And

$$\bar{M}\,\overline{\alpha_r^2}\, e^{2\beta m_{bB} + 2\beta^2\sigma_{dB}^2}(1+\xi^1) = e^{2m_M + \sigma_M^2}\,[e^{\sigma^{2m}} - 1] \quad (34)$$

The solution is

$$\sigma_M^2 = ln\left[\frac{\overline{\alpha_r^2}\,(1+\xi')e^{\beta^2 \alpha_{dB}^2}}{\bar{M}\,(\overline{\alpha_r})^2(1+\xi)^2} + 1\right] \quad (35)$$

And

$$m_M = ln\,[\bar{M}\,\overline{\alpha_r}\,(1+\xi)] + \beta m_{dB} + 1/2\,(\beta^2\sigma_{dB}^2 - \sigma_M^2) \quad (36)$$

Using these parameters, the blocking probability formula for the lognormal approximation is

$$B_{CDMA} = Pr\{Z > Z_O\} \approx Pr\{e^{m+\sigma_M G} > Z_0\} = Q\left(\frac{lnZ_{0} - m_M}{\sigma_M}\right) \quad (37)$$

Substituting the expressions for m_M and σ_M, we obtain general expressions for the CDMA blocking probability under the lognormal approximation for the interference statistic, given by

$$B_{CDMA} = Q\left(\frac{ln\left[\frac{W}{R_b}(1-\eta_0)\right] - ln[\bar{M}\,\overline{\alpha_r}\,(1+\xi)] - \beta m_{dB}}{\sqrt{ln\left[\frac{\overline{\alpha_r^2}(1+\xi')e^{\beta^2\sigma_{dB}^2}}{M(\overline{\alpha})^2(1+\xi)^2}+1\right]}}\;\frac{\frac{1}{2}\left\{\beta^2\sigma_{dB}^2 - ln\left[\frac{\overline{\alpha_r^2}(1+\xi')e^{\beta^2\sigma_{dB}^2}}{M(\overline{\alpha})^2(1+\xi)^2}+1\right]\right\}}{\sqrt{ln\left[\frac{\overline{\alpha_r^2}(1+\xi')e^{\beta^2\sigma_{dB}^2}}{M(\overline{\alpha})^2(1+\xi)^2}+1\right]}}\right) \quad (38)$$

in which the Erlang capacity is $\bar{M}$. Because the interference parameter η is η = 1 – X, we may convert the threshold η_0 into a loading threshold $X_0 = 1 - \eta_0$.

Thus with this, we have derived Erlang capacity formulas for CDMA cellular system under two separate approximations for the interference statistic: Gaussian approximation, by invoking the CLT, and lognormal approximations, on the assumption that the sum of M lognormal RVs is also a lognormal RV. The blocking probabilities derived is expressed as a function of the interference parameter threshold η_0 and the cell loading threshold X_0.

The blocking probabilities will be plotted as a function of Erlang capacity $\bar{M}$ parametric in median values of $m_{dB}=E_b/N_0$ taking values of 5, 6, and 7dB. Plots showing other variants will also be taken to provide ingredients for analysis.

3. RESULTS

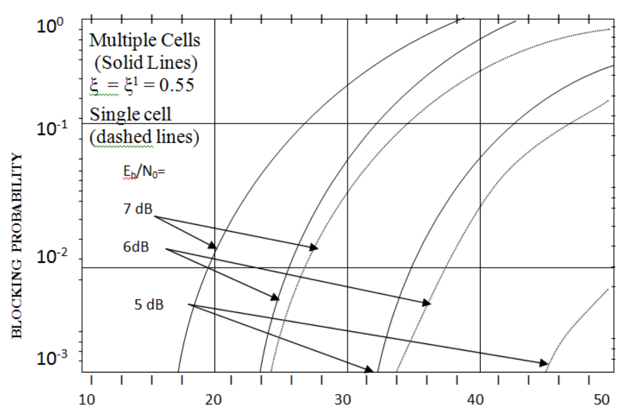

Figure 1 CDMA blocking probability (Gaussian approximation) versus average number of mobile users, SNR requirement varied

