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Newton Gregory Formulae for Modeling Biomechanical Systems

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ABSTRACT

This paper shows how Average value based approach and Newton Gregory Formulae can be used in a successful way to model the locomotion of human knee joint. A Novel modeling technique based on Average Value algorithm has been developed for a healthy human knee joint locomotion. The developed mathematical model will become a guide line for the design of drive mechanism having similar motion. The requirements of this approach are a set of reading from the real time system. In this paper we considered subjects performing gait on normal floor. The knee joint locomotion is modeled from the data acquired by means of calculating the base value and the variational components. The entire procedure is achieved through the video picture of the locomotion captured by single or multiple cameras with proper resolution. The results obtained from the proposed model and actual results of locomotion of human knee joint were giving close results.

Keywords: *Average Value based approach, Newton Gregory Formulae, Locomotion*

1. INTRODUCTION

The objective of the paper is to develop a mathematical model for the knee joint locomotion, which will become a guide line for the design of drive mechanism having similar motion. This drive mechanism will find application in developing assistive limb. The assistive limb mechanism is useful for the people whose movements are disturbed due to profound muscle weakness or impaired motor control.

Gait analysis is gaining prominence for evaluation of disability of handicapped people on comparison of such analysis of healthy people with the disabled. This is motivated by study conducted by Tommy Oberg and et.al [1] on 233 healthy subjects (116men and 117 women) up to 79 years of age. With reference to their study significant differences were observed on subjects with difference both in age and sex. Also significant changes were noticed on increasing the gait speed. Berbyuk and et.al [2] has made a mathematical modeling of the dynamics of human gait in a saggital plane as an optimal control problem for a nonlinear multidimensional mechanical system with phase constraints given by experimental data. The optimality criterion was chosen as a freshet non differential function used to estimate energy consumption in human walking. The constraints on the phase coordination of the system were given on the basis of non experimental data. To solve the optimal control problem the method based on Fourier-spline approximation of independently varying functions, concept of inverse problem of dynamics and minimizing the objective function over maximally likely directions were attempted [2]. Nishchenko and et.al [3] proposed a new formulation of the problem of designing a femoral prosthesis. The essence of the procedure consists of formulating and solving the problem of energetically optimal parametric control of a nonlinear multidimensional mechanical system that models the controlled movement of the locomotors system of a person with prosthesis. Optimal control method proposed in this article included a number of special restrictions on phase coordinates and controlling efforts.

Jerry E.Pratt and et.al [4] developed a one degree of freedom exoskeleton called RoboKnee. User intent is determined through the knee joint angle and ground reaction forces. The Roboknee allows the wearer to climb stairs and perform deep knee bends while carrying a significant load in a backpack. The Roboknee shows that a simple control algorithm can significantly enhance one's capability. However, the Roboknee is too bulky and has too short of a lifetime between battery recharge. Hayashi and et.al [5] developed Hybrid Assistive Limb, a robot suit as an assistive device for lower limbs. This method uses biological and motion interaction. HAL produces torque corresponding to muscle contraction torque by referring to the myoelectricity that is the biological information to control operator's muscles. Yamamoto and et.al [6] proposes a stand alone wearable power assisting suit which gives nurses the extra muscle they need to lift their patients and avoid back injuries. The muscle forces are sensed by a new muscle hardness sensor utilizing a sensing tip mounted on a force sensing film device. The embedded microprocessor calculates the necessary joint torque for maintaining a position according to the equations derived from static body mechanics using joint angles, and the necessary joint torque is combined with the output signals of the muscle sensors to make control signals. Joint torques was calculated using Lagrange's method. In addition, the muscle hardness sensor was developed and it measured its characteristics.

D Jin and et.al [7] investigated the advantages of the mechanism as used in the prosthetic knee from the kinematic and dynamic points of view. The results show that the six-bar mechanism, as compared to the four-bar mechanism, can be designed to better achieve the expected trajectory of the ankle joint in swing phase. Moreover, a six-bar linkage can be designed to have more instant inactive joints than a four-bar linkage, hence making the prosthetic knee more stable in the standing phase. In the dynamic analysis, the location of the moment controller was determined for minimum value of the control moment. Takahiko Nakamura and et.al [8] propose a wearable antigravity muscles support system to support activities of physically weak persons. In this system, Posture-based control algorithm is implemented to a wearable antigravity muscles support device. In this algorithm, joint support moments are calculated based on user's posture without biological signals. Wearable Walking Helper-KH is developed as a wearable support device Experimental results show the effectiveness of the proposed system.

