iJET EXPLORE International Journal of Engineering & Technology
CENTRE OF PROFESSIONAL RESEARCH PUBLICATIONS
ISSN 2049-3444

International Journal of Engineering and Technology Volume 3 No. 5, May, 2013

Numerical Study for Free Convection Heat Transfer inside an Inclined Cavity with Cylindrical Obstacles

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ABSTRACT

The laminar natural convection heat transfer inside an inclined rectangular cavity containing two cylindrical obstacles has been numerically studied. The cavity was differentially heated and the values of angle of inclination was ranged from $\overset{\circ}{0} \leq \alpha \leq \overset{\circ}{90}$ while values of the Rayleigh number was ranged form 10^3 to 10^6. A grid generation system was adopted to deal with the complexity of the studied configuration. The transformed algebraic governing equations of vorticity and energy was solved by an alternated difference scheme(ADI) while that of stream function was with iteration method. The flow and thermal characteristics are studied for different parameters such as the angle of inclination, the distance between obstacles and the Rayleigh number. The obtained results show that the rate of heat transfer is increased as angle of inclination increases. It was found the angle of inclination was a controlling factor on forming the vortices.

Keywords: *free convection, cavities, obstacles*

NOMENCLATURE

a	thermal diffusivity
e	dimensionless distance between obstacles(c/H)
g	gravitational acceleration, m/s^2
H	height of the cavity wall, m
J	Jacobian of the transformation
Nu	local Nusselt number
Nu_{av}	average Nusselt number
R	radius of the obstacle, m
Ra	Rayleigh number
T_c	cold wall temperature, Ċ
T_h	hot wall temperature, Ċ
u,v	velocity components, m/s
x,y	Cartesian coordinates, m
X, Y	dimensionless Cartesian coordinates
α, β ,γ,τ,σ	Transformation parameters in grid generation
ξ,η	coordinates in the transformed domain
ψ	dimensionless stream function
ω	dimensionless vorticity
ρ	density, Kg/m^3
α	angle of inclination
a	thermal diffusivity, m^2/s
θ	dimensionless temperature

1. INTRODUCTION

The natural convection heat transfer in enclosures or cavities is considered one of the important targets for many researchers due to its application in many industrial systems as cooling of electronic equipments, solar collectors and heat exchangers. The enhancement of heat transfer in these cavities or enclosures made the researchers to search for many additional factors such adding obstacles with different configurations. However some of the shapes of these obstacles made the cavity to be a complex and hence a grid generation technique is needed. The present study is aimed to enhance the related studies in this field. Many experimental and numerical studies was dealt with studying the laminar natural convection in cavities with obstacles. The heat transfer characteristics inside a square cavity filled with a fixed amount of conducting solid material was studied by Edimilson and Marcel [1]. In their study, they verified that the average Nusselt number for cylindrical rods were slightly lower than those of the square rods. Arnab and Amaresh [2] studied the natural convection around a tilted heated square cylinder kept in an enclosure. They verified that the Stream-function vorticity formulation of the Navier-stockes equations was solved numerically using finite difference method in non-orthogonal body-fitted coordinates system numerically. The natural convection heat transfer for a heated cylinder placed in a square enclosure was studied by Roychowdhury et al. [3]. They used different thermal boundary conditions. Marcel and Antonine [4] made a numerical study for natural convection heat transfer of air from two vertical walls of finite conductance and horizontal walls at heat sink temperature. The results were obtained for Rayleigh number up to 10^6. The two dimensional natural convection from a heated square cylinder placed inside a cooled circular enclosure was studied by Sambamurthly et al. [5] . A correlation between Ra, aspect ratio and conductivity was reported. The natural convection heat transfer from a horizontal cylinder enclosed in a rectangular cavity was studied by Cesiniet et al. [6]. The temperature distribution in the air and the heat transfer coefficients were measured by a holographic interferometer and compared with numerical predictions obtained by finite element based on stream-vorticity formulation of the momentum equations.The natural convection in a cavity with a solid block obstacles was investigated by many researchers such as Merrikh and large [7], Das and Reddy[8] and Laguerre et al. [9]. A numerical study on natural convection heat transfer in a vertical square cavity in the presence of a hot conductivity body situated in the center was introduced by House et al. [10]. They results that the heat transfer across the enclosures was increased by the body as the conductivity less than one.

