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Plastic Buckling Analysis of an Isotropic C-SS-SS-SS Plate under In-plane Loading using Taylor's Series Displacement Function

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ABSTRACT

Solutions to numerous plate buckling problems have been found using trigonometric series. However, the use of trigonometric series in formulating the displacement function of plates with certain boundary conditions may be rigorous. This paper presents a technique for the plastic buckling analysis of a thin, rectangular, isotropic plate under uniform in-plane compression in the longitudinal direction. The plate was bounded by two simply supported loaded edges, one simply supported unloaded edge and one clamped unloaded edge. The deformation theory of plasticity based on Stowell's approach was applied in deriving the governing equation. The study involved a theoretical derivation based on Taylor's series and application of a work principle. The approximate displacement function formulated from the Taylor's series was truncated at the fifth term which resulted to a peculiar displacement function for the boundary conditions. The displacement function was substituted in the governing equation and results for the plate buckling coefficient were obtained for aspect ratios ranging from 0.1 to 2.0 at intervals of 0.1, with values for the ratio of the tangent modulus to the secant modulus (E_t/E_s) equal to 0.5, 0.6, 0.7, 0.8 and 0.9. The results for E_t/E_s equal to 0.9 compared favourably with the elastic buckling values with an average percentage difference of −2.274%. This difference shows that the technique from the present study can be used to analyze the plastic buckling of thin isotropic plates with C-SS-SS-SS boundary conditions.

Keywords: *boundary conditions, critical load, deformation theory, inelastic buckling, shape function, Stowell's theory, Taylor's series, thin plate, uniaxial compression*

NOMENCLATURE

a	length of plate
A	amplitude of the displacement function
b	width of plate
C	clamped edge
C-SS-SS-SS	rectangular plate with three simply supported edges and one longitudinal clamped edge
D	plate flexural rigidity in the elastic range
$\overline{D}$	plate flexural rigidity in the plastic range
E	Young's modulus
E_s	secant modulus
E_t	tangent modulus
H	buckling curve expression
J, K	unknown constants in the power series
k	plate buckling coefficient
m	number of half-waves of the buckling mode along the x-direction
n	number of half-waves of the buckling mode along the y-direction
N_x	uniaxial in-plane compressive load on x-plane
$(N_x)_{cr}$	critical buckling load
p	aspect ratio
R, Q	non-dimensional form of the x and y coordinates respectively
SS	simply supported edge
t	thickness of plate
w	transverse deflection
w'^{Q}	first derivative of the deflection in the Q coordinate
w''^{R}, w''^{Q}	second derivative of the deflection in the R and Q coordinates respectively
x,y	Cartesian coordinates in the horizontal and vertical directions respectively
ν	Poisson ratio
σ_x	in-plane compressive stress in the x-direction

1. INTRODUCTION

Plates are straight, plane, two-dimensional structural components of which the thickness is much smaller than the other dimensions (Szilard, 2004). Plate elements are used to transmit in-plane and/or lateral loads. Applications of thin rectangular plates in the field of engineering include ships, aircrafts, lock gates and offshore structures. When a thin rectangular plate is subjected to in-plane compressive loads and the loads are gradually increased, the plate begins to buckle at a critical value of the compressive loads even though transverse loads may not be present. Unlike the case of columns where bending occurs in one plane, the buckling behaviour of thin rectangular plates involves bending in two planes and two boundary conditions on each edge of the plate. In plate buckling, quantities such as deflections and bending moments are functions of two independent variables, and plate buckling analyses are generally more complicated than those of one-dimensional elements. Based on the stress-strain relationship, plate buckling may be categorized as elastic buckling or plastic buckling. In elastic buckling analysis, a linear stress-strain relationship is adopted in deriving the governing equations, and it is assumed that the buckling stress is less than the proportional limit of the plate material. On the other hand, Hooke's law is invalid in plastic buckling analysis because the stresses are assumed to exceed the proportional limit before buckling occurs. Elastic buckling loads are generally greater than the buckling loads in the plastic range. It is therefore important to know the plastic buckling characteristics in order to accurately predict the critical buckling loads in the plastic range.

