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Analysis of Natural Convection in a Porous Cavity with Cylindrical Obstacles

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ABSTRACT

A numerical simulation is fulfilled to analyze the free convection heat transfer inside a cavity filled with porous media and containing two cylindrical obstacles. The Darcy model was used to convert the Navier Stockes equations to stream function. The effect of angle of inclination and the distance between the obstacles was studied for different values of Rayleigh numbers. A curvilinear coordinates system was used to deal with complexity of obstacled boundary conditions. The results show that the average Nusselt number is increased with increasing the distance between the obstacles and Rayleigh number. It was found that these parameters affected the size and location of the resulting vortices.

Keywords: *free convection, cavities, porous media, obstacles.*

NOMENCLATURE

α	thermal diffusivity
e	dimensionless distance between obstacles(c/H)
g	gravitational acceleration, m/s^2
H	height of the cavity wall, m
J	Jacobian of the transformation
Nu	local Nusselt number
Nu_{av}	average Nusselt number
R	radius of the cylider, m
Ra	Rayleigh number
T_c	cold wall temperature, Ċ
T_h	hot wall temperature, Ċ
u,v	velocity components, m/s
x,y	Cartesian coordinates, m
X, Y	dimensionless Cartesian coordinates
$\alpha, \beta, \gamma, \tau, \sigma$	Transformation parameters in grid generation
ξ, η	coordinates in the transformed domain
ψ	dimensionless stream function
ω	dimensionless vorticity
ρ	density, Kg/m^3
θ	dimensionless temperature

1. INTRODUCTION

The free convection phenomena is found in multiple engineering and technological applications such as cooling of electronic equipments, solar collectors and heat exchangers. The researchers introduced many investigations to cover the free convection in cavities or enclosures containing porous media or without porous media. However this topic is still needs to more investigations to enhance the rate of heat transfer. Adding obstacles to these cavities is considered one of the useful tools. However a grid generation is needed to capture the resulted complexity in the physical domain A theoretical study on laminar natural convection and conduction in enclosed enclosure with vertical partitions was studied by Kangni *et. al.*[1]. In their study, they verified that the heat transfer was decreased as the thickness and number of partitions increased. Sharif [2] made a numerical study on mixed convection heat transfer in an inclined lid-driven enclosure filled with viscous fluid. He chose non-porous medium and observed that the average Nusselt number increases with increasing the cavity inclination angle. The natural convection in porous media filled right-angle triangular enclosure was investigated by Yasin *et. al.* [3].

The governing equations were obtained using Darcy model and solved by a finite difference methode. The study was performed for Rayleigh number up to 1000 and the aspect ratio ranging from 0.25 to 1. They showed that the heat transfer was increased with decreasing of aspect ratio and multiple cells were formed at high Rayleigh numbers. Hajiri *et. al.* [4]studied double-diffusive natural convection inside a triangular cavity . They found that for small values of the buoyancy ratio, there is little increase in the heat and mass transfer over that due to conduction. Asan and Namli [5] studied the laminar free convection heat transfer in triangular-shaped roofs with different Rayleigh numbers and angles of inclination. Th results showed that both Rayleigh number and aspect ratio affected the flow and thermal fields. Also they showed that the rate of heat transfer was decreased with the increase of aspect ratio. Tanmay *et. al.* [6] presented a penalty finite element analysis with bi-quadratic elements to investigate the effects of uniform and non-uniform heating of inclined walls on natural convection flows inside an isosceles triangular enclosure. The study was made for $10^3 \leq Ra \leq 10^6$ and $0.026 \leq Pr \leq 1000$. They verified that for small Prandtl numbers, geometry does not have much influence on the flow structure. The natural convection heat transfer in enclosures with a partition was studied by Dagtekin and Oztop [7]. The obtained results showed that the presence of a partition was the effective parameter on heat transfer. Al-Amiri [8] investigated the momentum and energy transfer in a lid-driven cavity filled with a porous medium. In his study he used both the inertia and viscous effects through the general formulation of momentum and energy transfer. Date [8] performed a computational study on natural convection heat transfer on non-rectangular complex enclosures.The governing equations were transformed to body-fitted coordinates system. The study was validated with theoretical and experimental published results. A numerical study on unsteady natural convection in differentially heated cavity with a fin on a side wall was investigated by Xu *et. al.* [9]. Different lengths for $Ra=3.8\times10^9$ were performed. They showed that the fin length significantly influenced the transient thermal flow around the fin and heat transfer through the finned side wall in the early stage of the transient flow development. A numerical study on natural convection heat transfer in a vertical square cavity in the presence of a hot conductivity body situated in the center was introduced by House et al. [10]. They verified that the heat transfer across the enclosures was increased by the body as the conductivity less than one. The two dimensional natural convection from a heated square cylinder placed inside a cooled circular enclosure was investigated by Sambamurthly et al. [11] . A correlation between Ra, aspect ratio and conductivity was reported.