From the literature no documented study on inclined rectangular cavity with two cylindrical obstacle was found. In this work, the laminar natural convection heat transfer and fluid flow inside an inclined rectangular cavity containing two cylindrical obstacles is numerically studied. As shown in Fig.1. The bottom wall and cylindrical obstacles is hot while the upper wall is cold. The two vertical walls are insulated. The grid generation method proposed by Thompson[11] was used to transform the physical domain to a computational one. The cavity angle of inclination is changed between $0^{\circ} \le \alpha \le 90^{0}$ while the distance between obstacles is form 0.48 to 1.013. The study is performed for a Rayleigh number range $10^{3} \le Ra \le 10^{6}$.

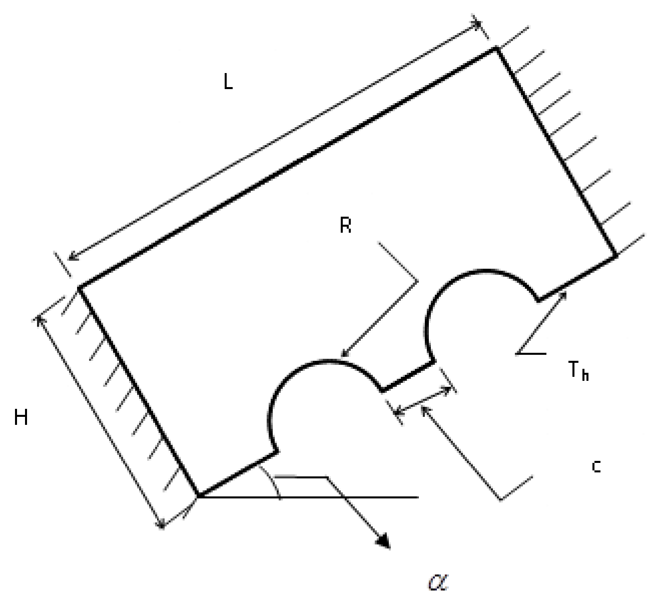

Fig.1 Schematic diagram of the studied problem, **L=2m, H=1m, R=0.25m**

2. MATHEMATICAL FORMULATION AND NUMERICAL ANALYSIS

The governing equations of continuity, momentum and energy are described as follows. The viscous and inertia forces are neglected while the Boussinesque approximation is valid.

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial Y^2} = -\omega \quad (1)$$

$$\frac{\partial\Omega}{\partial t}+u\frac{\partial\Omega}{\partial x}+v\frac{\partial\Omega}{\partial y}=\mu\left(\frac{\partial^2\Omega}{\partial x^2}+\frac{\partial^2\Omega}{\partial y^2}\right)+\rho g\beta\left(\sin\theta\frac{\partial T}{\partial y}+\cos\theta\frac{\partial T}{\partial x}\right) \quad (2)$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \quad (3)$$

The mentioned equations (1) to (3) can be written in dimensionless stream and vorticity method after using the following dimensionless parameters.

$$X=\frac{x}{H},\quad Y=\frac{y}{H},\quad \psi=\frac{\varphi}{a},\quad \theta=\frac{T-T_0}{T_H-T_C},$$

$$T_0=(T_H+T_C)/2,\ \tau=\frac{at}{H^2},\ \omega=\frac{\Omega H^2}{a},\ \beta=\frac{1}{T_0}$$

$$\Pr=\frac{a}{\upsilon},\ Ra=\frac{g\beta(T_H-T_C)H^3}{a\upsilon}$$

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial Y^2}=-\omega \quad (4)$$

$$\frac{\partial\omega}{\partial t}+\frac{\partial\psi}{\partial Y}\frac{\partial\omega}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\omega}{\partial Y}=\Pr\left(\frac{\partial^2\omega}{\partial X^2}+\frac{\partial^2\omega}{\partial Y^2}\right)+Ra\Pr\left(\sin\theta\frac{\partial\theta}{\partial Y}+\cos\theta\frac{\partial\theta}{\partial X}\right) \quad (5)$$

$$\frac{\partial\theta}{\partial\tau}+\frac{\partial\psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\theta}{\partial Y}=\frac{1}{Rapr}\left(\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}\right) \quad (6)$$

