

## Issues in Solving Vehicle Routing Problem with Time Window and its Variants using Meta heuristics - A Survey

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### ABSTRACT

Vehicle Routing Problem with Time Window (VRPTW) is an extension of classical Vehicle Routing Problem (VRP). It is NP hard, combinatorial optimization problem. The optimization method helps in finding the solution that minimizes the total cost. Recent meta-heuristics like simulated annealing is not an efficient and effective technique to solve VRPTW. Several hybrid meta-heuristics were proposed but they are complex. Also, some real life problems cannot be modeled exactly as VRPTW, but it's variant. In this paper, we present the issues related to meta heuristics in VRPTW and also highlights the remedies for future research.

**Keywords:** optimization, vehicle routing problem, time windows, meta heuristics

### 1. INTRODUCTION

Vehicle routing problem (VRP) is one of the most challenging areas of combinatorial optimization and it's a name given to broad class of problems. VRP is a generalization of Traveling Salesman Problem (TSP) [16]. Introduction of different capacities for each vehicle and service requirements for customers to multiple travelling salesman problem leads to VRP. More than 50 years ago Danting and Ramser [4] introduced the VRP and till now many variants of VRP have been proposed. Attachment of time span or a time window to each customer in VRP forms vehicle routing problem with time windows (VRPTW) [6, 7, 15]. In addition to the capacity constraint, now vehicle must visit the customer within a certain time limit. A vehicle may arrive, before time window opens, but in that case it has for service till the window open. However, it cannot arrive after the closing of time window.

Solving the VRP [1, 5, 12, 14] is a key to efficient transportation management and supply chain coordination. It can be defined in a broad terms as follows. Given a number of identical vehicles with a given capacity, all are located at a central depot. They are required to service a set of customers having demand for goods. Each customer has fixed distance from the depot i.e. their travel costs are provided. The goal is to design a least cost route such that all customers are visited once and only once with all constraints preserved.

VRP is a NP hard problem. Therefore, many methods some exact and approximate (heuristics) were developed to solve the instances. Exact methods [17] cannot solve instances with more than 100 customers in a reasonable time. One, therefore, has to resort to approximate methods

[17]. In approximate methods, guarantee of finding optimal solutions are scarified for the sake of getting good solution in a reduced amount of time. Meta Heuristics [3, 8, 13], a new kind of approximate methods, try to explore the search space efficiently and effectively by combining heuristics in a large framework. Meta heuristics combines Meta and heuristics. Meta means beyond, in an "upper level" and heuristics means "to find".

In section 2 VRPTW is introduced. Section 3 highlights the related work. Open research areas are introduced in section 4. Section 5 will present future work. Finally some concluding remarks are given in section 6.

### 2. VEHICLE ROUTING PROBLEM WITH TIME WINDOWS (VRPTW)

VRPTW is an acronym used for vehicle routing problem with time windows. In this scenario, a vehicle is required to visit a particular customer within a specified time window. Fig 1 illustrates a graphical model of a simple VRPTW problem.

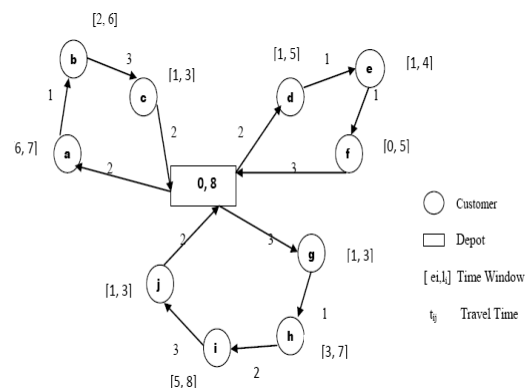


Fig 1: Vehicle routing problem with time window

The terms and notations used in the graph for VRPTW [18, 15] are given as follows:

- **Graph:** The VRPTW is represented by a connected undirected graph  $G=(V,E)$  consisting of  $V=\{v_0,v_1,\dots,v_n\}$  vertices, where  $v_0$  represents the depot and  $v_1,v_2,\dots,v_n$  represents the customers. The set  $E=\{(v_i,v_j) : v_i,v_j \in V, i < j\}$  represents the all possible connections between the customers including depot.
- **Depot:**  $v_0$  is a vertex from where a route starts and terminates. Arc of every route originates and terminates at  $v_0$  and a time window is attached with it.
- **Customers:** The problem is defined in terms of  $N$  customers represented by  $v_1,v_2,\dots,v_n$  which has non negative demand,  $q_i \geq 0$ .
- **Vehicles:** VRPTW is defined in terms of vehicles that serve the customers. These vehicles have capacity  $Q$  and can serve unlimited customers provided  $q_i \leq Q$  and must reach the depot back within the time specified.
- **Traveling Cost:** Traveling cost between customers  $i$  and  $j$  is represented by  $c_{ij}$ . It satisfies the triangular inequality  $c_{ij} + c_{jk} \leq c_{ik}$ . It is calculated using Euclidian distance between two customers.
- **Routes:** Route is a sequence of nodes each starting and ending at the depot. It is commonly represented as  $r = \{v_0, v_1, v_2, \dots, v_n, v_0\}$ . The size of route  $|r|$  is the number of customers in the route.
- **Route Cost:** Route cost, denoted by  $t(r)$ , is the sum of cost of visiting all the customers in the route including the cost of visiting to and from depot i.e.  $t(r) = cv_{01} + cv_{12} + \dots + cv_{n0}$ .
- **Time Window:** The customers and depot has time windows, denoted by  $[e_i, l_i]$ , where  $e_i$  is the earliest arrival time also refereed as opening time and  $l_i$  is the latest arrival time also called as closing time. A vehicle must arrive at the customer before latest time. It can arrive before  $e_i$ , but customer will not be served. Time window with depot  $[e_0, l_0]$  represents the time when all vehicles leave the depot and must come back to the depot.
- **Decision Parameters:** The model has two decision parameters  $x$  and  $s$ . For each arc  $(i,j)$ , where  $i \neq j, i \neq n, j \neq 0$ , and each vehicle  $k$ , we define decision variable  $x_{ij}^k$  as

$$x_{ij}^k = \begin{cases} 1, & \text{if vehicle } k \text{ drives directly from vertex } i \text{ to } j \\ 0, & \text{otherwise} \end{cases}$$

The decision variable  $s_{ik}$  is defined for each customer  $i, i \in C$  and each vehicle  $k, k \in V$  and denotes the time vehicle  $k$  starts to service customer  $i$ . If vehicle  $k$  does not service  $i$ , then  $s_{ik}$  has no meaning and therefore  $s_{0k} = 0$ , for all  $i$  for such  $k$ .

The aim of the VRPTW is to service all the  $C$  customers using the  $V$  vehicles such that the following objectives are met and the following constraints are satisfied.

**Objectives:**

- Minimize the total number of vehicles used to service the customers.
- Minimize the distance travelled by all the vehicles.

**Constraints:**

- Each customer is serviced only once.
- Time window constraints need to be met.
- Total demand of any route does not exceed capacity constraints of vehicle.
- Each vehicle must start and end at the depot.

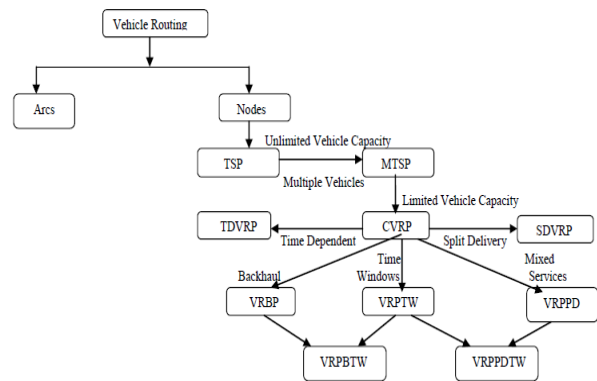


Fig 2: Variants of VRP

**3. LITERATURE REVIEW**

Vehicle routing problem [5, 6, 7, 12, 14, 15] lies at the heart of distribution management. Its solution is of most important to the companies that are engaged in transportation of goods and people. Several variants of VRP exists because conditions changes from one environment to other. Basically routing and scheduling problems are classified as Arc and Node routing problems. Vehicle routing problem falls under node routing where service demand is associated with nodes. Fig 2 shows various variants of VRP along with the various constraints.