In this work, a computational study is performed to analyze the free convection heat transfer in a cavity filled with porous media and containing two solid partitions. As shown in Fig.1, the bottom wall and the sold cylindrical partitions are assumed to be a hot while the upper wall is cold. The two vertical walls are considered insulated. The grid generation method proposed by Thompson [12] was used to transfer the physical model to a computational domain. The problem was tested for different parameters such as the distance between obstacles $0.48 \leq e \leq 1.013$ and Rayleigh number $150 \leq Ra \leq 250$.

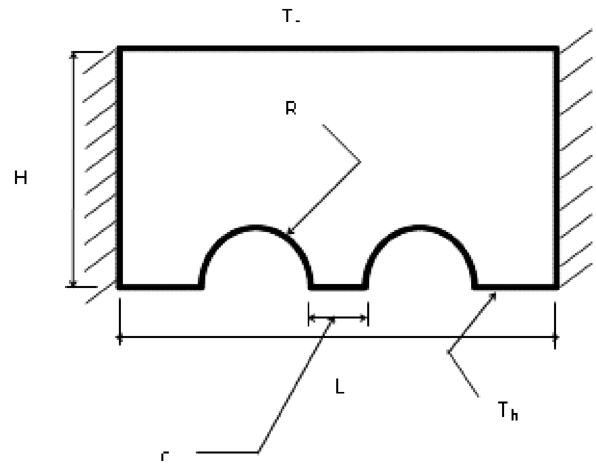

Fig.1 Schematic diagram of the studied problem, L=2m, H=1m, R=0.25m

2. MATHEMATICAL FORMULATION AND NUMERICAL SIMULATION

The laminar natural convection heat transfer and fluid flow inside a cavity filled with porous media and containing solid cylindrical partitions has been investigated. The fluid is assumed to be standard Boussinesque and the viscous and inertia effects are negligible. With these assumptions, the governing equations of mass continuity, Darcy and energy can be described as follows].

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=-\frac{g\beta}{\upsilon}\frac{\partial T}{\partial x} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

This is called Darcy model and the equations can be written in dimensionless form after using the following parameters.

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x},\ X = \frac{x}{H},\ \ Y = \frac{y}{H},\ \psi = \frac{\varphi}{\alpha},$$

$$\theta = \frac{T - T_c}{T_h - T_c},\ Ra = \frac{g\beta K(T_h - T_c)H}{\alpha\upsilon},\ \tau = \frac{\alpha t}{H^2}$$

$$\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2} = -Ra\frac{\partial\theta}{\partial X} \quad (4)$$

$$\frac{\partial\theta}{\partial\tau} + \frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X} - \frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y} = \nabla^2\theta \quad (5)$$

The transformation of the new dependent variables (ζ,η) defined in the previous section leads to replacement of $\psi(x,y)$ in to $\psi(\zeta,\eta)$ and $\omega(x,y)$ to $\omega(\zeta,\eta)$[12].