The transformation of the new dependent variables (ζ,η) leads to replacement of $\psi(x,y)$ in to $\psi(\zeta,\eta)$, $\omega(x,y)$ to $\omega(\zeta,\eta)$ and $\theta(x,y)$ to $\theta(\zeta,\eta)$ [11].

$$\lambda\psi_\zeta+\sigma\psi_\eta+\alpha\psi_{\zeta\zeta}-2\beta\psi_{\zeta\eta}+\gamma\psi_{\eta\eta}=-J^2\omega \quad (7)$$

$$\omega_\tau+\left(-\psi_\zeta\omega_\eta+\psi_\eta\omega_\zeta\right)\Big/J=\left(\lambda\,\omega_\zeta+\sigma\,\omega_\eta+\alpha\,\omega_{\zeta\zeta}-2\beta\,\omega_{\zeta\eta}+\gamma\,\omega_{\eta\eta}\right)\Big/J^2\Pr+$$

$$Ra\Pr\cos\theta/J[(\theta_\zeta Y_\eta-\theta_\eta Y_\zeta)+(\theta_\eta X_\zeta-\theta_\zeta X_\eta) \quad (8)$$

$$\theta_\tau+\left(-\psi_\zeta\theta_\eta+\psi_\eta\theta_\zeta\right)\Big/J=\left(\lambda\theta_\zeta+\sigma\theta_\eta+\alpha\theta_{\zeta\zeta}-2\beta\theta_{\zeta\eta}+\gamma\theta_{\eta\eta}\right)\Big/J^2 Ra\Pr \quad (9)$$

Where

$$\lambda=\left(X_\eta D_y-y_\eta D_x\right)\Big/J \quad (10)$$

$$\sigma=\left(Y_\zeta D_x-X_\zeta D_y\right)\Big/J \quad (11)$$

$$D_x=\alpha Y_{\zeta\zeta}-2\beta Y_{\zeta\eta}+\gamma Y_{\eta\eta} \quad (12)$$

$$D_Y=\alpha X_{\zeta\zeta}-2\beta X_{\zeta\eta}+\gamma X_{\eta\eta} \quad (13)$$

2.1 Boundary Conditions

In order to solve the mathematical model, the following boundary conditions were used.

$U=V=0,\ \theta=0,\ \psi=0,\ \omega=\frac{-\alpha}{J^2}\psi_{\zeta\zeta}$ on the upper cold wall

$U=V=0,\ \theta=1,\ \psi=0,\ \omega=\frac{-\alpha}{J^2}\psi_{\zeta\zeta}$ on the lower hot wall

$U = V = 0$, $\frac{\partial \theta}{\partial \eta} = 0$, $\psi = 0$, $\omega = \frac{-\alpha}{J^2}\psi_{\eta\eta}$ on the two insulated walls

The mentioned boundary condition for the vorticity was imposed according to the Woods formula [13]. The local and average Nusselt number along the hot bottom wall is calculated as follows.

$$Nu = -\int_0^L \frac{d\theta}{dx} \quad (14)$$

$$Nu_{av} = \frac{1}{L}\int_0^L Nu dx \quad (15)$$

The resulting partial differential equations are descretized to algebraic equations by using a finite difference method. The resulting algebraic equations of stream function and energy was studied by (ADI) method while that of stream function by iteration method with successive over relaxation scheme(SOR). A built home computer program using Fortran 90 language was designed to solve the descretized equations. Different mesh densities(151×51), (181×51) and (201×51) were used to check the grid independency and the grid 201×51 was selected.

3. RESULTS AND DISCUSSION

The computational results are obtained for Rayleigh number $10^3 \leq Ra \leq 10^5$, angle of inclination $0^{\circ} \leq \alpha \leq 90^0$ and distance between cylindrical obstacles 0.48 to 1.013.

Fig.2 shows the contours of stream function for different values of angle of inclination at e=0.48 and Ra= 10^4. It can be seen that the angle of inclination played an important role in forming and distribution the resulting vortices. At $\alpha = 0$, the vortices were positioned above the two obstacles and when the angle increases to 30^0, the two vortices are elongated and changed their location. When the angle of inclination increases to 60^0, the position and size of vortices are changed. For $\alpha = 90^0$, four vortices are appeared and this trend is expected to affect the rate of heat transfer as shown in the next section. The distribution of isotherm lines for different angles of inclination, e=0.48 and Ra= 10^4 is depicted in Fig.3. It is shown that the inclination of isotherm lines and density of these lines on the cylindrical obstacles is changed with increasing angle of inclination and as mentioned in Fig.2, the angle 90^0 exhibits a large variation in isotherm lines especially at the region between the two obstacles.