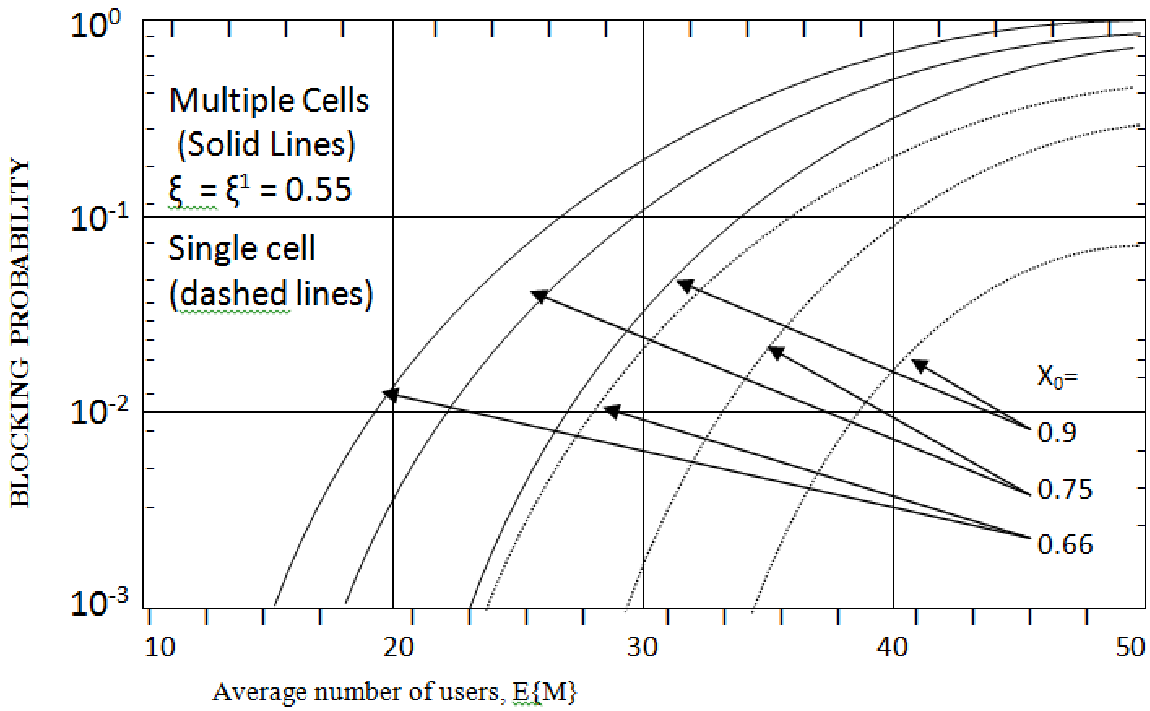

Figure 2 CDMA blocking probability (Gaussian approximation) versus average number of mobile users, loading threshold varied

$$\overline{\alpha_r} = 0.4, \quad \overline{\alpha_r^2} = 0.31, \quad X_0 = 0.9, \frac{W}{R_b} = 128$$

