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Vibration Analysis of Plate with One Free Edge Using Energy Method (CCCF Plate)

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ABSTRACT

The study was carried out through a theoretical formulation based on Taylor series and application of Ritz method. The paper presents a theoretical formulation based on Taylor series shape function and application of Ritz method. In this study, the free vibration of plate with three edges clamped and one freed was analysed using Taylor series shape function derived was substituted into the potential energy functional, which was minimized to obtain the fundamental natural frequency. This research focused on aspect ratio from 0.1 to 2.0 with 0.1 increments. The value obtained was compared previous research works, for example, for aspect ratio of 1, values of non dimensional natural frequency parameter, λ is 24.6475

Keywords: *fundamental natural frequency; CCCF plate; CCFC plate; one free edge panel; Ritz method.*

1. INTRODUCTION

The free-vibration analysis of a rectangular plate is of interest in the field of mechanics, civil, and aerospace engineering back in 1823, by using a double trigonometric series, Navier obtained the exact solution of bending of a rectangular plate with all edges simply supported. By using a single Fourier series, Levy[1] developed a method for solving the rectangular plate bending problems with two opposite edges simply supported and the two remaining opposite edges with arbitrary conditions of supports. For the free vibration analysis of rectangular plates, accurate analytical results were presented for the cases having two opposite sides simply supported, whereas the other cases with the possible combinations of clamped, simply supported, and free edge conditions were analyzed by using the Ritz method by Leissa [2]. In addition, the method of superposition was proposed by Gorman [3] to examine free-vibration analysis of cantilever plates and that of rectangular plates with a combination of clamped and simply supported edge conditions. More recently, many papers on the vibration analysis of rectangular plates have been published. The free-vibration analysis of isotropic and anisotropic rectangular thin plates subjected to general boundary conditions was conducted by using a modified Ritz method by Narita [4]. For centuries, however, an exact solution for a fully clamped rectangular plate has not yet been obtained, and it is currently considered that an exact solution is not achievable for the rectangular plate problem of this type. There has not been any exact solution on the vibration of rectangular plates with one free edge. Over the years, problems have been treated by the use of trigonometric series as the shape function of the deformed plate or by using method of superposition. Research has been carried out on the problems from equilibrium approach and others solve the problems from energy and numerical approaches. However, no matter the approach used, trigonometric series has been the most used method

2. FORMULATION OF FUNDAMENTAL NATURAL FREQUENCY

Chakraverty (2009) gave the maximum kinetic energy functional as

$$K_{max} = \frac{\lambda^2}{2}\iint \rho h W^2(x,y)\partial x \partial y \qquad (1)$$

Making use of the non dimensional parameters, R and Q, equation (1) becomes

$$K_{max} = \frac{ab\lambda^2 \rho h}{2}\iint w^2 \partial R \partial Q \qquad (2)$$

Where ρ is the weight per unit area of the plate, h is the plate thickness and λ is the frequency of plate vibration. Ibearugbulem [5] as follow gave the maximum strain energy functional for a thin rectangular isotropic plate under vibration:

$$U_{max} = \frac{D}{2}\iint [(W''^{x})^2 + 2(W''^{xy})^2 + (W''^{y})^2]\partial x \partial y \qquad (3)$$

Adding equations (2) and (3) gave the total potential energy functional of rectangular plate under lateral vibration as

$$\prod max = \frac{aDb}{2}\iint \left[\frac{1}{a^4}(W''^{R})^2 + \frac{2}{a^2b^2}(W''^{RQ})^2 + \frac{1}{b^4}(W''^{Q})^2\right]\partial R \partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2 \partial R \partial Q \qquad (4)$$

Factorizing b/a^3 outside

$$\prod max = \frac{Db}{2a^3}\iint\left[(W''^R)^2 + \frac{2a^2}{b^2}(W''^{RQ})^2 + \frac{a^4}{b^4}(W''^Q)^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q$$

If P= a/b

$$\prod max = \frac{Db}{2a^3}\iint [(W''^R)^2 + 2P^2(W''^{RQ})^2 + P^4(W''^Q)^2]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \qquad (5)$$