The effect of Rayleigh number on stream function distribution is shown in Fig.4., For $\alpha = 90^0$, the distribution of stream lines besides to size of resulting vortices are slightly changed. Also this trend is found in Fig.5 where the inclination of isotherm lines is changed as Rayleigh number increases and that expected to increase the rate of heat transfer. When the Rayleigh number increases, the gravity force increases and the convection currents consequently increases. Fig.6 shows variation of the local Nuseelt number on the obstacle and hot wall for e=0.48 and Ra= 10^4. As the figure shows, the Nusselt number increases as the Rayleigh number increases because the convection currents increase due to increase the gravity forces. The effect of angle of inclination on variation of local Nusselt number is shown in Fig.7. It is shown that the local Nusselt number is increased as angle of inclination increases. However these increases appeared above the obstacles and before and after these obstacles. The effect of distance between obstacles on variation of Nusselt number for $\alpha = 60^0$ is depicted in Fig.8. It is shown that the Nusselt number at the region between the obstacles is increased as the distance between obstacles increases while a wavy fluctuation of Nusselt number is appeared as a general trend. The validation of the present numerical code is done through comparison with the available published results as shown in Fig.9. The comparison indicated a good agreement

4. CONCLUSION

The laminar natural convection heat transfer inside an inclined cavity containing two cylindrical obstacles is numerically studied. From this study, the following conclusions can be obtained.

1. The local Nusselt number is increased as the cavity angle of inclination increases.
2. As the distance between the obstacles increases, the Nusselt number increases
3. The angle $\alpha = 90^0$ indicated the maximum local Nusselt number.

The strength and shape of vortices is changed as Raleigh number and angle of inclination increases.

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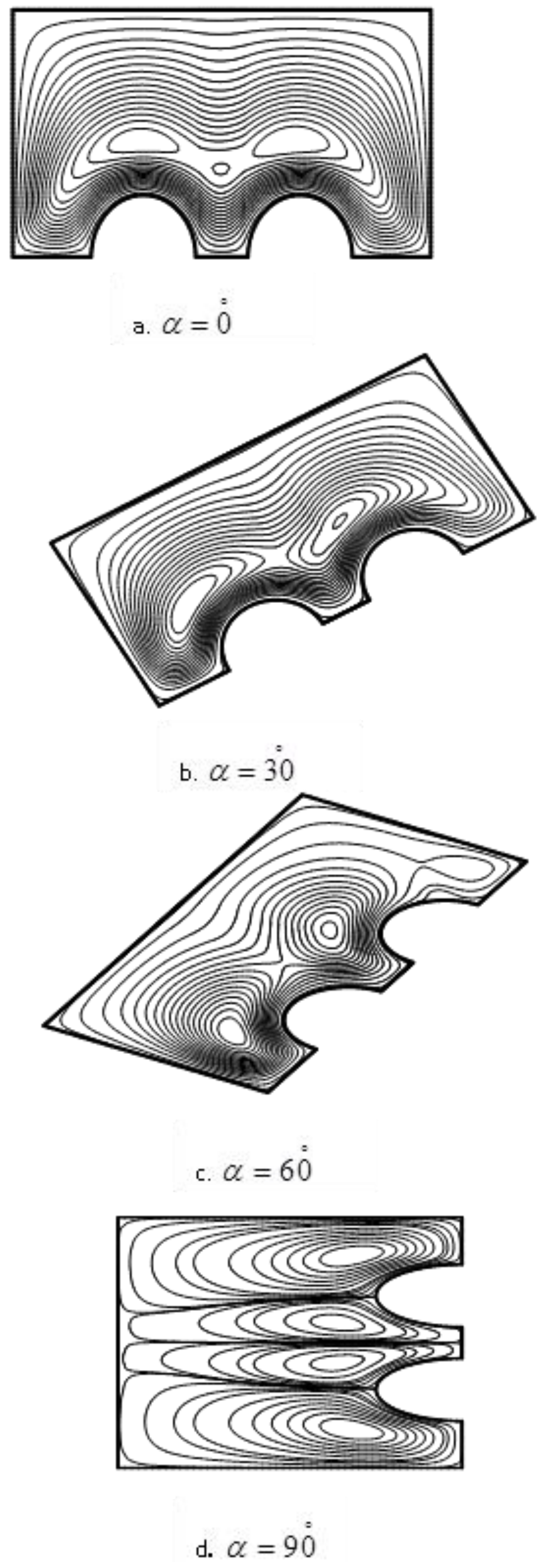