Satio and et.al [9] proposed wearable walking support system In this paper to validate the usefulness of proposed system, the Standing up motion, one of the hardest activities of daily life is analyzed and the usefulness of proposed method is discussed. Experimental results show the validity of the system. Sunil K Agarwal and et.al [10] proposes a gravity balancing lower extremity exoskeleton a simple mechanical device composed of rigid links, joints and springs, which is adjustable to the geometry and inertia of the leg of a human subject wearing it. This passive exoskeleton does not use any motors or controllers, yet can still unload the human leg joints of the gravity load over the full range of motion of the leg. The exoskeleton was tested on five healthy human subjects and a patient with right hemisparesis following a stroke. The evaluation of this exoskeleton was performed by comparison of leg muscle EMG recordings, joint range of motion using optical encoders, and joint torques measured using interface force-torque sensors. In the walking experiments, there was a significant increase in the range of motion at the hip and knee joints for the healthy subjects and the stroke patient. For the stroke patient, the range increased by 45 % at hip joint and by 85 % at the knee joint. Jane Courtney and et.al [11] presents a new, user-friendly, portable motion capture and gait analysis system for capturing and analyzing human gait, designed as a telemedicine tool to monitor remotely the progress of patients through treatment. The system requires minimal user input and simple single-camera filming. This system can allow gait studies to acquire a much larger data set and allow trained gait analysts to focus their skills on the interpretation phase of gait analysis. The design uses a novel motion capture method derived from spatiotemporal segmentation and model-based tracking. Testing is performed on four monocular, saggital-views, sample gait videos. Results of modeling, tracking, and analysis stages are presented with standard gait graphs and parameters compared to manually acquired data. The system was tested on four different video clips made under very different conditions. In all cases, the system compares well with manual measurements and with other published results for equivalent systems. This system will allow patients gait to be recorded in a relaxed and convenient environment without the need for a trained therapist to be present. Thus, therapists can use their expertise to diagnose and treat gait rather than spending time mastering and using marker systems [11].

Even though the above factors are true much detail regarding the dynamic characteristics of human locomotion are still essential. The dynamic characteristics of developed systems are expected to be exactly or at least as close as possible to that of the human locomotion. In fact, the closer the characteristic the better will be the performance of the system. It is towards this aspect that this paper is oriented. As an initial step an attempt towards the development of an empirical mathematical model to describe the dynamics of human locomotion is attempted. The model developed will help us in implementing a cost effective assistive limb mechanism. As the first step to achieve the above objective the dynamic modeling of the locomotion of the knee of a healthy person is carried out, which ultimately resulted in the proper development of a assistive device. The subsequent sections deal with this aspect in detail.

2. SYSTEM DESCRIPTION

2.1 Introduction

The study and research related to biomechanics of human body will be much simple by considering the human body as a block diagram shown in fig 1(A), which is popularly known as stick diagram [13] which is much useful for biomechanical analysis. The human body will be further classified as different subsystems as indicated in fig 1(B). Each one of them is considered in brief as under.

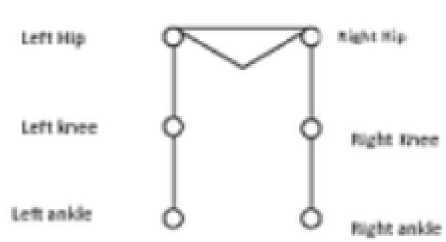

Fig 1(A) Human Body Block Diagram

2.1.1 Biomechanical Description

In Biomechanical engineering the knee is considered as a hinge joint consisting of two parts each is considered as lever like structure, whose various movements are explained with the help of fig 3(A,B,C). For easy understanding in biomechanical engineering point of view without losing the conceptual reality, the human leg can be divided into two segments called the thigh of length (l1) as shown in fig 3(A), shank of length (l2) as shown in fig 3(B). They are connected together by a hinge as shown in fig 3(C).The dynamics of this hinge joint is a representation of dynamics of the knee joint in physiology. The objective is to make the biomechanical system dynamics exactly similar to that of the physiology system dynamics. Here in this paper the concentration is for the development of the general mathematical model of the physiological system, which can be further utilized for developing an electronically controlled drive mechanism of the hinge joint as shown in fig 3(C) having the exactly similar dynamics.

The considered dynamics is having two parts 1. With linear distance and 2. With angular displacement. Both are considered here in detail.

Fig 3(A) Thigh   Fig 3 (B) Shank   Fig 3(C) Biomechanical Structure of Knee

This is the brief operation and description of the human locomotion and the development of mathematical model of the contribution towards the dynamics by subsystem3 are considered as under in section3.

3. SYSTEM DYNAMICS AND MATHEMATICAL MODEL DEVELOPMENT

3.1 Experimental Observations

From the gait video analysis it is observed that for persons with identical characteristics the knee flexion varies linearly for a particular activity under identical conditions. The variation of the human knee joint angle is taken for a finite period of time as shown in Appendix A. This characteristic is assumed to be similar for the further periods with similar conditions. The objective of the work is to develop a linear mathematical model for the above observed dynamics. This dynamics is formulated on the assumption that the dynamic variable is a linear combination of a base average value and infinite number of hierarchically considered variational terms [17]. These variational terms are called variational part in base average value, second variational part in the base average value and so on. The procedure for deriving these factors is explained in details as follows.

3.1.1 Primary Average Value sector

A close examination of the observed values corresponding to the knee joint angle as indicated in Appendix A resulted in the fact that the time 0-20 secs can be divided into different sets of gait cycle i.e. 1.3 secs such that the average values (knee joint angles) of these sectors are almost identical. These sectors are called primary sectors.

3.1.2 Secondary Sectors

For further experimentation with the observed values inside the primary sectors, inner secondary sectors exists which yields the average values which are slightly and independently different from the average values of the primary sectors. They are named as secondary averages.

