Estimation of Smoothing Constant with Optimal Parameters of Weight in the Medical Case of Blood Extracorporeal Circulation Apparatus

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ABSTRACT

In industries, how to improve forecasting accuracy such as sales, shipping is an important issue. There are many researches made on this. In this paper, a hybrid method is introduced and plural methods are compared. Focusing that the equation of exponential smoothing method (ESM) is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method is proposed before by us which satisfies minimum variance of forecasting error. Generally, smoothing constant is selected arbitrarily. But in this paper, we utilize above stated theoretical solution. Firstly, we make estimation of ARMA model parameter and then estimate smoothing constants. Thus theoretical solution is derived in a simple way and it may be utilized in various fields. Furthermore, combining the trend removing method with this method, we aim to improve forecasting accuracy. An approach to this method is executed in the following method. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is carried out to the sum total medical data of production and imports of Blood extracorporeal circulation apparatus for three cases (Hemodialysis apparatus, Dialyzer, and Blood circuit).

The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. The new method shows that it is useful for the time series that has various trend characteristics and has rather strong seasonal trend. The effectiveness of this method should be examined in various cases.

Keywords: minimum variance, exponential smoothing method, forecasting, trend, blood extracorporeal circulation apparatus

1. INTRODUCTION

Many methods for time series analysis have been presented such as Autoregressive model (AR Model), Autoregressive Moving Average Model (ARMA Model) and Exponential Smoothing Method (ESM)¹¹–¹⁴. Among these, ESM is said to be a practical simple method.

For this method, various improving method such as adding compensating item for time lag, coping with the time series with trend¹⁵, utilizing Kalman Filter¹⁶, Bayes Forecasting¹⁷, adaptive ESM¹⁸, exponentially weighted.

Moving Averages with irregular updating periods⁹, making averages of forecasts using plural method¹⁰ are presented. For example, Maeda¹⁶ calculated smoothing constant in relationship with S/N ratio under the assumption that the observation noise was added to the system. But he had to calculate under supposed noise because he could not grasp observation noise. It can be said that it does not pursue optimum solution from the very data themselves which should be derived by those estimation. Ishii¹¹ pointed out that the optimal smoothing constant was the solution of infinite order equation, but he didn’t show analytical solution. Based on these facts, we proposed a new method of estimation of smoothing constant in ESM before¹². Focusing that the equation of ESM is equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in ESM was derived.

In this paper, utilizing above stated method, a revised forecasting method is proposed. In making forecast such as production data, trend removing method is devised. Trend removing by the combination of linear and 2nd order non-linear function and 3rd order non-linear function is executed to the sum total medical data of production and imports of Blood extracorporeal circulation apparatus for three cases (Hemodialysis apparatus, Dialyzer, and Blood circuit). The weights for these functions are set 0.5 for two patterns at first and then varied by 0.01 increment for three patterns and optimal weights are searched. For the comparison, monthly trend is removed after that. Theoretical solution of smoothing constant of ESM is calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting is executed on these data. This is a revised forecasting method. Variance of forecasting error of this newly proposed method is assumed to be less than those of previously proposed method. The rest of the paper is organized as follows. In section 2, ESM is stated by ARMA model and estimation method of smoothing constant is derived using ARMA model identification. The combination of linear and non-linear function is introduced for trend removing in section 3. The Monthly Ratio is referred in section 4. Forecasting is executed in section 5, and estimation accuracy is examined.
2. DESCRIPTION OF ESM USING ARMA MODEL.