$$\lambda\psi_\zeta + \sigma\psi_\eta + \alpha\psi_{\zeta\zeta} - 2\beta\psi_{\zeta\eta} + \gamma\psi_{\eta\eta} = -J\ Ra(\theta_\zeta Y_\eta - \theta_\eta Y_\zeta) \quad (6)$$

$$\theta_\tau + \left(-\psi_\zeta\theta_\eta + \psi_\eta\theta_\zeta\right)\Big/J = \left(\lambda\theta_\zeta + \sigma\theta_\eta + \alpha\theta_{\zeta\zeta} - 2\beta\theta_{\zeta\eta} + \gamma\theta_{\eta\eta}\right)\Big/J^2 \quad (7)$$

Where

$$\lambda = \left(X_\eta D_y - y_\eta D_x\right)\Big/J \quad (8)$$

$$\sigma = \left(Y_\zeta D_x - X_\zeta D_y\right)\Big/J \quad (9)$$

$$D_x = \alpha Y_{\zeta\zeta} - 2\beta Y_{\zeta\eta} + \gamma Y_{\eta\eta} \quad (10)$$

$$D_Y = \alpha X_{\zeta\zeta} - 2\beta X_{\zeta\eta} + \gamma X_{\eta\eta} \quad (11)$$

2.1 Boundary Conditions

In order to solve the mathematical model, the following boundary conditions are used.

$\theta = 0$, $\psi = 1$ on the cold wall

$\theta = 1$, $\psi = 0$ on the hot wall

$\frac{\partial\theta}{\partial\eta} = 0$, on the two insulated walls

The local and average Nusselt number along the hot bottom wall is calculated as follows.

$$Nu = -\int_0^L \frac{d\theta}{dx} \quad (12)$$

$$Nu_{av} = \frac{1}{L}\int_0^L Nu dx \quad (13)$$

The mentioned governing partial differential equations are discretised using finite difference technique and the resulting algebraic equations are solved by using iteration method along with successive under relaxation method (SOR).To ensure that the flow and thermal field are not affected by the mesh, different mesh densities(151×51), (181×51) and (201×51) were used to check the grid independency and the grid 201×51 was selected. The computational procedure is repeated until the following convergence criteria is achieved $\left|\phi^k - \phi^{k-1}\right| \le 10^{-5}$.

The validation of this numerical code is achieved through a comparison of with the published studies as shown in the next sections which show a good agreement.

3. RESULTS AND DISCUSSION

The numerical results are obtained for Rayleigh number range 150 ≤ Ra≤ 500. The dis obtained results arte demonstrated as follows.

Fig.2 illustrates the stream function distribution for different values of distance between the two cylindrical obstacles. It is evident that increasing the distance between the obstacles affect the formation of the resulting vortices. When e=0.48, two vortices are formed above the two cylinders and the stream lines are very thick near the solid cylinders and in the region between these cylinders. This means that the convection currents are faster. Also it can be seen that these lines remote when moving upward towards the upper wall and that indicates the flow is slower. Another insight to this figure(b. e=0.73), the convection currents become faster when moving towards the upper wall besides to increase the boundary layer penetration towards the bottom hot wall and consequently expecting to increase the rate of heat transfer. It can be mention here that with increasing the distance (e), the two vortices are decomposed to one elongated vorticty at the region between the cyliders.. This behavior is dominated for other values of Rayleigh number.

The distribution of isotherm lines is found in Fig.3 for different values of the distance between the two cylinders.