3.1.3 Tertiary Sectors

Similarly, some tertiary sectors are identified inside each of the above secondary sectors which further yields slightly different average values from the respective average values of the secondary sector. They are termed as tertiary averages.

3.2 Derivation of Input Output Equations

Observation of the experimentally observed characteristics of the variation of values at the output side and that of the input side is done. It is always possible to have a relationship between the input and output dynamics. Assuming the system as a linear one, the following empirical relation is developed.

As said above, it will be quite conforming to think that the output dynamics is a linear combination of component

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involving a base average value bavg, a variational part over the above average value, first change in base avg Δbavg, a factor containing variational part in Δbavg i.e. Δ^2 bavg and so on [17]. It can be stated in another way that the output is a combination of base average value and infinite number of hierarchically considered variational terms. To make it more clear, these variational terms must be multiplied by factors (A1,A2,A3--------). The input output relationship can be represented empirically as

A1*(Base Avg) + A2*(First change in Base Avg)+A3*(Second change in Base Avg) and so on = output (angular distance covered) (1)

Here the output is the total angular distance covered. This approach is somewhat similar to the dynamic models describe in [17]. The main difference is that the present argument is based upon an average value whereas the one in [17] is treated mainly taking time as the independent variable. In [17] the input variable is a function of time whereas in the proposed algorithm it is a function of average value. Again it is assumed that the variational terms at the input is restricted upto two terms which are derived or formulated from directly measured knee joint angles i.e.equation 1 gets modified to

$$A1*(bavg) + A2*(\Delta bavg)+A3*(\Delta^2 bavg) = \text{output (angular distance covered)} \quad (2)$$

In the light of the above explanations, the input output dynamics is stated as under

$$A1*(bavg) + A2*(\Delta bavg)+A3*(\Delta^2 bavg) = \text{output (angular distance covered) primary sector1} \quad (3)$$

$$A1*(bavg) + A2*(\Delta bavg)+A3*(\Delta^2 bavg) = \text{output (angular distance covered) primary sector2} \quad (4)$$

$$A1*(bavg) + A2*(\Delta bavg)+A3*(\Delta^2 bavg) = \text{output (angular distance covered) primary sector3} \quad (5)$$

3.3 Experimental Setup and Procedure

The entire procedure of determining the linear mathematical model of human knee is explained by conducting the experiments on subjects on normal walking.

Normal Walking: The subjects are now allowed to walk along a strip of paper or paint in a straight line for a finite distance.The video is captured and the knee angle is measured with the help of trace paper/slide from the video taken. The angles are measured and is tabulated as shown in [Appendix A].

The above method is repeated on five subjects and Average based knee locomotion algorithm is applied for one subject and the linear mathematical model for the dynamics of knee variation in the locomotive movements can be detemined and tested.

3.4 Algorithm

The above described values i.e. the average values and the variational terms are calculated from the observed values of the output shown in Appendex A. The base value and the variational components are displayed in for the considered or observational time period. The proposed algorithm is applied on the data collected as follows. Overall algorithm can be explained as follows

1. Videos taken are used for the analysis of knee joint angle

2. Measure the knee joint angle of both legs with the help of goniometer by tracing the image on the slide from the video/screen.

3. Advance the video frame by frame and draw the picture of the body. Presently our concentartion /study is on knee joint angle.

4. Repeat the procedure in steps 2 & 3 for the entire video and tabulate the reading seperatly for different subjects.

5. Reading are tabulated for both types 1)Normal walking (linear distance) 2)Normal walking (angular distance)

6. Divide the entire time into equal parts of same duration.

7. Find the average joint knee angle of the entire period. Let this be the base Average.

8. find the average joint knee angle of the individual parts of the entire period. Find the difference between them (individual avg ~ base avg). find out the average of the above values. Let this be the first change in base average.

9.For each part of the full period. Consider the average as the base of that particular part and repeat step(8)

10. Find the Average of the values obtained in step(9). Let this be the second change in base Average.
11. Form the differential equation in the following form. Angular Distance covered= A1(Base Avg) + A2(First change in Base Avg)+A3(Second change in Base Avg).

12. Formulate required number of equations of the form in step(11) and solve for the unknown coefficients.

13. With the coefficients obtained so, check the validity of the equations formulated.

3.5 Applications

Case study 1: Normal Walking (Linear Distance)

The proposed algorithm is applied to the normal walking case and the mathematical model of the variation of joint knee angle is developed as below[18].

The normal walking video is captured for a finite time i.e. 20 seconds. The entire data base is divided into four sets of 5 seconds each. The proposed method is applied and variational terms are obtained as shown in table 2

Table 1 Base Values and variational values

TIME	BASE AVG Value	FIRST CHANGE	SECOND CHANGE	TYPE OF DATA
0-5(subject 1)	154	5.2	15.3	EQUATION-1
5-10(subject 1)	153	6.4	10.88	EQUATION-2
10-15(subject 1)	150	3.2	14.4	EQUATION-3
15-20(subject 2)	152	5.8	11.62	TESTING

The entire distance covered in 20 seconds is 14.4 M and for each set of data of duration 5 seconds the distance covered is 3.6M.