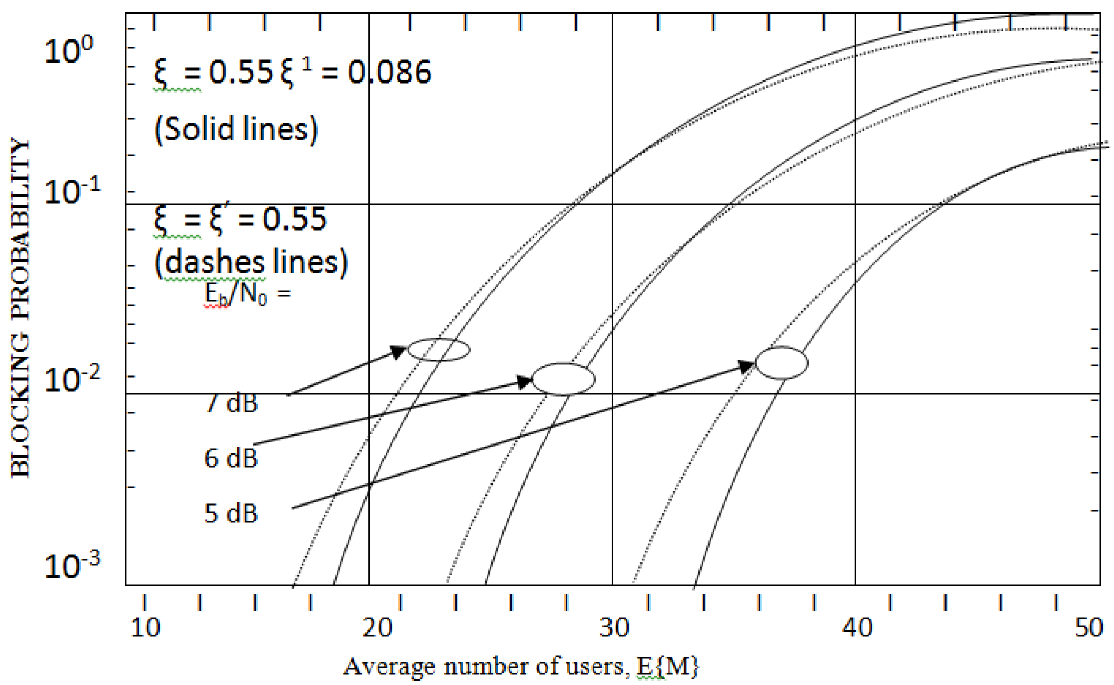

Figure 3 CDMA blocking probability (Gaussian approximation) versus average number of mobile users, reuse fractions varied

$$\overline{\alpha_r} = 0.4, \bar{\alpha} = 0.31, X_0 = 0.9, W/R_b = 128$$

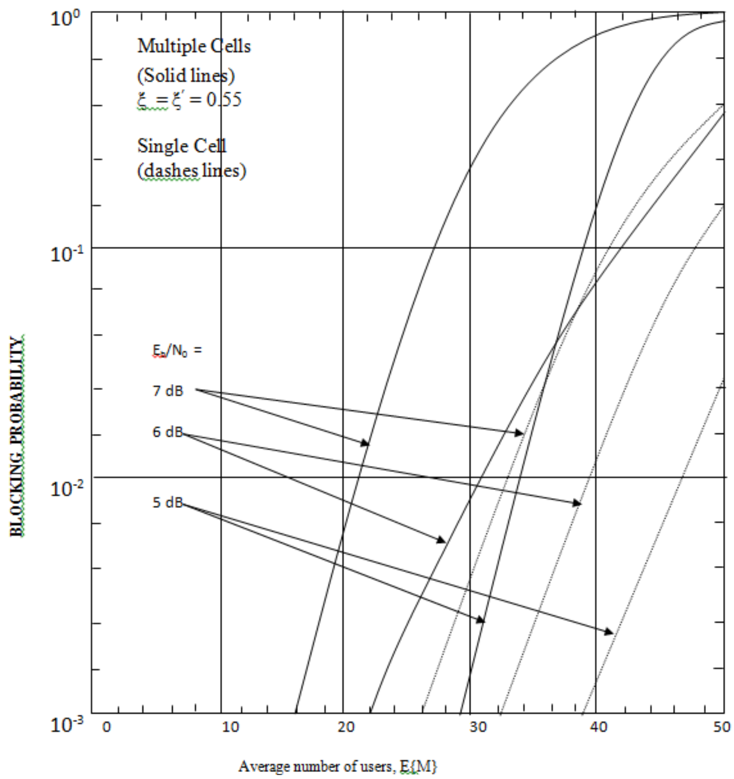

Figure 4 CDMA blocking probability (lognormal approximation) versus average number of mobile users, SNR requirement varied.