Numerous experimental and theoretical studies on plastic buckling analysis of thin rectangular plates have been carried out for several decades. Despite the fact that such studies have been conducted extensively, several aspects of plastic plate buckling are still controversial primarily because of the difficulty in the proper representation of the stress-strain relationship (Szilard, 2004). Various plasticity theories have been proposed to account for the plastic behaviour of plates but the two main theories are the deformation theory developed by Ilyushin (1947), and the incremental or flow theory propounded by Handelman and Prager (1948). The incremental theory has a stronger theoretical formulation but when applied to the plastic buckling of thin homogenous isotropic plates, it predicts results which are unreasonably higher than results obtained from experiments. Results obtained by the deformation theory tend to be in better agreement with experimental evidence. Hence, investigators have continued to use the deformation theory in solving plastic buckling problems of plate elements despite its weak mathematical background.

Solutions to plastic buckling problems of thin rectangular isotropic plates for various loading and boundary conditions have been found using exact, energy and numerical methods (Stowell, 1948; Iskason & Pifko, 1969; Guran-Savadkuhi, 1981; Iyengar, 1988; Shen, 1990; Wang, et al., 2004; Wang, et al., 2005). Most of these solutions were obtained by applying Fourier series or trigonometric series irrespective of the analytical method or plasticity theory used. The application of trigonometric series in the energy method for formulation of the displacement function for certain boundary conditions may be very difficult or even impossible (Ugural, 1999; Ventsel & Krauthammer, 2001). Because of these limitations, the Taylor's series can be used to estimate the displacement function of the deformed plate. The use of Taylor's series displacement function in solving plate buckling problems has attracted very little attention. From available literature, the Taylor's series has not been used in the energy method to formulate the displacement function for the plastic buckling of plates with three simply supported edges and one unloaded clamped edge in the longitudinal direction.

This study aims at providing a solution for the plastic buckling of a thin, rectangular, isotropic plate with C-SS-SS-SS boundary conditions subjected to uniform compressive in-plane loading in the longitudinal direction (x-axis). The deformation theory (based on Stowell's approach) and a variational technique were used to determine the plastic buckling behaviour. The displacement function used in estimating the deformed shape of the plate was formulated using Taylor-Maclaurin series.

2. PROBLEM FORMULATION AND THEORETICAL DERIVATIONS

2.1 Plastic Buckling Equation

Stowell (1948) derived the governing equation for the plastic buckling of a thin, flat, rectangular plate under uniform in-plane compression along the x-direction. He assumed that the plate is isotropic and incompressible and he adopted a numerical value of 0.5 for the Poission ratio. The equation was expressed as

$$\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{t\sigma_x}{\bar{D}}\frac{\partial^2 w}{\partial x^2} \qquad (1)$$

Where

$$\sigma_x = N_x/t \qquad (2)$$

$$\bar{D} = E_s t^3/9 \qquad (3)$$

Expressing Equation (1) in terms of non-dimensional coordinates with consideration to Equation (2),

$$\frac{1}{p^4}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 w}{\partial R^4}+\frac{2}{p^2}\frac{\partial^4 w}{\partial R^2\partial Q^2}+\frac{\partial^4 w}{\partial Q^4}-\frac{N_x b^2}{\bar{D}p^2}\frac{\partial^2 w}{\partial R^2}=0 \qquad (4)$$

In Equation (4),

$$p = a/b \qquad (5)$$

$$R = x/a; \;\; Q = y/b \qquad (6)$$

Eziefula (2013) applied a technique based on Ibearugbulem, et al. (2013) where Equation (4) was transformed using the principle of conservation of work in a static continuum and obtained

$$N_x = \frac{\frac{\bar{D}p^2}{b^2}\int_0^1\int_0^1\left[\frac{H}{p^4}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)\frac{\partial^4 H}{\partial R^4}+\frac{2H}{p^2}\frac{\partial^4 H}{\partial R^2\partial Q^2}+H\frac{\partial^4 H}{\partial Q^4}\right]\partial R\partial Q}{\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\partial R\partial Q} \qquad (7)$$

where

$$w = AH \qquad (8)$$

2.2 Boundary Conditions

The thin rectangular isotropic plate in this study has three simply supported edges and one clamped longitudinal edge as illustrated in Figure 1. For the clamped edge, the deflection and slope are zero. For the simply supported edges, the deflection and bending moment varnish.