The following three equations are formulated from the data collected as shown in table 5(b).

$$154*B1 + 5.2*B2 + 15.3*B3 = 3.6 \quad (6)$$

$$153*B1 + 6.4*B2 + 10.88*B3 = 3.6 \quad (7)$$

$$150*B1 + 3.2*B2 + 14.4*B3 = 3.6 \quad (8)$$

Here the relation between a dependent variable and independent variable is not accurately available from theory. The optimum form of relation and the ‘best ‘ set of numerical coefficients, given a set of measured data is found using MATLAB. The three unknown coefficients B1,B2 and B3 are 0.0267, -0.0452 and -0.0183 respectively.

$$0.0267*(bavg)+(-0.0452)*(\Delta bavg)+(-0.0183)*(\Delta^2 bavg) = \text{distance covered} \quad (9)$$

Normal walking (Angular Distance)

The proposed algorithm is applied to the normal walking case considering angular distance covered in one gait cycle as the output variable, the mathematical model of the variation of knee joint angle is developed as below.

From the video analysis the time taken for one gait cycle is 1.3secs of which stance phase is for 800 msecs and swing phase for 500 msecs.

The normal walking video is captured for a finite time i.e. 20 seconds. The data base is divided into 10 sets each of one gait cycle. The proposed method is applied and variational terms are obtained for the swing phase of each set as shown in table 2.

The angular distance covered for each gait cycle is calculated and the following three equations are formulated from the data collected as shown in table2.

Table 2 Normal Walking (angular distance) variatinal terms

SET NO /SubSet	BASE AVG Value	FIRST CHANGE	SECOND CHANGE	TYPE OF DATA
1	144	8.1667	10.833	TESTING
2 (0.8-1.3secs)	130	3.2778	4.722	TESTING
3	161	9.111	9.222	TESTING
4	152	5.8333	7.8333	TESTING
5 (2.1-2.6 secs)	128	1.944	5.3889	TESTING
6	148	11.5	10.5	TESTING

7	140	11.5	11.1667	EQUATION-1
8 (3.4-3.9 secs)	126	3	4	EQUATION-2
9	155	12.22	10.111	EQUATION-3
10	142	6.333	8	TESTING
11 (4.7-5.2 secs)	129	15.1667	31.833	TESTING
12	152	9.7778	5.5556	TESTING
13	146	4.2778	5.8333	TESTING
14 (6-6.5 secs)	132	2.8333	0.5	TESTING
15	150	7.111	5.444	TESTING
16	140	8.6667	6	TESTING
17 (7.3-7.8 secs)	129	1.2778	7.0556	TESTING
18	155	4.9444	10.9444	TESTING
19	155	8.1667	8.1667	TESTING
20 (8.6-9.1 secs)	126	1.2778	2.9444	TESTING
21	136	6.0556	8.7222	TESTING
22	147	7.1667	5.5	TESTING
23(9.9- 10.4 secs)	128	5.9444	4.7222	TESTING
24	134	4.1667	4.5	TESTING
25	145	10.6111	8.9444	TESTING
26 (11.2-11.7 secs)	125	0.7778	3.4444	TESTING
27	149	11.1222	11.6111	TESTING
28	156	4.4444	6.1111	TESTING
29 (12.5-13 secs)	128	3.5556	1.8889	TESTING
30	145	6.6667	15.3333	TESTING

$$140*B1 + 11.5*B2 + 11.1667*B3 = 979 \quad (10)$$

$$126*B1 + 3*B2 + 4*B3 = 879 \quad (11)$$

$$155*B1 + 12.22*B2 + 10.111*B3 = 1085 \quad (12)$$

Here the relation between a dependent variable and independent variable is not accurately available from theory. The optimum form of relation and the 'best ' set of numerical coefficients, given a set of measured data is found using MATLAB. The three unknown coefficients B1,B2 and B3 are 6.9777, 0.8490 and -0.6844 respectively.

$$6.9777*(bavg)+(0.8490)*(\Delta bavg)+(-0.6844)*(\Delta^2 bavg) = \text{angular distance covered} \quad (13)$$

4. MODEL VALIDATION

The above developed mathematical model is validated by comparing the calculated dynamics through equations 9 and 13 with the experimentally detected values from the real system. The details of the validation procedure are shown as under.

Now the correctness of the developed equation using the proposed method is checked using the remaing set of gait cycles i.e set of data. From the reading mentioned in the table 2 for the above values 9 and 13 becomes

$$0.0267*(152)+(-0.0452)*(5.8)+(-0.0183)*(11.62) = 3.58 \quad (14)$$

$$6.9777*(132)+(0.8490)*(2.8333)+(-0.6844)*(0.5) = 922 \quad (15)$$

The detailed error[Table 3] is found to be in reasonable limits so as to establish the validity of the model.The performance evaluation can be summarised for both the cases in the following table with respect to the error in the angular distance covered.