$$\overline{\alpha_r} = 0.4, \overline{\alpha_r^2} = 0.31, E_b/N_0 = 6dB, W/R_b = 128$$

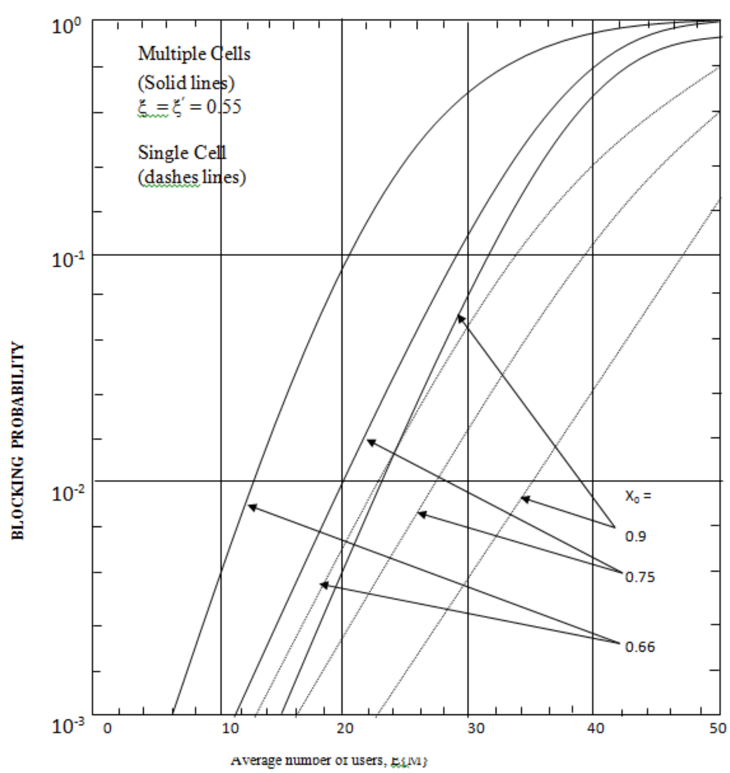

Figure 5 CDMA blocking probability (lognormal approximation) versus average number of mobile users, loading threshold varied. $\overline{\alpha_r} = 0.4, \overline{\alpha_r^2} = 0.31, X_0 = 0.9dB, \quad W/R_b = 128$

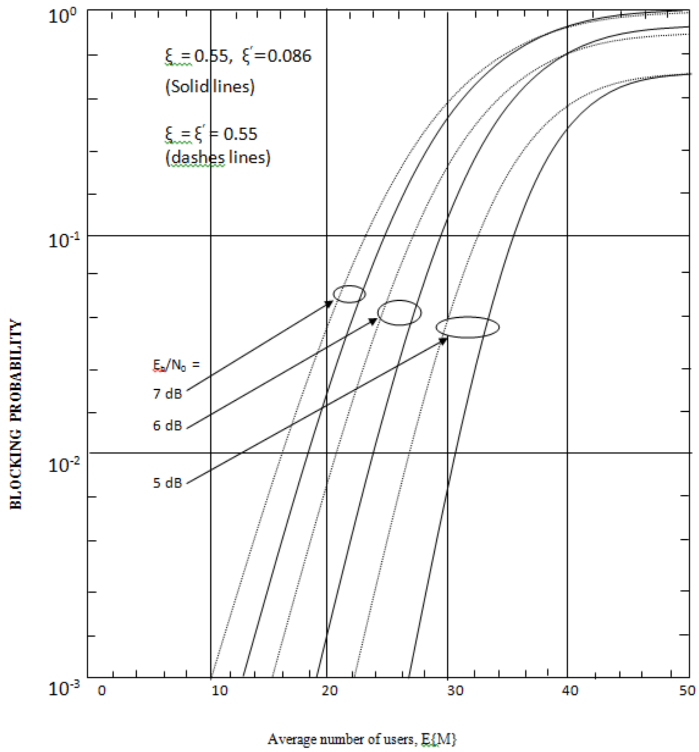

Figure 6 CDMA blocking probability (lognormal approximation) versus average number of mobile users, reuse fraction varied.