The boundary conditions for the C-SS-SS-SS plate are

$$w(R=0)=0; \;\; w''^{R}(R=0)=0 \qquad (9)$$

$$w(R=1)=0; \;\; w''^{R}(R=1)=0 \qquad (10)$$

$$w(Q=0)=0; \;\; w'^{Q}(Q=0)=0 \qquad (11)$$

$$\mathrm{w}(\mathrm{Q}=1)=0; \;\; \mathrm{w}''^{\mathrm{Q}}(\mathrm{Q}=1)=0 \qquad (12)$$

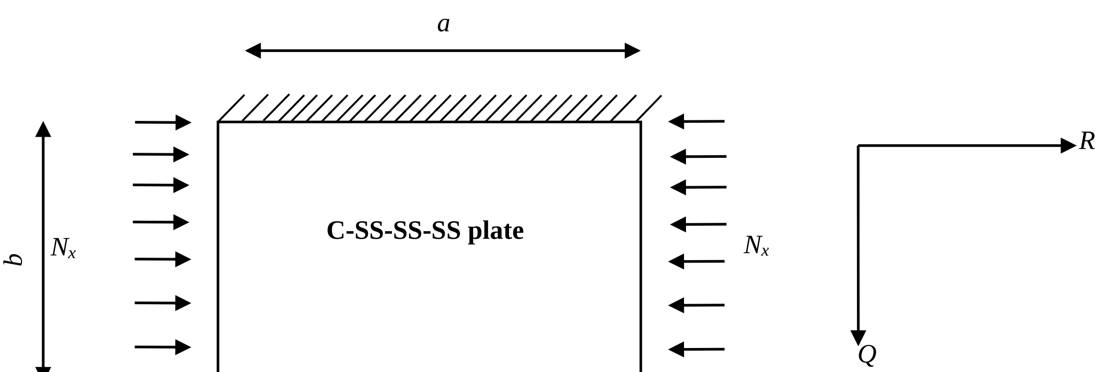

Figure 1.Schematic diagram of C-SS-SS-SS plate under uniaxial in-plane compression

2.3 Taylor's Series Displacement Function

Ibearugbulem (2012) expanded the displacement function using Taylor's series and he assumed the displacement function to be continuous and differentiable. He truncated the infinite power series at m = n =4 and obtained

$$w = \sum_{m=0}^{4}\sum_{n=0}^{4} J_m K_n \, R^m Q^n \qquad (13)$$

Equations (9), (10), (11) and (12) are applied in Equation (13). Substituting Equation (9) into Equation (13) gave

$$J_0 = J_2 = 0$$

Substituting Equation (10) into Equation (13) and solving the resulting simultaneous equations gave

$J_1 = J_4; J_3 = -2J_4$

Substituting Equation (11) into Equation (13) yielded

$K_0 = K_1 = 0$

Similarly, substituting Equation (12) into Equation (13) gave

$K_2 = 1.5K_4; K_3 = -2.5K_4$

Substituting the values of J_0, J_1, J_2, J_3, J_4, K_0, K_1, K_2, K_3 and K_4 into Equation (13) gave the peculiar displacement function of the C-SS-SS-SS plate. This displacement function is expressed as

$$w = J_4K_4[(R - 2R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)] \qquad (14)$$

Where

$$A = J_4K_4 \qquad (15)$$

$$H = (R - 2R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4) \qquad (16)$$

2.4 Application of Variational Principle

Partial derivatives of Equation (16) with respect to R, Q or both R and Q gave

$$H\frac{\partial^4 H}{\partial R^4} = 24(R - 2R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)^2 \qquad (17)$$

$$H\frac{\partial^4 H}{\partial Q^4} = 24(R - 2R^3 + R^4)^2(1.5Q^2 - 2.5Q^3 + Q^4) \qquad (18)$$

$$H\frac{\partial^4 H}{\partial R^2 \partial Q^2} = 36(-R + R^2)(1 - 5Q + 4Q^2)(R - 2R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4) \quad (19)$$

$$H\frac{\partial^2 H}{\partial R^2} = 12(-R + R^2)(R - 2R^3 + R^4)(1.5Q^2 - 2.5Q^3 + Q^4)^2 \quad (20)$$

Equations (17), (18), (19) and (20) were expanded and integrated partially with respect to R and Q respectively in a closed domain. The results in five significant figures were