As the figure shows, the removal of the heat from the solid hot wall is expected to increase as the distance(e) increases due to increase the heat transfer area. Also it can be noted the inclination of isother lines is increased as the distance between the cylinders increases and this verified increasing the movements of convection currents. The cause is when increasing the distance between the cylinders the semi core region above them is distorted and that gives more heat losses

The effect of Rayleigh number on the distribution of stream function for e=0.48 is depicted in Fig.4. As the figure shows, with increasing the Rayleigh, the resulting two vortices are decomposed to one elongated vorticity and the size and location of this vorticity is changed with any further increase in Rayleigh number value. The effect of Rayleigh number on isotherm lines is shown in Fig.5. It can be seen that the slope of isotherm lines are increased as Rayleigh number increases. The average Nusselt number versus the distance between obstacles is presented by Fig.6. It can be seen that the average Nusselt number is linearly increased as the distance (e) increase for $0.48 \leq e \leq 1.013$. After that a rapid linearly increase is found. The cause is due to increase the heat transfer exposed area exposed to convection currents. Fig.7 shows the effect of Rayleigh number on the variation of average for e=0.48. It is evident that the average Nusselt number is increased linearly as Rayleigh number increases. The increase in Rayleigh number increases the convection currents due to increase the gravity force.

The present numerical code is validated with the available published results as shown in Fig.8. The comparison verified and acceptable agreement.

4. CONCLUSION

A numerical study to predict the laminar free convection heat transfer in a cavity filled with porous media and containing two solid obstacles has been accomplished. The following concluding remarks may obtained.

1. The average Nusselt number is increased as the distance between the cylindrical obstacles increases.
2. The distance between the cylindrical obstacles was a controlling parameter on controlling the size and location of the vortices.

The average Nusselt number is increased by increasing the value of Rayleigh number.

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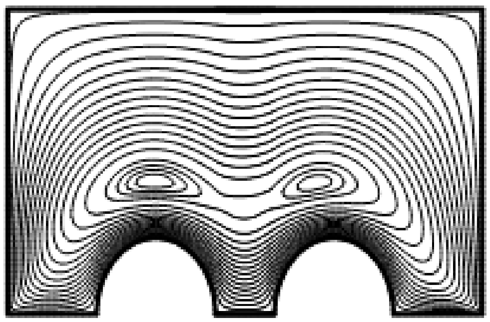
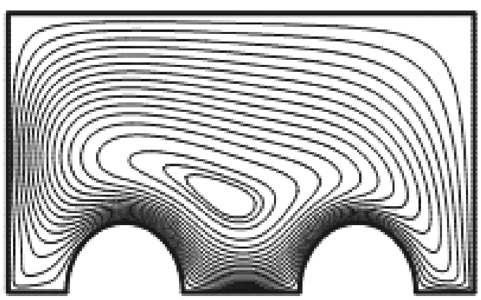
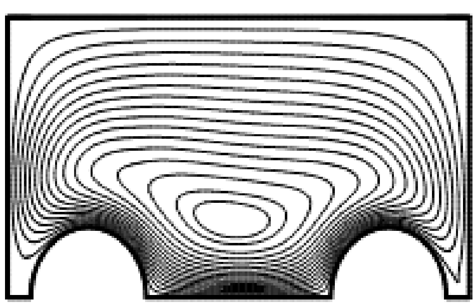