4. DISCUSSION

4.1 Graphical Plots and Analysis of CDMA Blocking Probability Formula for Gaussian Assumptions

Using (28), we take a plot of B_{CDMA} versus $\bar{M}$ for a single cell $(\xi = \xi' = 0)$ and for multiple cells $(\xi \quad = \xi'$=0.55) and the following typical parameter values:

σ_{dB}=2.5dB; m_{dB}=7dB; W=1.2288MH; R_b=9.6kbps; X_0= 0.9; $\overline{\propto_r}$ =0.4; $\overline{\alpha_r^2}$ =0.31

The plots are parametric in $m_{dB} = E_b/N_0$, which takes the values 5,6 and 7dB

We observe that the value of E_b/N_0 needed for link operations affects the average number of users that can be accommodated at a given level of blocking. Raising the E_b/N_0 requirement increases the blocking probability for the same value of M or decreases the capacity for the same probability. For example, when the blocking probability is chosen to be B_{CDMA} = 10^{-2} = 0.01 = 1%, Figure 1 shows for multiple cells that the corresponding value of the Erlang capacity *M* is 18 for m_{dB} = 7 dB, 24 for m_{dB} = 6dB, and 33 for m_{dB} = 5 dB.

If we denote the Erlang capacity for a single cell by $\bar{M}_c$ and the capacity for multiple cells by $\bar{M}$, for B_{CDMA} = 1.0% and E_b/N_0 = 6 dB, we then read from figure 1 that $\bar{M}_c$ = 37.5 and $\bar{M}$ = 24.1. The ratio of these values is 37.5/24.1 = 1.56, a value that is approximately equal to the assumed value of the reuse factor, F = 1 + ξ = 1.55. This is consistent with the definition of the reuse factor.

Assuming that m_{dB} = E_b / N_0 = 6 dB, the effect of varying the loading threshold X_0 on B_{CDMA} and $\bar{M}$ is illustrated in Figure 2, in which X_0 takes the values X_0 = 0.66, 0.75, and 0.9. These values correspond to the multiple access interference power being twice, three times, and nine times as strong as the thermal noise. Raising the loading threshold has the effect of relaxing the system requirements, and is seen in Figure 2 to result in either a decrease in the blocking probability for the same value of $\bar{M}$, or an increase in $\bar{M}$ for the same value of $B_{CDMA.}$ If we substitute specific numerical parameter values into the general expression (28), such as σ_{dB} = 2.5 dB, m_{dB} = 7dB, *W = 1.2288 MHz, R_b = 9.6kbps, X_0 = 0.9,* $\overline{\alpha_r}$ = 0.4, $\overline{\alpha_r^2}$ = 0.31, and we obtain

$$B_{CDMA} = Q\left(\frac{115.2-2.37\,(1+\xi)\bar{M}}{3.89\sqrt{(1+\xi)\bar{M}}}\right) \qquad (39)$$

Because Q(0) = 0.5, we infer from (39) that the blocking probability is 50% when $\bar{M}$ = *a/b*. This high a blocking probability is of course unacceptable, so we know that an acceptable value of blocking probability is realized only when $\bar{M}$ is much less than *a/b*. It is interesting therefore to note by comparing (28) and (39) that the upper limit on $\bar{M}$ based on having a small blocking probability is

$$\bar{M} < \frac{a}{b} = \frac{\frac{w}{R_b} X_0}{\overline{\alpha_r}\, Pmed\, e^{\frac{1}{2}\beta^2\sigma^2} dB\,(1+\xi)} = \frac{PG}{E_b/N_0} \cdot \frac{1}{\propto_r F} \cdot \frac{X_0}{e^{\frac{\beta^2\sigma_{dB}^2}{2}}} \quad (40)$$