Table 3 Normal Walking (angular distance) output

SET NO /SubSet	From real system	From Model $6.9777*BAVG+0.8490*\Delta BAVG+(-0.6844)*\Delta^2 BAVG$
1	1007	1005
2 (0.8-1.3)secs	907	907
3	1124	1124
4	1064	1060
5 (2.1-2.6)secs	893	893
6	1184	1180
7	979	979
8 (3.4-3.9)secs	879	879
9	1085	1085
10	990	852
11(4.7-5.2)secs	900	891
12	1060	1062
13	1018	1022
14(6-6.5)secs	921	922
15	1046	1049
16	980	981
17(7.3-7.8)secs	900	902
18	1087	1082
19	1082	1084
20(8.6-9.1)secs	879	879
21	949	949
22	1032	1026
23 (9.9-10.4)secs	898	893
24	937	935
25	1012	1012
26 (11.2-11.7)secs	877	872
27	1046	1039
28	1089	1089
29 (12.5-13)secs	897	894
30	1015	1012

Figure 5 and 6 below show the time based plot of the real system and model developed respectively.

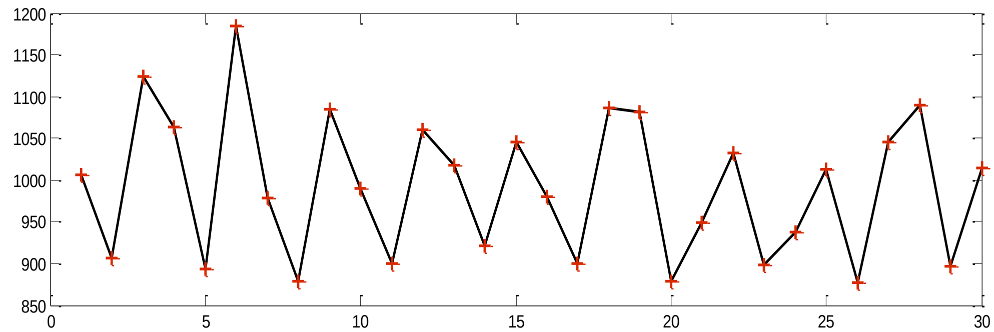

Fig 5 Time base plot of real system

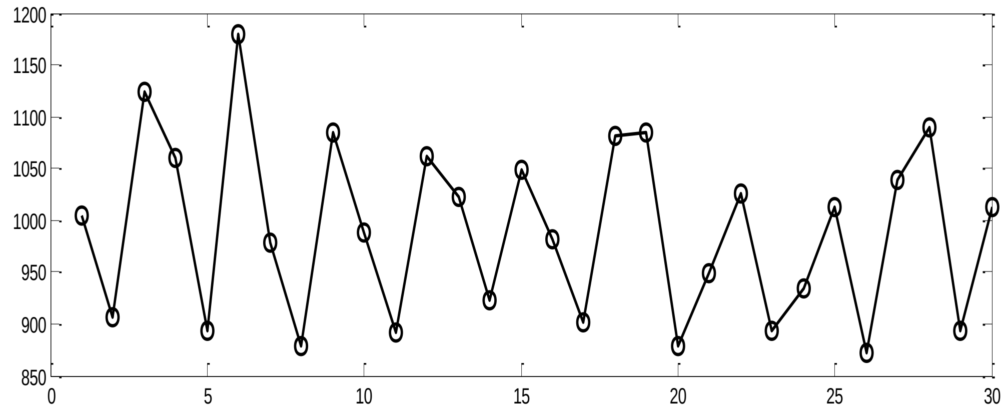

Fig 6 Time base plot of model

5. NEWTON GREGORY FORMULAE

The proposed algorithm in section 3.4 i.e. Average value Algorithm is applied to the normal walking case, considering angular distance covered in one gait cycle as the output variable the mathematical model of the variation of knee joint angle is developed as below.Here in the video analysis the time taken for one gait cycle is 1.3 secs of which stance phase is for 800 msecs and swing phase for 500 msecs. The normal walking video is captured for a finite time i.e. 20 secs. The database is divided into 10 sets each of one gait cycle. The proposed method is applied and variational terms are obtained for the swing phase of each set as shown in table 3

Once again for convenience the graphical function is tabulated as a function of time as follows

Table 4 Cummulative angular displacement vs gait cycle for real system and model

X	F(X) for real time system	F(X) for model	X	F(X) for real time system	F(X) for model	X	F(X) for real time system	F(X) for model
1	1007	1005	11	900	891	21	949	949
2	907	907	12	1060	1062	22	1032	1026
3	1124	1124	13	1018	1022	23	898	893
4	1064	1060	14	921	922	24	937	935
5	893	893	15	1046	1049	25	1012	1012
6	1184	1180	16	980	981	26	877	872
7	979	979	17	900	902	27	1046	1039
8	879	879	18	1087	1082	28	1089	1089
9	1085	1085	19	1082	1084	29	897	894
10	990	852	20	879	879	30	1015	1012

From the table 4 by applying Newton's forward difference interpolation technique or Newton Gregory Formulae, the above values are stated as the general function i.e. both model and real system is expressed as a general polynomial form.

5.1 Newtons Forward Difference Interpolation

From theory we know, interpolation is the process of approximating a given function, whose values are known at N+ 1 tabular point, by a suitable polynomial $P_N(x)$ of degree N which takes the values y_i at $x=x_i$ for i=0,1,2,…… Note that if the given data has errors, it will also be reflected in the polynomial so obtained.