If p= b/a

$$\prod max = \frac{Db}{2a^3}\iint\left[(W''^R)^2 + \frac{2}{P^2}(W''^{RQ})^2 + \frac{1}{P^4}(W''^Q)^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \qquad (6)$$

From equation (4) factorizing a/b^3 outside the bracket then

$$\prod max = \frac{Da}{2b^3}\iint\left[\frac{b^4}{a^4}(W''^R)^2 + \frac{2b^2}{a^2}(W''^{RQ})^2 + (W''^Q)^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q$$

If P = a/b

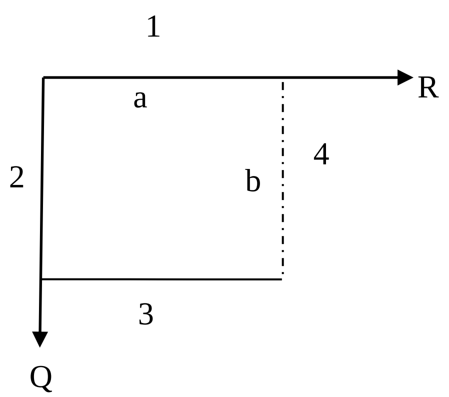
$$\prod max = \frac{Da}{2b^3}\iint\left[\frac{1}{P^4}(W''^R)^2 + \frac{2}{P^2}(W''^{RQ})^2 + (W''^Q)^2\right]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \qquad (7)$$

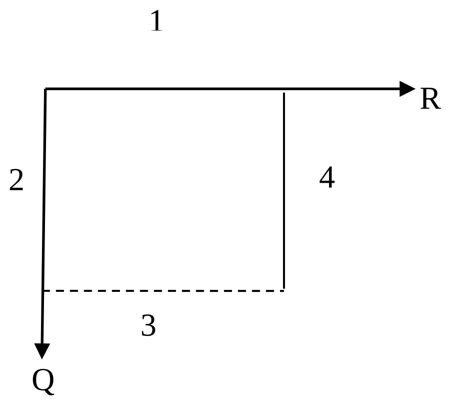
If P = b/a

$$\prod max = \frac{Da}{2b^3}\iint [P^4(W''^R)^2 + 2P^2(W''^{RQ})^2 + (W''^Q)^2]\partial R\partial Q - \frac{ab\lambda^2\rho h}{2}\iint w^2\partial R\partial Q \qquad (8)$$

3. TAYLOR-MCLAURIN'S SERIES' SHAPE FUNCTION

Displacement Function for CCCF and CCFC Plate





Displacement Equation

$$\mathbf{W} = (a_0 + a_1R + a_2R^2 + a_3R^3 + a_4R^4)(b_0 + b_1Q + b_2Q^2 + b_3Q^3 + b_4Q^4)$$

Boundary Condition	**Boundary condition**
$W(R=0)=0, W(Q=0)=0$	$W(Q=0)=0, W(R=0)=0$
$W'(R=0)=0, W'(Q=0)=0$	$W'(Q=0)=0, W'(R=0)=0$
$VR(R=1)=0, W'(Q=1)=0$	$W'(R=1)=0, VQ(Q=1)=0$

$W^R = a_4(4R^2 - 4R^3 + R^4)$ (9)

For W^Q

$W^Q = b_4(Q^2 - 2Q^3 + Q^4)$ (10)

Therefore the displacement function of CCCF plate $W = A(4R - 4R^3 + R^4)(Q^2 - 2Q^3 + Q^4)$ (11) while displacement function of CCFC PLATE gives $W = A(R^2 - 2R^3 + R^4)(4Q^2 - 4Q^3 + Q^4)$

Total Potential Energy for CCCF and CCFC Plate

Integrating the squares of the differential equation (11) with respect to R and Q

$$\int_0^1\int_0^1 (W^{iiR})^2\partial R\partial Q = A^2(12.8)(1.587301587 \text{ x } 10^{-3}) = 0.02031746032\ A^2$$

$$\int_0^1\int_0^1 (W^{iRiQ})^2 \partial R \partial Q = A^2(1.219047619)(0.01904761905) = 2.664489796\ A^2$$

$$\int_0^1\int_0^1 (W^{iiQ})^2 \partial R \partial Q = A^2(0.4063492063)(1.8) = 0.3250793651\ A^2$$

$$\int_0^1\int_0^1 (W)^2 \partial R \partial Q = A^2(0.4063492063)(1.587301587 \times 10^{-3}) = 6.4499874 \times 10^4\ A^2$$