In ESM, forecasting at time $t+1$ is stated in the following equation.

$$\hat{x}_{t+1} = \hat{x}_t + \alpha (x_t - \hat{x}_t)$$

$$= \alpha x_t + (1 - \alpha) \hat{x}_t$$

(1)

Here,

- $\hat{x}_{t+1}$: forecasting at $t + 1$
- $x_t$: realized value at $t$
- $\alpha$: smoothing constant $(0 < \alpha < 1)$

(1) is re-stated as

$$\hat{x}_{t+1} = \sum_{j=0}^{\infty} \alpha (1 - \alpha)^j x_{t-j}$$

(2)

By the way, we consider the following $(1,1)$ order ARMA model.

$$x_t - x_{t-1} = e_t - \beta e_{t-1}$$

(3)

Generally, $(p,q)$ order ARMA model is stated as

$$x_t + \sum_{i=1}^{p} a_i x_{t-i} = e_t + \sum_{j=1}^{q} b_j e_{t-j}$$

(4)

Here, $\{x_t\}$:

Sample process of Stationary Ergodic Gaussian Process $x(t)$ $t = 1, 2, \ldots, N, \ldots$

$\{e_t\}$: Gaussian White Noise with $0$ mean $\sigma_e^2$ variance

MA process in (4) is supposed to satisfy convertibility condition. Utilizing the relation that

$$E[e_t | e_{t-1}, e_{t-2}, \ldots] = 0$$

we get the following equation from (3).

$$\hat{x}_t = x_{t-1} - \beta e_{t-1}$$

(5)

Operating this scheme on $t + 1$, we finally get

$$\hat{x}_{t+1} = \hat{x}_t + (1 - \beta) e_t$$

$$= \hat{x}_t + (1 - \beta) (x_t - \hat{x}_t)$$

(6)

If we set $1 - \beta = \alpha$, the above equation is the same with (1), i.e., equation of ESM is equivalent to $(1,1)$ order ARMA model, or is said to be $(0,1,1)$ order ARIMA model because 1st order AR parameter is $-1$.

Comparing with (3) and (4), we obtain

From (1), (6),

Therefore, we get

$$\begin{cases} a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases}$$

(7)

From above, we can get estimation of smoothing constant after we identify the parameter of MA part of ARMA model. But, generally MA part of ARMA model become non-linear equations which are described below. Let (4) be

$$\hat{x}_t = x_t + \sum_{i=1}^{p} a_i x_{t-i} + \sum_{j=1}^{q} b_j e_{t-j}$$

(8)

$$\tilde{x}_t = e_t + \sum_{j=1}^{q} b_j e_{t-j}$$

(9)

We express the autocorrelation function of $\tilde{x}_t$ as $\tilde{r}_k$ and from (8), (9), we get the following non-linear equations which are well known.

$$\tilde{r}_k = \sigma_e^2 \sum_{j=0}^{q} b_j b_{k+j} \quad (k \leq q)$$

$$0 \quad (k \geq q + 1)$$

(10)

For these equations, recursive algorithm has been developed. In this paper, parameter to be estimated is only $b_1$, so it can be solved in the following way.

From (3) (4) (7) (10), we get

$$\begin{cases} q = 1 \\ a_1 = -1 \\ b_1 = -\beta = \alpha - 1 \end{cases}$$

(11)

If we set

$$\rho_k = \frac{\tilde{r}_k}{\tilde{r}_0}$$

(12)

the following equation is derived.

$$\rho_1 = \frac{b_1}{1 + b_1^2}$$

(13)

We can get $b_1$ as follows.

$$b_1 = \frac{1 \pm \sqrt{1 - 4 \rho_1^2}}{2 \rho_1}$$

(14)

In order to have real roots, $\rho_1$ must satisfy

$$\rho_1 \geq 1$$
\[ |\rho_1| \leq \frac{1}{2} \]  
(15)

From invertibility condition, \( b_1 \) must satisfy
\[ |b_1| < 1 \]
(16)

From (13), using the next relation,
\[
\begin{align*}
(1-b_1)^2 & \geq 0 \\
(1+b_1)^2 & \geq 0
\end{align*}
\]
(17)

(15) always holds. As
\[ \alpha = b_1 + 1 \]
(18)

\( b_1 \) is within the range of
\[ -1 < b_1 < 0 \]
(19)

Finally we get
\[ b_1 = \frac{1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \]
(20)

\[ \alpha = \frac{1 + 2\rho_1 - \sqrt{1 - 4\rho_1^2}}{2\rho_1} \]
(21)

which satisfy above condition. Thus we can obtain a theoretical solution by a simple way.