In the following, we shall use forward differences to obtain polynomial function approximating y=f(x) when the tabular points x_i 's are equally spaced. Let

$$f(x) \approx P_N(x),$$

Where the polynomial $P_N(x)$ is given in the following form:

$$P_N(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \cdots + a_k(x-x_0)(x-x_1)\cdots(x-x_{k-1}) + a_N(x-x_0)(x-x_1)\cdots(x-x_{N-1}). \quad 5.6$$

for some constants $a_0, a_1, a_2, \ldots$. to be determined using the fact that $P_N(x_i)$ for i=0,1,2,….,N

So, for i=0, substitute $x=x_0$ in (5.6) to get $P_N(x_0) = y_i$ This gives us $a_0 = y_0$ Next,

$$P_N(x_1) = y_1 \Rightarrow y_1 = a_0 + (x_1 - x_0)a_1.$$

So, $a_1 = \frac{y_1 - y_0}{h} = \frac{\Delta y_0}{h}.$ for $i=2, \quad y_2 = a_0 + (x_2 - x_0)a_1 + (x_2 - x_1)(x_2 - x_0)a_2,$ or equivalently

$$2h^2 a_2 = y_2 - y_0 - 2h\left(\frac{\Delta y_0}{h}\right) = y_2 - 2y_1 + y_0 = \Delta^2 y_0.$$

Thus, $a_2 = \frac{\Delta^2 y_0}{2h^2}.$ now, using mathematical induction, we get

$$a_k = \frac{\Delta^k y_0}{k!\, h^k} \quad \text{for} \quad k = 0, 1, 2, \ldots, N.$$

Thus,

$$P_N(x) = y_0 + \frac{\Delta y_0}{h}(x-x_0) + \frac{\Delta^2 y_0}{2!\, h^2}(x-x_0)(x-x_1) + \cdots + \frac{\Delta^k y_0}{k!\, h^k}(x-x_0)\cdots(x-x_{k-1}) + \frac{\Delta^N y_0}{N!\, h^N}(x-x_0)\ldots(x-x_{N-1}).$$

As this uses the forward differences, it is called NEWTON'S FORWARD DIFFERENCE FORMULA for interpolation, or simply, forward interpolation formula.
For the sake of numerical calculations, we give below a convenient form of the forward interpolation formula.

Let

$$u = \frac{x - x_0}{h},$$

Then

$$x - x_1 = hu + x_0 - (x_0 + h) = h(u-1), x - x_2$$

With this transformation the above forward interpolation formula is simplified to the following form:

$$P_N(u) = y_0 + \frac{\Delta y_0}{h}(hu) + \frac{\Delta^2 y_0}{2!\,h^2}\{(hu)(h(u-1))\} + \cdots + \frac{\Delta^k y_0 h^k}{k!\,h^k}[u(u-1)\cdots(u-k+1)]$$

$$+ \cdots + \frac{\Delta^N y_0}{N!\,h^N}\Big[(hu)\big(h(u-1)\big)\cdots\big(h(u-N+1)\big)\Big].$$

$$= y_0 + \Delta y_0(u) + \frac{\Delta^2 y_0}{2!}(u(u-1)) + \cdots + \frac{\Delta^k y_0}{k!}\Big[u(u-1)\cdots(u-k+1)\Big]$$

$$+ \cdots + \frac{\Delta^N y_0}{N!}\Big[u(u-1)\ldots(u-N+1)\Big]. \qquad (5.7)$$

If N =1, we have a linear interpolation given by

$$f(u) \approx y_0 + \Delta y_0(u). \qquad (5.8)$$

For N=2, we get a quadratic interpolating polynomial:

$$f(u) \approx y_0 + \Delta y_0(u) + \frac{\Delta^2 y_0}{2!}[u(u-1)] \qquad (5.9)$$

and so on.

It may be pointed out here that if f(x) is a polynomial function of degree N then $P_N(x)$ coincides with f(x) on the given interval. Otherwise, this gives only an approximation to the true values of f(x)

From the table 4 by applying Newton's forward difference interpolation technique or Newton Gregory Formulae, the above values are stated as the general function i.e. both model and real system is expressed as a general polynomial form. The readings in the table are divided into six 5 point tables and a fourth order polynomial is derived for each of the six sets. Any value in between the points can be approximated by the corresponding polynomial derived. The procedure for derivation of the polynomial function for both real system and model is as follows

For the data available we have to form the forward difference table as below

X_i	y_i	Δy_i	$\Delta^2 yi$	$\Delta^3 y_i$	$\Delta^4 y_i$
1	1007	-100	317	-594	760
2	907	217	-277	166	
3	1124	-60	-111		
4	1064	-171			
5	893				

Here x0=1; h=1; u = (x-x0)/h

$P_4(x)$ = 1007+u*(-100)+u(u-1)/2! (317) + u(u-1)(u-2)/3! (-594) + u(u-1)(u-2)(u-3)/4! (760) (a)

In the similar lines for the other sets of readings polynomial is expressed as follows

X_i	y_i	Δy_i	$\Delta^2 yi$	$\Delta^3 y_i$	$\Delta^4 y_i$
6	1184	-205	-105	411	1018

7	979	-100	306	-607	
8	879	206	-301		
9	1085	-95			
10	990				

Here x0=6; h=1; u = (x-x0)/h

$P_4(x)$ = 1184+u*(-205)+u(u-1)/2! (-105) + u(u-1)(u-2)/3! (411) + u(u-1)(u-2)(u-3)/4! (1018) (b)