a. e=0.48

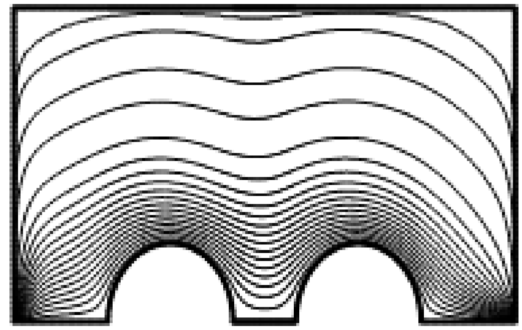


b. e=0.73

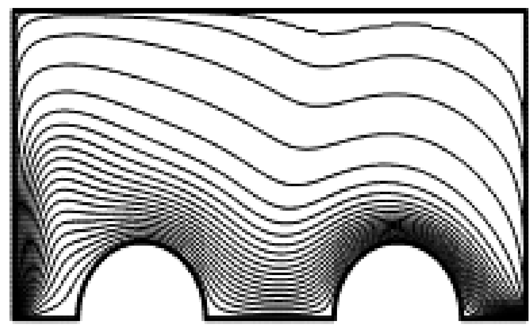


c. e=1.013

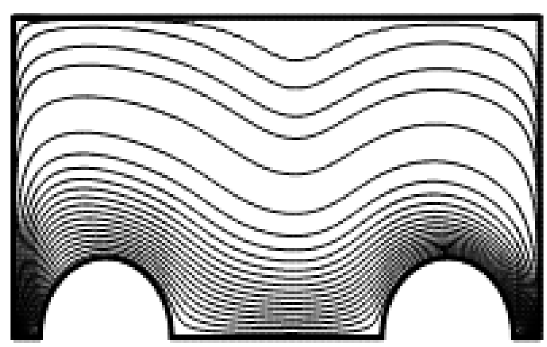
Fig.2 Contours of stream function distribution for different distances between obstacles, Ra=150



a. e=0.48

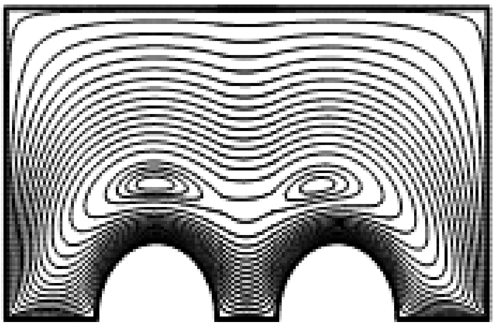


b. e=0.73

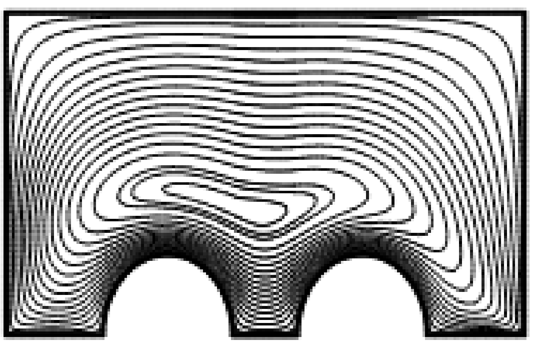


c. e=1.013

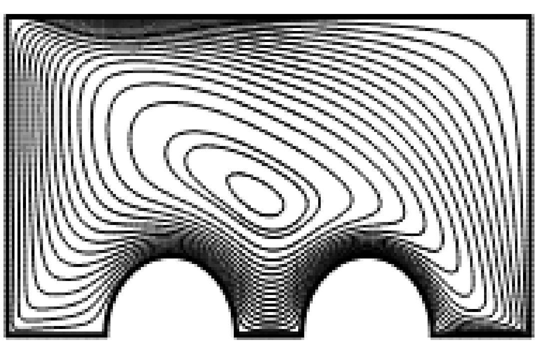
Fig.3 Contours of isotherm lines for different distances between obstacles, Ra=150