Substituting the above into integral (5), (6), (7) and (8)

If P = a/b

$$\prod max = \frac{DbA^2}{2a^3}[0.02031746 + 0.0464399\,P^2 + 0.325079\,P^4] - \frac{ab\lambda^2 phA^2}{2}[6.44998 \times 10^{-4}] \qquad (12)$$

If p = b/a

$$\prod max = \frac{DbA^2}{2a^3}\left[0.02031746 + \frac{0.0464399}{P^2} + \frac{0.325079}{P^4}\right] - \frac{ab\lambda^2 phA^2}{2}[6.44998 \times 10^{-4}] \qquad (13)$$

If P = a/b

$$\prod max = \frac{DaA^2}{2b^3}\left[\frac{0.02031746}{P^4} + \frac{0.04643990}{P^2} + 0.3250793\right] - \frac{ab\lambda^2 phA^2}{2}[6.44998 \times 10^{-4}] \qquad (14)$$

If P=b/a

$$\prod max = \frac{DaA^2}{2b^3}[0.02031746P^4 + 0.0464399P^2 + 0.325079] - \frac{ab\lambda^2 phA^2}{2}[6.44998 \times 10^{-4}] \qquad (15)$$

Minimizing (12), (13), (14), (15) by $\frac{\partial \prod max}{\partial A} = 0$ and making λ subject of formula fundamental natural frequency results .Figure 1 to 8 shows the graphical model of fundamental natural frequency results, for both X-X and Y-Y axis.

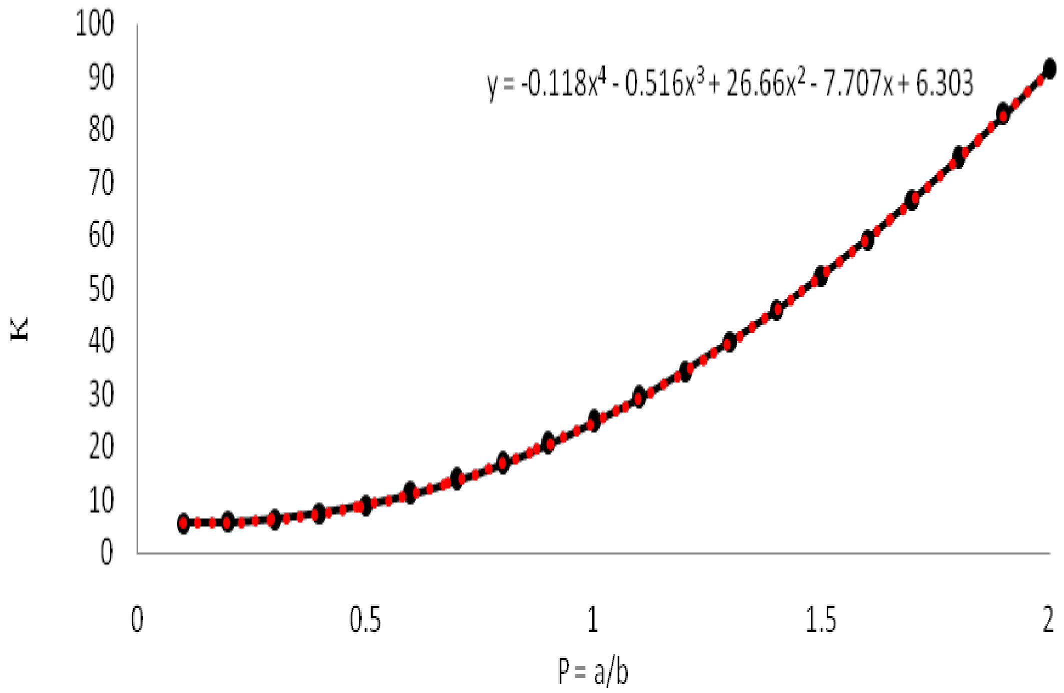


Figure 1: Graph of CCCF Plate P =a/b (note: y = k and x = aspect ratio, p)