Here \( \rho_1 \) must satisfy
\[ -\frac{1}{2} < \rho_1 < 0 \]
(22)

in order to satisfy \( 0 < \alpha < 1 \).

Focusing on the idea that the equation of ESM is equivalent to (1,1) order ARMA model equation, we can estimate smoothing constant after estimating ARMA model parameter. It can be estimated only by calculating 0th and 1st order autocorrelation function.

3. Trend Removal Method[12]

As trend removal method, we describe the combination of linear and non-linear function.

[1] Linear function
We set
\[ y = a_1 x + b_1 \]
(23)
as a linear function.

[2] Non-linear function
We set
\[ y = a_i x^2 + b_i x + c_i \]
(24)
as a 2nd and a 3rd order non-linear function.

[3] The combination of linear and non-linear function
We set
\[ y = \alpha_1 (a_1 x + b_1) + \alpha_2 (a_2 x^2 + b_2 x + c_2) \]
(25)

\[ y = \beta_1 (a_1 x + b_1) + \beta_2 (a_2 x^2 + b_2 x + c_2) \]
(26)

\[ y = \gamma_1 (a_1 x + b_1) + \gamma_2 (a_2 x^2 + b_2 x + c_2) + \gamma_3 (a_3 x^3 + b_3 x^2 + c_3 x + d_3) \]
(27)
as the combination of linear and 2nd order non-linear and 3rd order non-linear function. Here, \( \alpha_2 = 1 - \alpha_1 \), \( \beta_2 = 1 - \beta_1 \), \( \gamma_3 = 1 - (\gamma_1 + \gamma_2) \). Comparative discussion concerning (21), (22) and (23) are described in section 5.


For example, if there is the monthly data of L years as stated bellow:
\[ \{ x_{ij} \} (i = 1, \ldots, L) (j = 1, \ldots, 12) \]
(28)

Where, \( x_{ij} \in R \) in which \( j \) means month and \( i \) means year and \( x_{ij} \) is a shipping data of \( i \)-th year, \( j \)-th month. Then, monthly ratio \( \bar{x}_{ij} (j = 1,\ldots,12) \) is calculated as follows.

\[ \bar{x}_j = \frac{1}{L \cdot 12} \sum_{i=1}^{L} \sum_{j=1}^{12} x_{ij} \]
(29)

Monthly trend is removed by dividing the data by (24). Numerical examples both of monthly trend removal case and non-removal case are discussed in 5.

5. Forecasting the Shipping Data of Manufacturer

5.1 Analysis Procedure

Sum total data of Blood extracorporeal circulation apparatus for three cases (Hemodialysis apparatus, Dialyzer, and Blood circuit) from January 2007 to December 2009 are analyzed. These data are obtained from the Annual Report of Statistical Investigation on Statistical-Survey-on-Trends-in-Pharmaceutical-Production by Ministry of Health, Labour and Welfare in Japan. First of all, graphical charts of these time series data are exhibited in Fig. 1, 2, 3.

\[ y = a_1 x + b_1 \]
(30)

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method stated in 3. Then we calculate monthly ratio by the method stated in 4. After removing monthly trend, the method stated in 2 is applied and Exponential Smoothing Constant with minimum variance of forecasting error is estimated. Then 1 step forecast is executed. Thus, data is shifted to 2nd to 25th and the forecast for 26th data is executed consecutively, which finally reaches forecast of 36th data. To examine the accuracy of forecasting, variance of forecasting error is calculated for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Forecasting error is expressed as:

$$e_i = \hat{x}_i - x_i$$  \hspace{1cm} (25)$$

$$\bar{e} = \frac{1}{N} \sum_{i=1}^{N} e_i$$  \hspace{1cm} (26)$$

Variance of forecasting error is calculated by:

$$\sigma_e^2 = \frac{1}{N-1} \sum_{i=1}^{N} (e_i - \bar{e})^2$$  \hspace{1cm} (27)$$

5.2 Trend Removing

Trend is removed by dividing original data by (21), (22), (23). The patterns of trend removal are exhibited in Table 1.