a. Ra=150

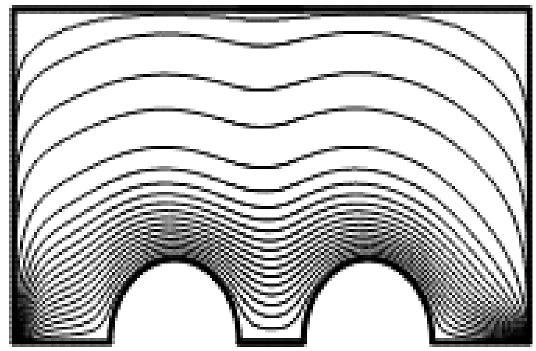


b. Ra=250

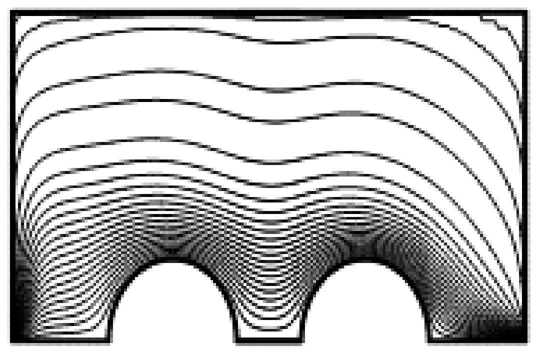


c. Ra=500

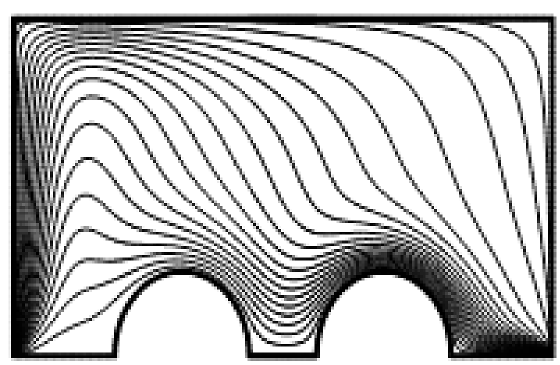
Fig. 4 Effect of Rayleigh number on stream function distribution for e=0.48



a. Ra=150



b. Ra=250



c. Ra=500

Fig. 5 Effect of Rayleigh number on temperature distribution for e=0.48

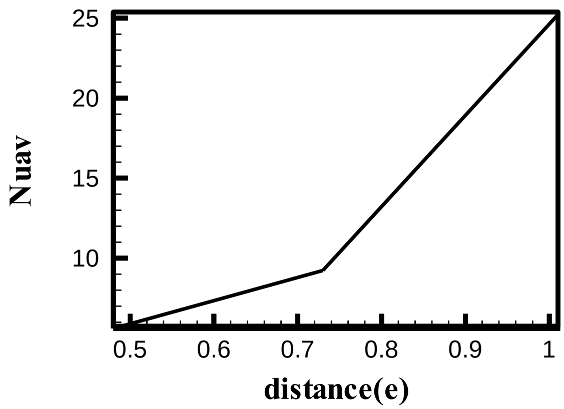


Fig.6 Effect of distance between obstacles on local Nusselt number distribution (on the hot wall) For Ra=150

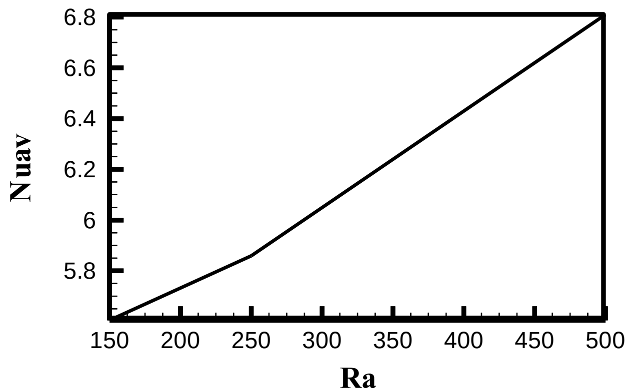


Fig.7 Effect of Rayleigh number local Nusselt number distribution(on the hot wall) for e=0.48

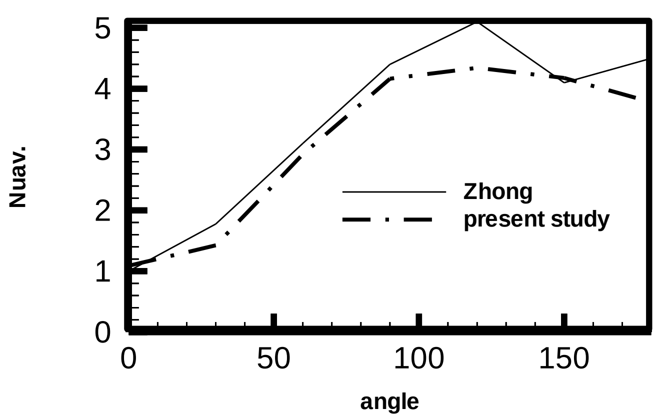


Fig .8 Validation of the present code with published results [13] at Ra = 10^5