The free edge of the plate is on X – X axis. With respect to length (a), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y = -0.118x^4 - 0.516x^3 + 26.66x^2 - 7.707x + 6.303$

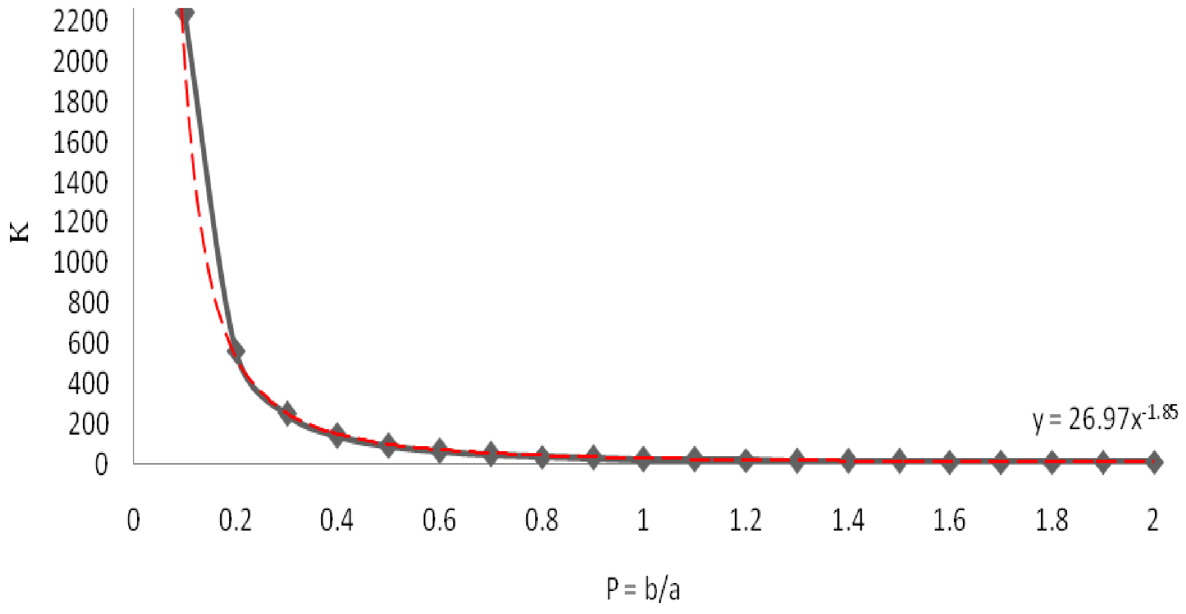


Figure 2: Graph of CCCF Plate P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the plate is on X – X axis. With respect to length (a), the natural frequency decreases as the aspect ratio increases, the power equation curve is represented by $y = 26.97x^{-1.85}$