<table>
<thead>
<tr>
<th>Pattern</th>
<th>(\alpha_1, \alpha_2) are set 0.5 in the equation (21)</th>
<th>(\beta_1, \beta_2) are set 0.5 in the equation (22)</th>
<th>(\alpha_1) is shifted by 0.01 increment in (21)</th>
<th>(\beta_1) is shifted by 0.01 increment in (22)</th>
<th>(\gamma_1) and (\gamma_2) are shifted by 0.01 increment in (23)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern1</td>
<td>(\alpha_1, \alpha_2) are set 0.5 in the equation (21)</td>
<td>(\beta_1, \beta_2) are set 0.5 in the equation (22)</td>
<td>(\alpha_1) is shifted by 0.01 increment in (21)</td>
<td>(\beta_1) is shifted by 0.01 increment in (22)</td>
<td>(\gamma_1) and (\gamma_2) are shifted by 0.01 increment in (23)</td>
</tr>
<tr>
<td>Pattern2</td>
<td>(\beta_1, \beta_2) are set 0.5 in the equation (22)</td>
<td>(\alpha_1) is shifted by 0.01 increment in (21)</td>
<td>(\beta_1) is shifted by 0.01 increment in (22)</td>
<td>(\gamma_1) and (\gamma_2) are shifted by 0.01 increment in (23)</td>
<td></td>
</tr>
<tr>
<td>Pattern3</td>
<td>(\alpha_1) is shifted by 0.01 increment in (21)</td>
<td>(\beta_1) is shifted by 0.01 increment in (22)</td>
<td>(\gamma_1) and (\gamma_2) are shifted by 0.01 increment in (23)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pattern4</td>
<td>(\gamma_1) and (\gamma_2) are shifted by 0.01 increment in (23)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pattern5</td>
<td>(\gamma_1) and (\gamma_2) are shifted by 0.01 increment in (23)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In pattern 1 and 2, the weight of \(\alpha_1, \alpha_2, \beta_1, \beta_2\) are set 0.5 in the equation (21), (22). In pattern 3, the weight of \(\alpha_1\) is shifted by 0.01 increment in (21) which satisfy the range \(0 \leq \alpha_1 \leq 1.00\). In pattern 4, the weight of \(\beta_1\) is shifted in the same way which satisfy the range \(0 \leq \beta_1 \leq 1.00\). In pattern 5, the weight of \(\gamma_1\) and \(\gamma_2\) are shifted by 0.01 increment in (23) which satisfy the range \(0 \leq \gamma_1 \leq 1.00, 0 \leq \gamma_2 \leq 1.00\). The best solution is selected which minimizes the variance of forecasting error. Estimation results of coefficient of (18), (19) and (20) are exhibited in Table 2. Estimation results of weights of (21), (22) and (23) are exhibited in Table 3.

Table 2: Coefficient of (18),(19) and (20)

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a_1)</td>
<td>(b_1)</td>
<td>(a_2)</td>
</tr>
<tr>
<td>Hemodialysis</td>
<td>450.538260869</td>
<td>1691131.8</td>
<td>250.064654</td>
</tr>
<tr>
<td></td>
<td>576</td>
<td>55</td>
<td>9</td>
</tr>
</tbody>
</table>
### Table 3: Weights of (21), (22) and (23)

<table>
<thead>
<tr>
<th></th>
<th>Monthly ratio</th>
<th>Pattern1</th>
<th>Pattern2</th>
<th>Pattern3</th>
<th>Pattern4</th>
<th>Pattern5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemodialysis apparatus</td>
<td>Used</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$\beta_1$</td>
<td>$\beta_2$</td>
<td>$\gamma_1$</td>
</tr>
<tr>
<td></td>
<td>Not used</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Dialyzer</td>
<td>Used</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.3</td>
</tr>
<tr>
<td></td>
<td>Not used</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Blood circuit</td>
<td>Used</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.74</td>
<td>0.26</td>
</tr>
<tr>
<td></td>
<td>Not used</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.7</td>
<td>0.26</td>
</tr>
</tbody>
</table>

Graphical chart of trend is exhibited in Fig. 4, 5, 6 for the cases that monthly ratio is used.