X_i	y_i	Δy_i	$\Delta^2 yi$	$\Delta^3 y_i$	$\Delta^4 y_i$
11	900	160	-202	-147	424
12	1060	-42	-55	277	
13	1018	-97	222		
14	921	125			
15	1046				

Here x0=11; h=1; u = (x-x0)/h

$P_4(x)$ = 900+u*(160)+u(u-1)/2! (-202) + u(u-1)(u-2)/3! (-147) + u(u-1)(u-2)(u-3)/4! (424) (c)

X_i	y_i	Δy_i	$\Delta^2 yi$	$\Delta^3 y_i$	$\Delta^4 y_i$
16	980	-80	267	-459	-453
17	900	187	-192	-6	
18	1087	-5	-198		
19	1082	-203			
20	879				

Here x0=16; h=1; u = (x-x0)/h

$P_4(x)$ = 980+u*(-80)+u(u-1)/2! (267) + u(u-1)(u-2)/3! (-459) + u(u-1)(u-2)(u-3)/4! (-453) (d)

X_i	y_i	Δy_i	$\Delta^2 yi$	$\Delta^3 y_i$	$\Delta^4 y_i$
21	949	83	-217	390	-527
22	1032	-134	173	-137	
23	898	39	36		
24	937	75			
25	1012				

Here x0=21; h=1; u = (x-x0)/h

$P_4(x)$ = 949+u*(83)+u(u-1)/2! (-217) + u(u-1)(u-2)/3! (390) + u(u-1)(u-2)(u-3)/4! (-527) (e)

X_i	y_i	Δy_i	$\Delta^2 yi$	$\Delta^3 y_i$	$\Delta^4 y_i$
26	877	169	-126	-109	654
27	1046	43	-235	545	
28	1089	-192	310		
29	897	118			
30	1015				

Here x0=26; h=1; u = (x-x0)/h

$P_4(x)$ = 877+u*(169)+u(u-1)/2! (-126) + u(u-1)(u-2)/3! (-109) + u(u-1)(u-2)(u-3)/4! (654) (f)

The above procedure for obtaining the polynomial for model can also be done on the similar lines and the polynomials can be obtained as

For set 1 i.e. x=1 to 5

$P_4(x)$ = 1005+u*(-98)+u(u-1)/2! (315) + u(u-1)(u-2)/3! (-596) + u(u-1)(u-2)(u-3)/4! (774) (g)

$P_4(x)$ = 1180+u*(-201)+u(u-1)/2! (101) + u(u-1)(u-2)/3! (205) + u(u-1)(u-2)(u-3)/4!(-950) (h)

$P_4(x)$ = 891+u*(171)+u(u-1)/2! (-211) + u(u-1)(u-2)/3! (151) + u(u-1)(u-2)(u-3)/4! (136) (i)

$P_4(x)$ = 981+u*(-79)+u(u-1)/2! (259) + u(u-1)(u-2)/3! (-437) + u(u-1)(u-2)(u-3)/4! (408) (j)

$P_4(x)$ = 949+u*(77)+u(u-1)/2! (-210) + u(u-1)(u-2)/3! (385) + u(u-1)(u-2)(u-3)/4! (-525) (k)

$P_4(x)$ = 872+u*(167)+u(u-1)/2! (-117) + u(u-1)(u-2)/3! (-128) + u(u-1)(u-2)(u-3)/4! (686) (l)

In general

$$\begin{aligned} P_N(u) &= y_0 + \frac{\Delta y_0}{h}(hu) + \frac{\Delta^2 y_0}{2!\,h^2}\{(hu)(h(u-1))\} + \cdots + \frac{\Delta^k y_0 h^k}{k!\,h^k}[u(u-1)\cdots(u-k+1)] \\ &\quad + \cdots + \frac{\Delta^N y_0}{N!\,h^N}\Big[(hu)\big(h(u-1)\big)\cdots\big(h(u-N+1)\big)\Big]. \\ &= y_0 + \Delta y_0(u) + \frac{\Delta^2 y_0}{2!}(u(u-1)) + \cdots + \frac{\Delta^k y_0}{k!}\Big[u(u-1)\cdots(u-k+1)\Big] \\ &\quad + \cdots + \frac{\Delta^N y_0}{N!}\Big[u(u-1)\ldots(u-N+1)\Big]. \end{aligned} \tag{6.0}$$

If N =1, we have a linear interpolation given by

$$f(u) \approx y_0 + \Delta y_0(u). \tag{6.1}$$

For N=2, we get a quadratic interpolating polynomial:

$$f(u) \approx y_0 + \Delta y_0(u) + \frac{\Delta^2 y_0}{2!}[u(u-1)] \tag{6.2}$$

The above equation gives smooth characteristics as shown in fig below. For the estimation of values in between the range a MATLAB code is written. From the software code more number of readings can be obtained by the general polynomial expression.