The ideal CDMA capacity was shown as

$$M = \frac{PG}{E_b N_0} \cdot \frac{1}{\alpha_r F} = \frac{PG}{E_b N_0} \cdot \frac{1}{\propto_r} \cdot Fe \quad (41)$$

which is valid under perfect power control and omnidirectional cell antenna assumptions. Note that under the conditions of perfect power control (σ_{dB} = 0 dB) and 100% cell loading in the ideal situation (X_0 = 1), then the Erlang capacity bound in (40) is equal to the ideal capacity in (41).

4.2 Sensitivity To ξ'

The sensitivity of the CDMA blocking probability to the value of the "second order" reuse fraction ξ is considered in Figure 3, in which ξ = 0.55 and E_b / N_0 = 5, 6, and 7 dB. The B_{CDMA} versus M curves for ξ' = 0.55 are seen to be slightly to the left of those for ξ' = 0.086 for small blocking probabilities. This indicates that the terms in the analysis that involve the mean square power are not critical and that use of assumption that ξ' = ξ does not incur a significant loss in accuracy, provided that the blocking probability is taken to be greater than 1%.

4.3 Graphical Plots and Analysis of CDMA Blocking Probability Formula for Lognormal Assumptions

Plots of equation (38) for B_{CDMA} versus $\bar{M}$ are shown in Figure 4 for a single cell ($\xi = \xi'$ = 0) and for multiple cells, using $\xi = \xi'$ = 0.55 and the parameter values σ_{dB} = 2.5dB, *W* = 1.2288 MHz, R_b = 9.6 kbps, X_0 = 0.9, $\overline{\alpha_r}$ = 0.4, and $\overline{\alpha_r^2}$ = 0.31. The plots are parametric in m_{dB} = E_b / N_0, which takes the values 5, 6, and 7 dB.

As for the Gaussian approximation in Figure 1, we observe that the value of E_b /N_0 needed for link operations greatly affects the value of $\bar{M}$ for a given value of B_{CDMA}. The higher the E_b/N_0, the lower the $\bar{M}$ value, for given blocking probability. For example, when B_{CDMA}= 10^{-2} = 0.01 = 1%, figure 4 shows for multiple cells that the corresponding value of the Erlang capacity $\bar{M}$ is 16 for E_b /N_0 = 7dB, 22 for E_b /N_0 = 6dB, and 29 for E_b /N_0 = 5 dB.

In Figure 5, the cell loading threshold X_o is varied for the lognormal approximation and the case of Eb/N_0 = 6dB. Again we see that raising or lowering X_0 has a significant effect on the Erlang capacity for a given value of the blocking probability. The amount of increase or decrease in Erlang capacity is greater than the amount of increase or decrease in Xo. For example, raising X_o by 20% from 0.75 to 0.90 increases the value of Erlang capacity at B_{CDMA}= 1% from about 16 to 22, or about 35%, indicating the high sensitivity of the Erlang capacity to the cell loading. Therefore, the threshold value of cell loading should be chosen very carefully. The sensitivity of the CDMA blocking probability to the value of the second-order reuse fraction ξ' under the lognormal approximation is shown in Figure 6, in which the first order reuse fraction ξ = 0.55 and E_b /N_0 = 5,6, and 7dB are used.