**Fig. 4: Trend of Hemodialysis apparatus**

**Table 3:** Weights of (21), (22) and (23)
5.3 Removing trend of monthly ratio

After removing trend, monthly ratio is calculated by the method stated in 4. Calculation result for 1st to 24th data is exhibited in Table 4 through 8.

Table 4: Monthly ratio (Pattern1)

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemodialysis apparatus</td>
<td>0.89</td>
<td>1.03</td>
<td>1.09</td>
<td>0.92</td>
<td>0.95</td>
<td>1.07</td>
<td>1.06</td>
<td>0.94</td>
<td>1.07</td>
<td>0.96</td>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>Dialyzer</td>
<td>1.03</td>
<td>1.08</td>
<td>0.96</td>
<td>1.06</td>
<td>0.94</td>
<td>1.04</td>
<td>0.96</td>
<td>0.95</td>
<td>0.89</td>
<td>1.06</td>
<td>0.96</td>
<td>1.05</td>
</tr>
<tr>
<td>Blood circuit</td>
<td>0.99</td>
<td>0.97</td>
<td>1.09</td>
<td>0.94</td>
<td>1.01</td>
<td>0.96</td>
<td>1.05</td>
<td>0.94</td>
<td>0.97</td>
<td>1.09</td>
<td>0.95</td>
<td>1.04</td>
</tr>
</tbody>
</table>
Table 5: Monthly ratio (Pattern 2)

<table>
<thead>
<tr>
<th>Month</th>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
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<td>Hemodialysis apparatus</td>
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<td>1.03</td>
<td>1.09</td>
<td>0.92</td>
<td>0.95</td>
<td>1.07</td>
<td>1.06</td>
<td>0.94</td>
<td>1.07</td>
<td>0.96</td>
<td>1</td>
<td>1.02</td>
</tr>
<tr>
<td>Dialyzer</td>
<td>1.04</td>
<td>1.08</td>
<td>0.97</td>
<td>1.06</td>
<td>0.94</td>
<td>1.04</td>
<td>0.96</td>
<td>0.95</td>
<td>0.89</td>
<td>1.06</td>
<td>0.96</td>
<td>1.04</td>
</tr>
<tr>
<td>Blood circuit</td>
<td>0.99</td>
<td>0.97</td>
<td>1.10</td>
<td>0.94</td>
<td>1.01</td>
<td>0.96</td>
<td>1.05</td>
<td>0.94</td>
<td>0.97</td>
<td>1.09</td>
<td>0.95</td>
<td>1.03</td>
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</table>

Table 6: Monthly ratio (Pattern 3)

<table>
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<tr>
<th>Month</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
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<tr>
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<td>0.89</td>
<td>1.03</td>
<td>1.09</td>
<td>0.92</td>
<td>0.95</td>
<td>1.07</td>
<td>1.06</td>
<td>0.94</td>
<td>1.07</td>
<td>0.96</td>
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</tr>
<tr>
<td>Dialyzer</td>
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<td>1.08</td>
<td>0.96</td>
<td>1.06</td>
<td>0.94</td>
<td>1.04</td>
<td>0.96</td>
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<td>1.06</td>
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<td>1.05</td>
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<tr>
<td>Blood circuit</td>
<td>0.98</td>
<td>0.97</td>
<td>1.10</td>
<td>0.94</td>
<td>1.01</td>
<td>0.96</td>
<td>1.05</td>
<td>0.95</td>
<td>0.97</td>
<td>1.09</td>
<td>0.95</td>
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Table 7: Monthly ratio (Pattern 4)