As it is intended to concentrate on VRPTW, literature survey will focus on VRPTW and its solution methods. In VRPTW each customer has an associated, fixed time interval during which service must be provided. A lot of work has been done in the field of VRPTW [2, 3]. As VRPTW is NP hard, techniques like exact optimization and heuristics approaches have been used for solving VRPTW. In [19], author categorized vehicle routing methods into three generations. The first generation was

simple heuristics, based on local search and sweep that terminates once local optima has been found. Second generation, mathematical programming based heuristic, were near-optimization algorithms that are very different from normal heuristics generating superior results than prior generation. Third generation that is currently at the center of research is the meta heuristics. In the following section third generation methods are briefly introduced.

Meta heuristics can be broadly classified into three categories: (i) local search, (ii) population search, and (iii) learning mechanisms.

### 3.1 Local Search

In the local search methods, an intensive exploration of the solution space is performed by moving at each step from the current solution to another promising solution in its neighborhood [8]. Simulated annealing (SA) and Tabu search (TS) are two prime examples of this principle. Simulated annealing searches the neighborhood in a defined order. The procedure simulates the annealing process in metallurgy. There is a time associated with the search process. During the search space, the algorithm takes bigger jumps initially, and then starts smaller jumps gradually as the time passes or in the terminology of metallurgy as time of cooling increases cooling takes place more slowly. On the other hand Tabu search is a memory based technique that chooses the best solution in the neighborhood that does not violate certain restrictions. These restrictions are maintained in the queue called as Tabu list.

### 3.2 Population Search

Population search consists of maintaining a pool of good parent solutions and recombining them to produce better offspring. Genetic algorithm (GS) and adaptive memory procedures (AMPs) are two classical examples of population search. A Genetic algorithm starts with a pool of the population of chromosomes, which are the representing bit strings in VRPTW. These chromosomes undergo crossovers and mutation to produce some children that are different from parents but inherit some of their characteristics. This process continues until population has turned homogeneous in the sense that no further improvement in the solution appears possible.

In [18] a hybrid multiobjective evolutionary algorithm (HMOEA) that incorporates various heuristics for local exploitation in the evolutionary search and the concept of Pareto's optimality for solving multiobjective optimization in VRPTW is proposed. The proposed HMOEA is featured with specialized genetic operators and variable length chromosome representation to accommodate the sequence-oriented optimization in VRPTW. Adaptive memory procedures (AMPs) can be

viewed as an extension of GS where several parents are used to produce several offspring.

### 3.3 Learning Mechanisms

Learning mechanisms include neural networks (NN) and ant colony optimization (ACO). Early attempts to apply neural networks to the VRP have been rather unsuccessful; Schumann and Retzko and this line of research seems to have been abandoned. Ant colony optimization heuristics [20] attempt to mimic the behavior of ants that detect paths containing pheromone and strengthen them with their own pheromone. This leads to the emergence of shortest paths on which pheromone accumulate faster. In meta heuristics pheromone represents the memory of the system and corresponds to edges appearing often in good solutions. Thus the algorithm remembers good edges and is more likely to include them in a solution.

A number of researchers have attempted to combine the above meta heuristics. In [20] a hybrid ant colony system (DSACA-VRPTW) is proposed. In this paper ant solution is improved by dynamic sweep algorithm. Local peaks are avoided by making communication between ants when they are in stagnation state.

Shih-Wei Lin et al. [10] applied a hybrid approach which takes the advantages of simulated annealing and tabu search. Furthermore, the greedy local search is used to find better neighborhood solutions for VRPTW.

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A hybrid simulated annealing, genetic approach and tabu search is proposed in [9] in new GTSA algorithm combines tabu Search algorithm and Simulated Annealing algorithm.