Fig. 2 Effect of angle of inclination on stream function distributionfor e=0.48 and Ra=10^4

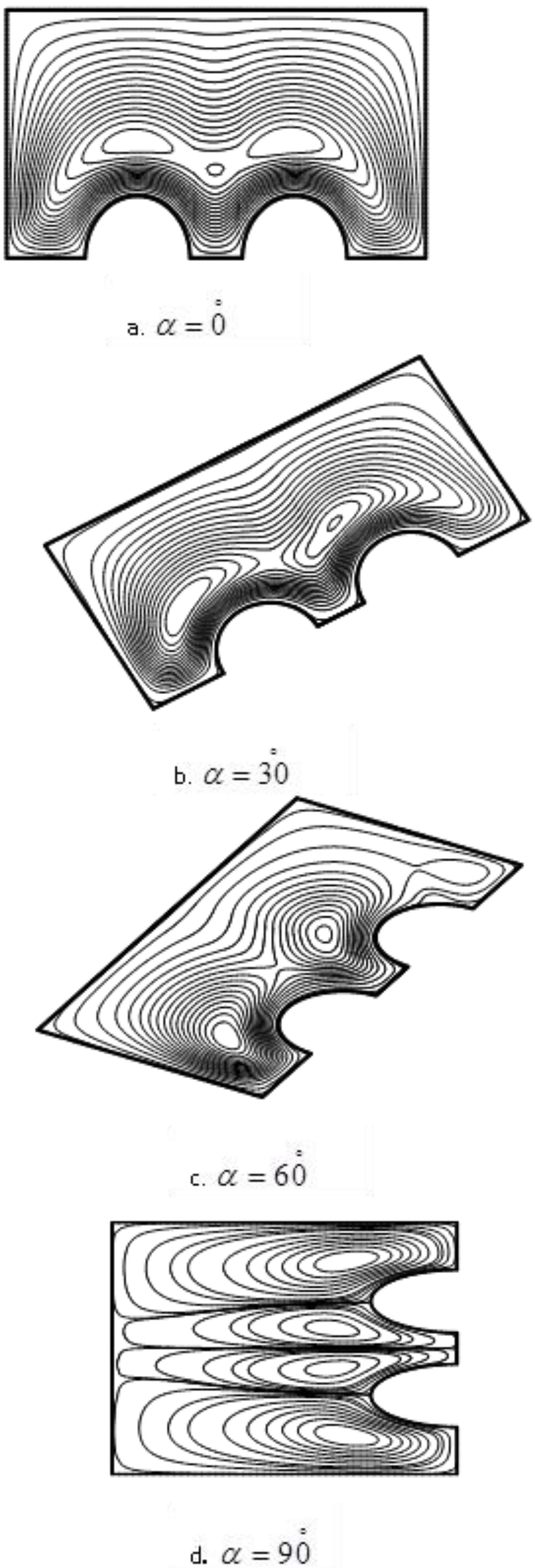


Fig. 2 Effect of angle of inclination on stream function distribution for e=0.48 and Ra=10^4.