<table>
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<th>3</th>
<th>4</th>
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<td>1.03</td>
<td>1.09</td>
<td>0.92</td>
<td>0.95</td>
<td>1.07</td>
<td>1.06</td>
<td>0.94</td>
<td>1.07</td>
<td>0.96</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>Dialyzer</td>
<td>1.04</td>
<td>1.08</td>
<td>0.97</td>
<td>1.06</td>
<td>0.94</td>
<td>1.04</td>
<td>0.96</td>
<td>0.95</td>
<td>0.89</td>
<td>1.06</td>
<td>0.96</td>
<td>1.05</td>
</tr>
<tr>
<td>Blood circuit</td>
<td>0.99</td>
<td>0.97</td>
<td>1.10</td>
<td>0.94</td>
<td>1.01</td>
<td>0.96</td>
<td>1.05</td>
<td>0.95</td>
<td>0.97</td>
<td>1.09</td>
<td>0.95</td>
<td>1.03</td>
</tr>
</tbody>
</table>

Table 8: Monthly ratio (Pattern 5)

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemodialysis apparatus</td>
<td>0.89</td>
<td>1.03</td>
<td>1.09</td>
<td>0.92</td>
<td>0.95</td>
<td>1.07</td>
<td>1.06</td>
<td>0.94</td>
<td>1.07</td>
<td>0.96</td>
<td>1.00</td>
<td>1.02</td>
</tr>
<tr>
<td>Dialyzer</td>
<td>1.04</td>
<td>1.08</td>
<td>0.97</td>
<td>1.06</td>
<td>0.94</td>
<td>1.04</td>
<td>0.96</td>
<td>0.95</td>
<td>0.89</td>
<td>1.06</td>
<td>0.96</td>
<td>1.05</td>
</tr>
<tr>
<td>Blood circuit</td>
<td>0.99</td>
<td>0.97</td>
<td>1.10</td>
<td>0.94</td>
<td>1.01</td>
<td>0.96</td>
<td>1.05</td>
<td>0.95</td>
<td>0.97</td>
<td>1.09</td>
<td>0.95</td>
<td>1.03</td>
</tr>
</tbody>
</table>

5.4 Estimation of Smoothing Constant with Minimum Variance of Forecasting Error

After removing monthly trend, Smoothing Constant with minimum variance of forecasting error is estimated utilizing (16). There are cases that we cannot obtain a theoretical solution because they do not satisfy the condition of (15). In those cases, Smoothing Constant with minimum variance of forecasting error is derived by shifting variable from 0.01 to 0.99 with 0.01 interval. Calculation result for 1st to 24th data is exhibited in Table 9.

Table 9: Estimated Smoothing Constant with Minimum Variance

<table>
<thead>
<tr>
<th>Monthly ratio</th>
<th>Pattern1</th>
<th>Pattern2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho_1$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Hemodialysis apparatus</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used</td>
<td>-0.4349</td>
<td>0.4175</td>
</tr>
<tr>
<td>Not used</td>
<td>-0.1805</td>
<td>0.8132</td>
</tr>
<tr>
<td>Dialyzer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used</td>
<td>-0.0087</td>
<td>0.9913</td>
</tr>
<tr>
<td>Not used</td>
<td>-0.1301</td>
<td>0.8676</td>
</tr>
<tr>
<td>Blood circuit</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used</td>
<td>-0.2649</td>
<td>0.7134</td>
</tr>
</tbody>
</table>
5.5 Forecasting and Variance of Forecasting Error

Utilizing smoothing constant estimated in the previous section, forecasting is executed for the data of 25th to 36th data. Final forecasting data is obtained by multiplying monthly ratio and trend. Variance of forecasting error is calculated by (27). Forecasting results are exhibited in Fig. 7, 8, 9 for the cases that monthly ratio is used.

<table>
<thead>
<tr>
<th>Monthly ratio</th>
<th>Pattern3</th>
<th>Pattern4</th>
<th>Pattern5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemodialysis apparatus</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used</td>
<td>-0.4352</td>
<td>0.4167</td>
<td>-0.4367</td>
</tr>
<tr>
<td>Not used</td>
<td>-0.1703</td>
<td>0.8245</td>
<td>-0.1703</td>
</tr>
<tr>
<td>Dialyzer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used</td>
<td>-0.0084</td>
<td>0.9916</td>
<td>-0.0106</td>
</tr>
<tr>
<td>Not used</td>
<td>-0.1286</td>
<td>0.8692</td>
<td>-0.1289</td>
</tr>
<tr>
<td>Blood circuit</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used</td>
<td>-0.1786</td>
<td>0.8154</td>
<td>-0.2016</td>
</tr>
<tr>
<td>Not used</td>
<td>-0.4093</td>
<td>0.4800</td>
<td>-0.4819</td>
</tr>
</tbody>
</table>