Finally it can be concluded that the dynamics of human locomotion is expressed as a polynomial as follows. The cumulative angular displacement verses displacement is expressed as a polynomial.

$$\begin{aligned} P_N(u) &= y_0 + \frac{\Delta y_0}{h}(hu) + \frac{\Delta^2 y_0}{2!\,h^2}\{(hu)(h(u-1))\} + \cdots + \frac{\Delta^k y_0 h^k}{k!\,h^k}[u(u-1)\cdots(u-k+1)] \\ &\quad + \cdots + \frac{\Delta^N y_0}{N!\,h^N}\Big[(hu)\big(h(u-1)\big)\cdots\big(h(u-N+1)\big)\Big]. \end{aligned}$$

$$= \quad y_0 + \Delta y_0(u) + \frac{\Delta^2 y_0}{2!}(u(u-1)) + \cdots + \frac{\Delta^k y_0}{k!}\left[u(u-1)\cdots(u-k+1)\right]$$

$$+ \cdots + \frac{\Delta^N y_0}{N!}\left[u(u-1)...(u-N+1)\right]. \qquad (6.0)$$

The values obtained by using the Newton's Forward difference method can also be validated as the procedure adapted previously. The above equation gives smooth characteristics as shown in fig 7 . For the estimation of values in between the range a MATLAB code is written. From the software code more number of readings can be obtained by the general polynomial expression.

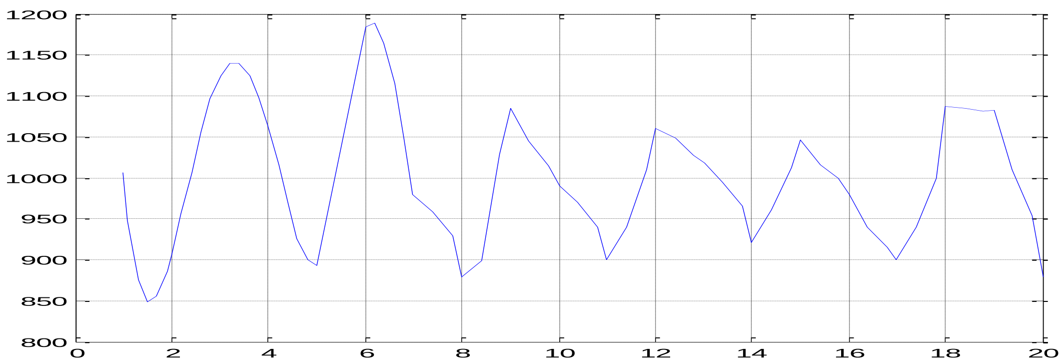

Fig 7 angular displacement verses sectors

6. CONCLUSION

A Novel modeling technique based on Average Value algorithm and Newton Gregory Formulae has been developed for a healthy human knee joint locomotion. The results obtained from the proposed model and actual results of locomotion of human knee joint were giving close results. Knee joint locomotion is observed and the corresponding knee joint angle at different time instants are recorded by conducting experiments on walking on normal floor with output variable as linear distance and angular displacement. A number of readings are taken in both the cases and a database is prepared from the real system. In this paper we considered subjects performing gait normal floor. The knee joint is modeled from the data acquired by means of calculating the base value and the variational components. The entire procedure is achieved through the video picture of the locomotion captured by single or multiple cameras with proper resolution. This method is only a simple empirical modeling technique. This algorithm in modeling knee locomotion will help us in developing a simple, easily implementable, cost effective assistive limb system for partially or completely paralysis people. The results obtained from the proposed model and actual results of locomotion of human knee joint were giving close results. This shows the validity of the proposed algorithm.

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APPENDIX A

FRAME NO	ANGLE (1 GAIT CYCLE)	ANGLE (2 GAIT CYCLE)	ANGLE (3 GAIT CYCLE)	ANGLE (4 GAIT CYCLE)	ANGLE (5 GAIT CYCLE)	ANGLE (6 GAIT CYCLE)	ANGLE (7 GAIT CYCLE)	ANGLE (8 GAIT CYCLE)	ANGLE (9 GAIT CYCLE)	ANGLE (10 GAIT CYCLE)
1 – 32 STANCE PHASE	**180**	**180**	**180**	**180**	**180**	**180**	**180**	**180**	**180**	**180**
33-53 SWING PHASE	158	164	158	155	156	158	160	163	151	162
	152	158	157	148	151	153	152	163	144	158
	150	155	151	144	145	142	143	163	139	153
	145	152	135	140	145	135	135	160	139	144
	142	150	130	135	145	131	130	153	139	135
	132	145	125	130	145	131	124	147	139	132
	128	140	123	126	135	131	125	135	139	128
	128	132	123	125	129	131	125	130	130	126
	126	125	123	128	128	126	122	125	125	124
	125	123	123	132	130	128	122	125	125	124
	128	125	123	132	130	126	122	125	120	124
	130	130	124	132	135	125	126	124	120	124
	135	128	128	137	135	131	126	125	125	125
	135	130	135	137	135	135	132	125	128	130
	142	134	135	149	142	145	139	125	132	132
	152	135	138	149	144	150	146	125	138	135
	158	138	148	155	144	150	154	135	138	139
	158	140	155	164	144	158	161	136	138	149
	165	148	162	171	152	158	171	136	138	156
	174	154	171	171	158	158	172	137	145	165
	175	163	176	172	165	168	173	155	155	170

Knee joint angle database of human locomotion for a finite period