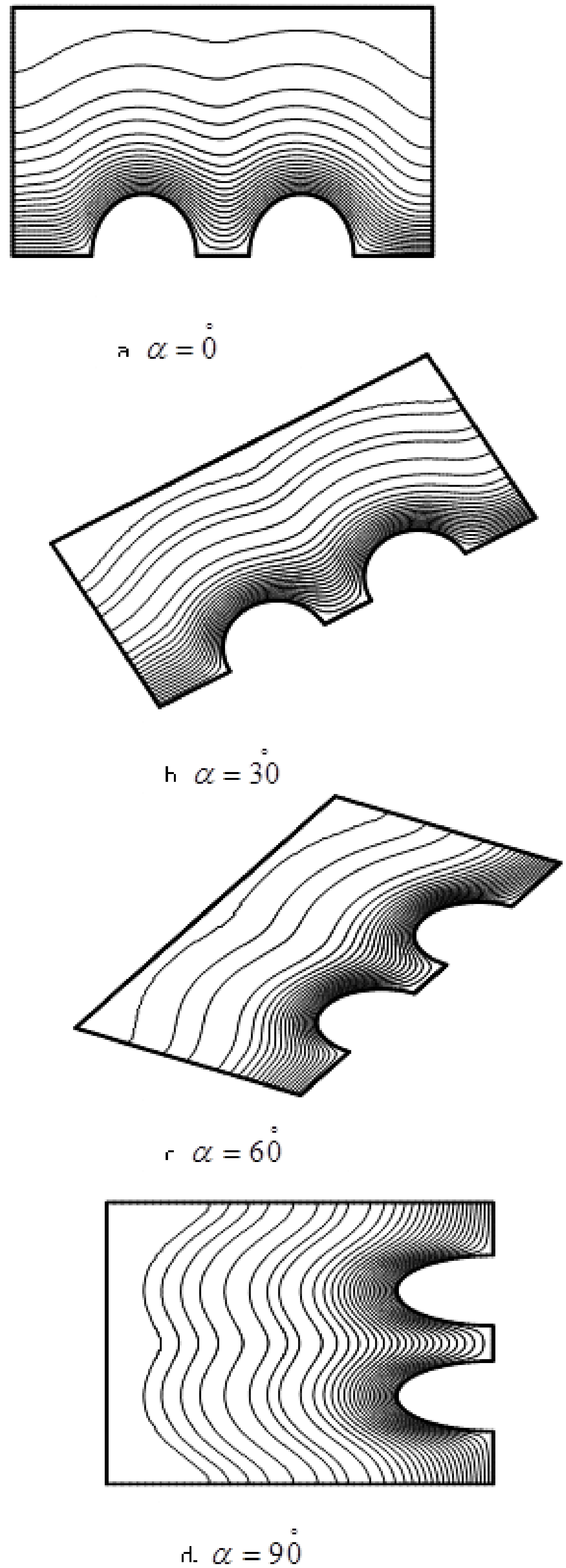
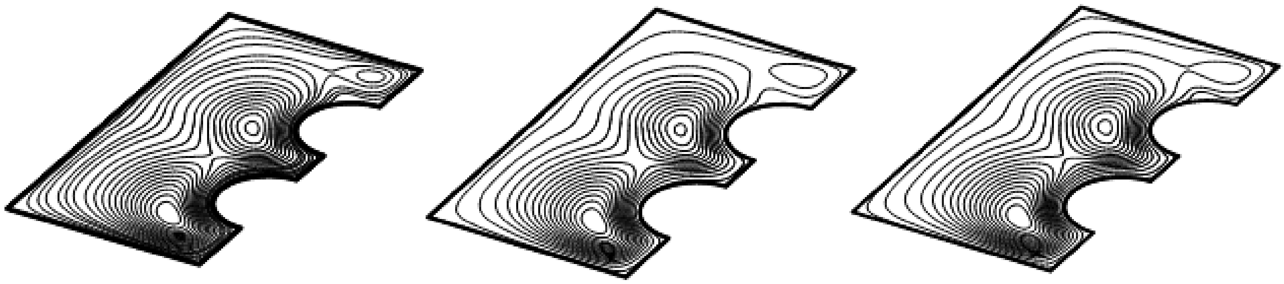
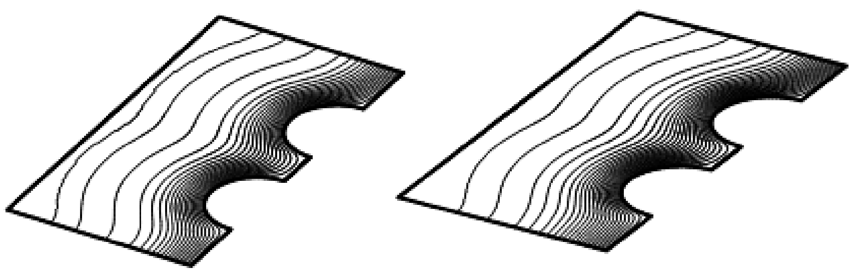