4.4 Erlang Capacity Comparisons of CDMA, FDMA, AND TDMA

Having found the equivalent number of channels in a CDMA system that corresponds to a value of blocking probability, we can now compare the capacities of CDMA, FDMA, and TDMA cellular systems. The Erlang capacities and equivalent values of $\bar{M}$ were read from the curves in the Figure 3, for the case of (ξ, ξ') = (0.55, 0.086), and the values of N are based on the Erlang B table. For example, for E_b/N_0 = 7 dB and *B* = B_{CDMA} = 1%, from Figure 3, we read the Erlang capacity $\bar{M}$ = 20 Erlangs. Now, for $\bar{M}$ = A, the offered load, we need to find the equivalent number of channels N that satisfies (41). The number of channels in Erlang B table is for a CDMA cellular system in one 1.25-MHz band (one Frequency Assignment). Assuming a noncontiguous 12.5-MHz cellular band, the CDMA system can use as many as nine 1.25-MHz FAs.

Thus, to compare the effective number of CDMA channels available in a particular sector with the number of channels available in the AMPS (FDMA) and TDMA (IS-54) cellular systems; the numbers for N must be multiplied by the number of FAs, which is nine. After this multiplication is done, the corresponding Erlang capacity can be obtained from an Erlang B Table or calculation by identifying the offered load with a multi-FA CDMA Erlang capacity denoted by $M_{multi-FA}$. Let us consider a numerical example. For B_{CDMA} = 1%, ξ = 0.55, ξ' = 0.086, and E_b / N_0 = 7 dB, we have N = 30 from Erlang table. Thus, the effective number of CDMA channels in a 9-FA system is N^1 = 30 x 9 = 270. We now need to find the offered load A = $\dot{M}_{multi\text{-}FA}$ corresponding to N^1 = 270 and 1% blocking. The offered load giving this numerical value according to the Erlang B table, with $\dot{M}$ replaced by $\dot{M}$multi-FA, is calculated to be 248 Erlangs. The offered load for an AMPS sector, which supports 19 channels, gives 12.3 Erlangs for a blocking probability of 2%, as shown in Erlang B table. For a single sector of an IS-54 TDMA system, which supports 19 x 3 = 57 channels, gives 46.8 Erlangs for the identical blocking probability of 1%. Thus, using the notation of $M_{CDMA} \triangleq \dot{M}_{multi\text{-}FA}$, the ratios of Erlang capacities are computed to be

$$\frac{M_{CDMA}}{A_{AMPS}} = \frac{248}{12.3} = 20.2 \text{ (CDMA advantage over AMPS) (42)}$$

$$\frac{M_{CDMA\square}}{IS-54} = \frac{248}{46.8} = 5.3 \text{ (CDMA advantage over IS-54) (43)}$$

Therefore, over a 12.5MHz band with nine FAs, the total number of Erlangs of traffic that a CDMA sector can service is over twenty times that of an AMPS (FDMA) sector and over five times that of an IS-54 (TDMA) sector.

To obtain this gain in capacity, however, requires careful implementation of sound engineering design principles peculiar to the spread-spectrum waveform techniques that are employed by the CDMA system.

5. CONCLUSION

The stochastic nature of call arrivals and departures were characterized using statistical means. Blocking occurred when the reverse link multiple access interference power reached a predetermined level that is set to maintain acceptable signal quality. When the total user interference at a base station receiver exceeded the set threshold, the system blocked the next user attempting to place a call. The number of users for which the CDMA blocking probability equaled 1% as chosen was taken to be the Erlang capacity of the network. Thus, a new CDMA blocking probability model is developed that enabled the estimation and analysis of Erlang capacity of CDMA networks. Graphic results for the blocking model generated showed the effect of variations in interference parameters on CDMA capacity. The Erlang capacity from the model was adapted into Erlang B formula to estimate capacity in terms of channels, and the number of subscribers a typical CDMA sector could accommodate.

Comparative capacity analysis showed that CDMA has a huge capacity advantage over TDMA and FDMA.

RECOMMENDATION

The following recommendation can make the work in this paper of immense practical usefulness. The model can be built into a handheld instrument that can be readily usable by network operators for planning and design.

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