Fig. 7: Forecasting Results of Hemodialysis apparatus

Fig. 8: Forecasting Results of Dialyzer
Fig. 9: Forecasting Results of Blood circuit

Variance of forecasting error is exhibited in Table 10.

Table 10: Variance of Forecasting Error

<table>
<thead>
<tr>
<th>Monthly ratio</th>
<th>Pattern1</th>
<th>Pattern2</th>
<th>Pattern3</th>
<th>Pattern4</th>
<th>Pattern5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hemodialysis apparatus</td>
<td>40,565,340,616.93270</td>
<td>17,142,890,782.39180</td>
<td>12,980,107,228.009</td>
<td>12,980,107,228.009</td>
<td>12,980,107,228.009</td>
</tr>
<tr>
<td>Not used</td>
<td>46,462,346,932.31830</td>
<td>50,121,615,807.23390</td>
<td>46,165,677,327.171</td>
<td>46,165,677,327.171</td>
<td>46,165,677,327.171</td>
</tr>
<tr>
<td>Not used</td>
<td>3,133,511,252,839.31000</td>
<td>3,200,809,355,726.0</td>
<td>3,049,927,858,670.810</td>
<td>3,049,927,858,670.810</td>
<td>3,049,927,858,670.810</td>
</tr>
<tr>
<td>Blood circuit</td>
<td>57,616,212,762.13040</td>
<td>65,695,798,380.37790</td>
<td>56,222,424,137.758</td>
<td>50,456,400,206.844</td>
<td>50,410,486,859.998</td>
</tr>
<tr>
<td>Not used</td>
<td>97,449,273,976.19490</td>
<td>105,765,945,326.02000</td>
<td>97,075,483,763.497</td>
<td>97,859,085,963.541</td>
<td>97,075,483,763.497</td>
</tr>
</tbody>
</table>

5.6 Remarks

Comparing the result of Table 10 with those of Table 3, we can observe the following. Hemodialysis apparatus had a good result in 1st order with the case that monthly ratio is used. Dialyzer in 1st+3rd order with the case that monthly ratio is not used. Blood extracorporeal circulation apparatus had good result in 1st+2nd+3rd order with the cases that monthly ratio is used.

6. CONCLUSION

Focusing on the idea that the equation of exponential smoothing method (ESM) was equivalent to (1,1) order ARMA model equation, a new method of estimation of smoothing constant in exponential smoothing method was proposed before by us which satisfied minimum variance of forecasting error. Generally, smoothing constant was selected arbitrarily. But in this paper, we utilized above stated theoretical solution. Firstly, we made estimation of ARMA model parameter and then estimated smoothing constants. Thus theoretical solution was derived in a simple way and it might be utilized in various fields.

Furthermore, combining the trend removal method with this method, we aimed to improve forecasting accuracy. An approach to this method was executed in the following method. Trend removal by a linear function was applied to the original Sum total data of Blood extracorporeal circulation apparatus for three cases (Hemodialysis apparatus, Dialyzer, and Blood circuit). The combination of linear and non-linear function was also introduced in trend removing. For the comparison, monthly trend was removed after that. Theoretical solution of smoothing constant of ESM was calculated for both of the monthly trend removing data and the non monthly trend removing data. Then forecasting was executed on these data. Comparing the result of Table 10 with those of Table 3, we can...
observe the following. Hemodialysis apparatus had a good result in 1st order with the case that monthly ratio is used. Dialyzer in 1st+3rd order with the case that monthly ratio is not used. Blood extracorporeal circulation apparatus had good result in 1st+2nd+3rd order with the cases that monthly ratio is used.

Various cases should be examined hereafter.

REFERENCES