Fig. 3 Effect of angle of inclination on isotherm lines distribution for e=0.48 and Ra=10^4



a. Ra=10^3 b. Ra=10^4 c. Ra=10^6

Fig. 4 Effect of Rayleigh number on stream function distribution for e=0.48 and $\alpha = 60^{\circ}$



a. Ra=10^3 b. Ra=10^4 c. Ra=10^6

Fig.5 Effect of Rayleigh number on isotherm lines distribution for e=0.48 and $\alpha = 60^{\circ}$

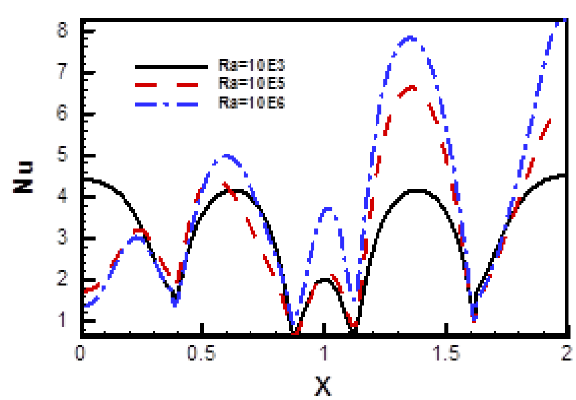


Fig.6 Effect of Rayleigh number local Nusselt number distribution(on the hot wall)

for e=0.48 and $\alpha = 60^{\circ}$

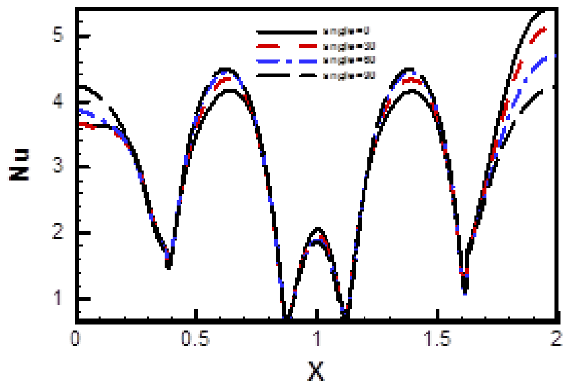


Fig.7 Effect of angle of inclination on local Nusselt number distribution(on the hot wall)

For e=0.48 and $Ra=10^4$

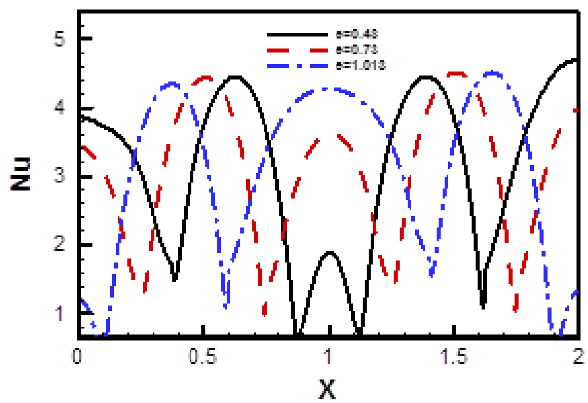


Fig.8 Effect of distance between obstacles on local Nusselt number distribution

(on the hot wall) For $Ra=10^4$ and $\alpha = 60^{\circ}$

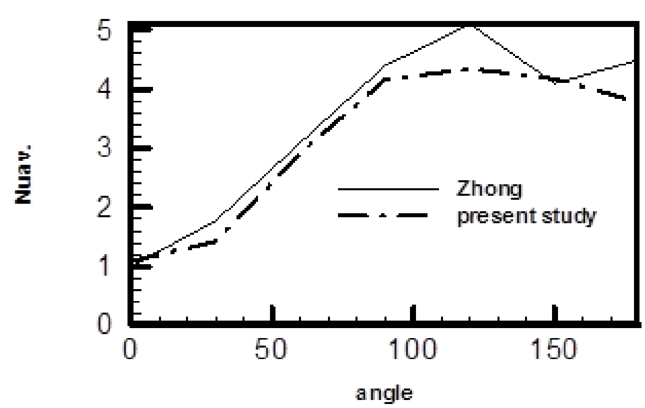


Fig .9 Validation of the present code with published results [12]

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