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial R^4}\partial R\partial Q = 0.036191 \qquad (21)$$

$$\int_0^1\int_0^1 H\frac{\partial^4 H}{\partial Q^4}\partial R\partial Q = 0.088571 \qquad (22)$$

$$\int_0^1\int_0^1 2H\frac{\partial^4 H}{\partial R^2\partial Q^2}\partial R\partial Q = 0.083265 \qquad (23)$$

$$\int_0^1\int_0^1 H\frac{\partial^2 H}{\partial R^2}\partial R\partial Q = 0.0036621 \qquad (24)$$

The numerical values of the integrals in Equations (21), (22), (23) and (24) were substituted into Equation (7). The plastic buckling equation is now expressed as

$$N_x = \frac{\frac{\bar{D}}{b^2}\left[\frac{0.036191}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+0.083265+0.088571p^2\right]}{0.0036621} \qquad (25)$$

The plastic buckling equation of plates may generally be expressed in the form

$$N_x = \frac{\bar{D}\pi^2}{b^2}k \qquad (26)$$

Writing Equation (25) in form Equation (26) gave

$$k = \left[\frac{1.00130}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+ 2.30374 + 2.45055p^2\right] \qquad (27)$$

3. RESULTS AND DISCUSSION

The critical plastic buckling load from the present study is expressed as

$$(N_x)_{cr} = \frac{\bar{D}\pi^2}{b^2}\left[\frac{1.00130}{p^2}\left(\frac{1}{4}+\frac{3}{4}\frac{E_t}{E_s}\right)+ 2.30374 + 2.45055p^2\right] \qquad (28)$$

The solution given by Ibearugbulem (2012) for the elastic stability of the same plate was

$$(N_x)_{cr} = \frac{\pi^2 D}{b^2}\left(\frac{1.001}{p^2}+ 2.304 + 2.45p^2\right) \qquad (29)$$

The solutions from both investigations are upper bound solutions and involved the use of Taylor's series. It can be noted that in Equation (28), $\bar{D}$ is used whereas D is used in Equation (29). $\bar{D}$ is a function of the secant modulus while D is a function of Young's modulus. D is expressed

$$D = \frac{Et^3}{12(1-\nu^2)} \qquad (30)$$

In calculating the values of E_s and E_t, an extensive knowledge of the stress-strain curve of the plate material in the plastic range is required. The factor E_t/E_s is equal to one in elastic buckling but its value is always less than unity in plastic buckling. Numerical values of E_t/E_s equal to 0.5, 0.6, 0.7, 0.8 and 0.9 were used in this study. Table 1 shows values of k obtained from the present study and Ibearugbulem (2012) for aspect ratios ranging from 0.1 to 2.0 at intervals of 0.1.

From Table 1, the average percentage difference between the elastic buckling coefficient and the plastic buckling coefficient for $E_t/E_s = 0.9$ is -2.274%. It may be observed that the difference between the two solutions improves as the aspect ratio increases from 0.1 to 2.0. Ibearugbulem (2012) compared his solution with results from Fok (1980) and Michelutti (1976). For p = 1.0, the percentage difference between Ibearugbulem (2012) and Fok (1976) was 6.25% while the difference between Ibearugbulem (2012) and Michelluti (1976) for p = 0.79 was 0.55% as cited in Ibearugbulem (2012). These results are close and the differences between these solutions are acceptable in statistics.

Table 1. Values of k for uniaxially compressed C-SS-SS-SS thin rectangular isotropic plate

p	k from PRESENT STUDY					k from IBEARUGBULEM (2012)	α
	E_t/E_s= 0.5	E_t/E_s= 0.6	E_t/E_s= 0.7	E_t/E_s= 0.8	E_t/E_s= 0.9		
0.1	64.909	72.419	79.929	87.439	94.948	102.429	−7.304
0.2	18.047	19.925	21.802	23.679	25.559	27.427	−6.811
0.3	9.478	10.312	11.147	11.981	12.815	13.647	−6.097
0.4	6.607	7.077	7.546	8.015	8.485	8.952	−5.217