In [8] authors presents the various meta heuristics along with their components, similarities and differences. Two important concepts that determines the behavior of meta heuristic are proposed namely: Intensification and diversification

Different diversification mechanisms have also been applied within the various meta heuristics to improve solution quality, so that the algorithm is directed towards exploring new areas of the search space. For example, in [21] the search is diversified using an adaptive memory, where routes taken from best solutions found during the search are stored. Some of these routes are combined to form solutions that will act as new starting solutions for the TS.

Intensification of the search around best solutions discovered during the search have also often been considered. For instance, in [22] the SA algorithm is forced to start from the current best solution several times, while the ACO in [20] uses global pheromone updating,

to further explore search areas around the best solutions obtained. A post-optimization phase is sometimes added to the basic meta-heuristic to further enhance best solutions. Fig 3 shows the classification of solution algorithms for VRP and its variants.

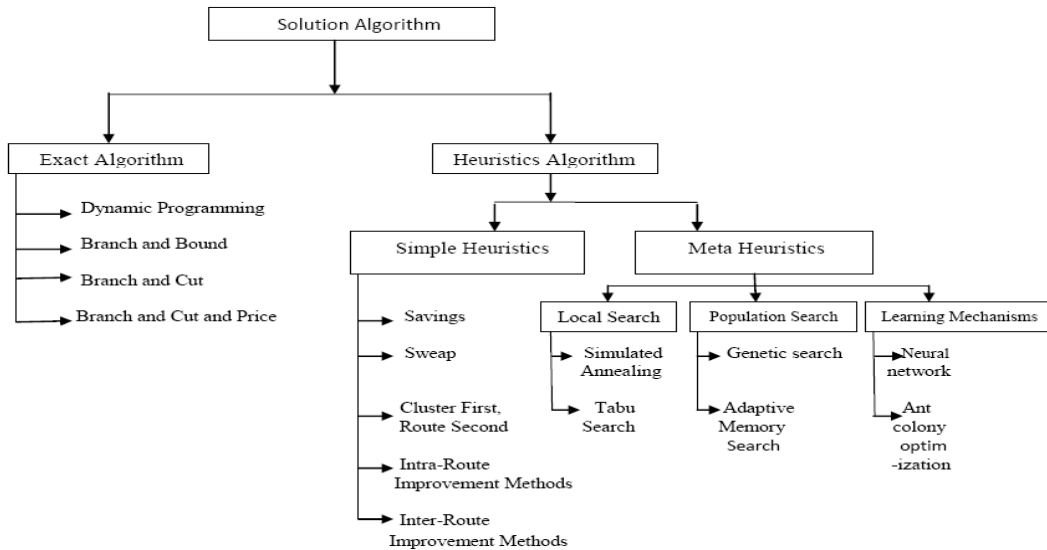


Fig 3: Solution algorithms for VRP and its variant

#### 4. RESEARCH AREAS

The main goal of VRPTW is to optimize the routing of multiple vehicles satisfying all the constraints. Key research issues are described below:

1. Straightforward heuristics and meta heuristics available for VRP cannot be applied for VRPTW because of additional constraint of time window.
2. Most researchers have focused on the comparison of meta heuristics for VRPTW on the basis of speed and accuracy, but flexibility and simplicity cannot be ignored.
3. Many researchers have used hybridization techniques to solve this problem which increases complexity whereas a simple and effective technique is generally preferable.
4. Meta heuristics that are independent of data sheet are to be designed.
5. Meta heuristics that can solve real life applications such as university bus routing, emergency services etc are to be designed.

#### 5. FUTURE WORK

Keeping in view the challenges stated above, the main objectives of research work will be enhancement of already existing meta heuristics to effectively solve

VRPTW and its variants. Design of robust hybrid meta heuristics while reducing complexity and to propose new meta heuristics for real life applications.

#### 6. CONCLUSION

In this paper a brief introduction to VRPTW and its solution techniques are presented. Some research areas and future work is also highlighted.

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