0.5	5.420	5.720	6.020	6.321	6.621	6.921	−4.335
0.6	4.924	5.133	5.342	5.550	5.759	5.967	−3.486
0.7	4.782	4.935	5.088	5.241	5.395	5.547	−2.740
0.8	4.850	4.967	5.085	5.202	5.319	5.436	−2.152
0.9	5.061	5.154	5.247	5.339	5.432	5.524	−1.665
1.0	5.380	5.455	5.530	5.605	5.680	5.755	−1.303
1.1	5.786	5.848	5.910	5.972	6.034	6.095	−1.001
1.2	6.267	6.319	6.371	6.424	6.476	6.527	−0.781
1.3	6.815	6.860	6.904	6.949	6.993	7.037	−0.625
1.4	7.426	7.464	7.503	7.541	7.579	7.617	−0.499
1.5	8.096	8.129	8.162	8.196	8.229	8.261	−0.387
1.6	8.822	8.851	8.880	8.910	8.939	8.967	−0.312
1.7	9.602	9.628	9.654	9.680	9.706	9.731	−0.257
1.8	10.437	10.460	10.483	10.506	10.529	10.551	−0.209
1.9	11.324	11.344	11.365	11.386	11.407	11.426	−0.166
2.0	12.262	12.281	12.300	12.319	12.337	12.354	−0.138

*α means percentage difference between k from PRESENT STUDY(E_t/E_s= 0.9) and IBEARUGBULEM (2012)

4. CONCLUSIONS

This study presented a solution for the plastic buckling analysis of a C-SS-SS-SS thin rectangular isotropic plate under uniaxial in-plane compression using Taylor's series displacement function. The governing equation was derived using Stowell's theory of plasticity and a work principle. From the present study, the following conclusions can be drawn:

1. A work principle based on the principle of conservation of work can be used to adequately analyze the plastic buckling of thin rectangular isotropic C-SS-SS-SS plates under in-plane compression.
2. Taylor's series truncated at m = n = 4 can be satisfactorily used to approximate the deformed shape of thin rectangular isotropic C-SS-SS-SS plates under in-plane compression.

REFERENCES

[1] Eziefula, U. G. (2013). Plastic buckling analysis of thin rectangular isotropic plates (M.Eng. thesis submitted to Postgraduate School). Federal University of Technology, Owerri, Nigeria.

[2] Guran-Savadkuhi, A. (1981). Inelastic buckling of plates by finite difference method (M.Eng. thesis). McGill University, Canada.

[3] Handelman, G. H. & Prager, W. (1948). Plastic buckling of rectangular plates under edge thrusts, NACA Technical Report, No. 946, 479-506.

[4] Ibearugbulem, O. M. (2012). Application of a direct variational principle in elastic stability analysis of thin rectangular flat plates (Ph.D. dissertation). Federal University of Technology, Owerri, Nigeria.

[5] Ibearugbulem, O. M., Ettu, L. O. & Ezeh, J. C. (2013). Direct integration and work principle as new approach in bending analyses of isotropic rectangular plates. The International Journal of Engineering and Science, 2(3), 28-36.

[6] Ilyushin, A. A. (1947). The elasto-plastic stability of plates. NACA Technical Memorandum, 1188, 1-30.

[7] Isakson, G. & Pifko, A. (1969). A finite element method for the plastic buckling analysis of plates. AIAA Journal, 7(10), 1950-1957.

[8] Iyengar, N. G. R. (1988). Structural stability of columns and plates. Chichester: Ellis Horwood.

[9] Shen, H. S. (1990). Elasto-plastic analysis for the buckling and postbuckling of rectangular plates under uniaxial compression. Applied Mathematics and Mechanics (English Edition), 11(10), 931-939.

[10] Stowell, E. Z. (1948). A unified theory of the plastic buckling of columns and plates. NACA Technical Report, 898, 127-137.

[11] Szilard, R. (2004). Theories and applications of plate analysis: Classical, numerical and engineering methods. New Jersey: Wiley and Sons.

[12] Ugural, A.C. (1999). Stresses in plates and shells (2nd ed.). New York: McGraw-Hill.

[13] Ventsel, E. & Krauthammer, T. (2001). Thin plates and shells: Theory, analysis and application. New York: Marcel Dekker.

[14] Wang, C. M., Chen, Y. & Xiang, Y. (2004). Plastic bucking of rectangular plates subjected to intermediate and end in-plane loads. International Journal of Solids and Structures, 41, 4279-4297.

[15] Wang, C.M., Wang, C.Y. & Reddy, J.N. (2005). Exact solutions for buckling of structural members. Boca Raton: CRC Press.