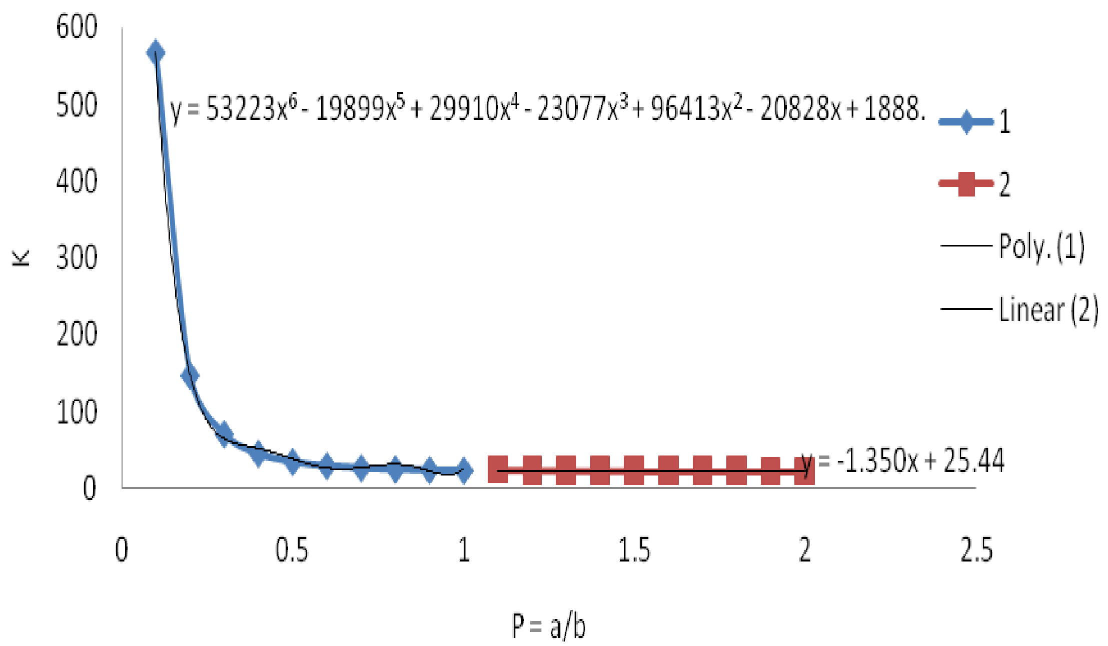


Figure 3: Graph of CCCF Plate P = a/b (note: y = k and x = aspect ratio, p)

The free edge of the plate is on X – X axis. With respect to width (b), the natural frequency decreases as the aspect ratio increases,

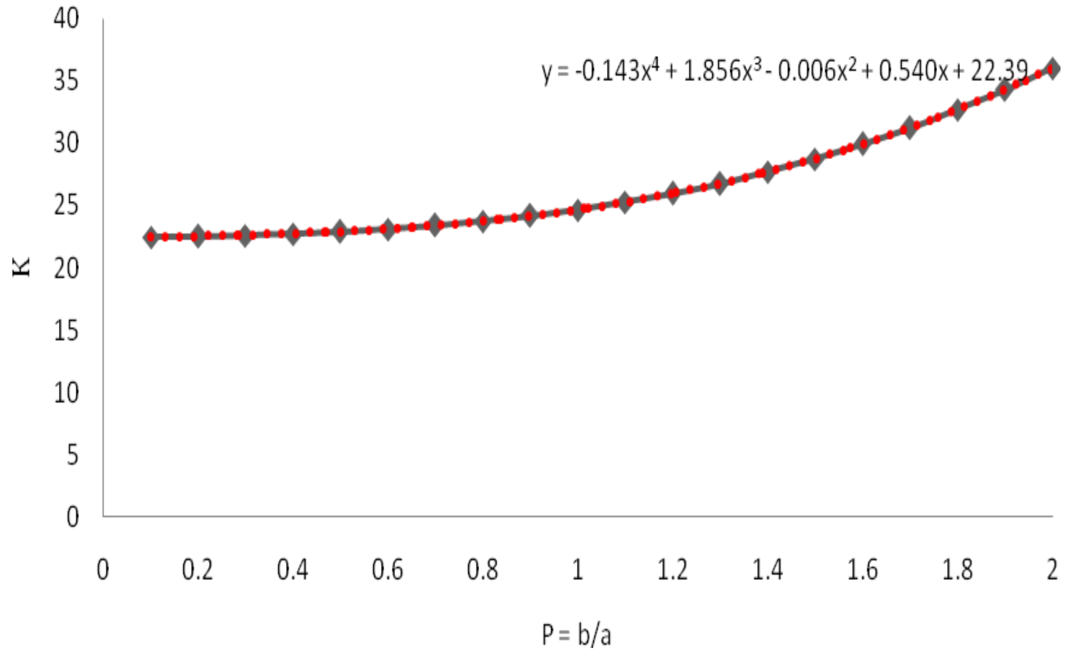


Figure 4: Graph of CCCF Plate P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the plate is on X – X axis. With respect to width (b), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y = -0.143x^4 + 1.856x^3 - 0.006x^2 + 0.540x + 22.39$

CCFC PLATE

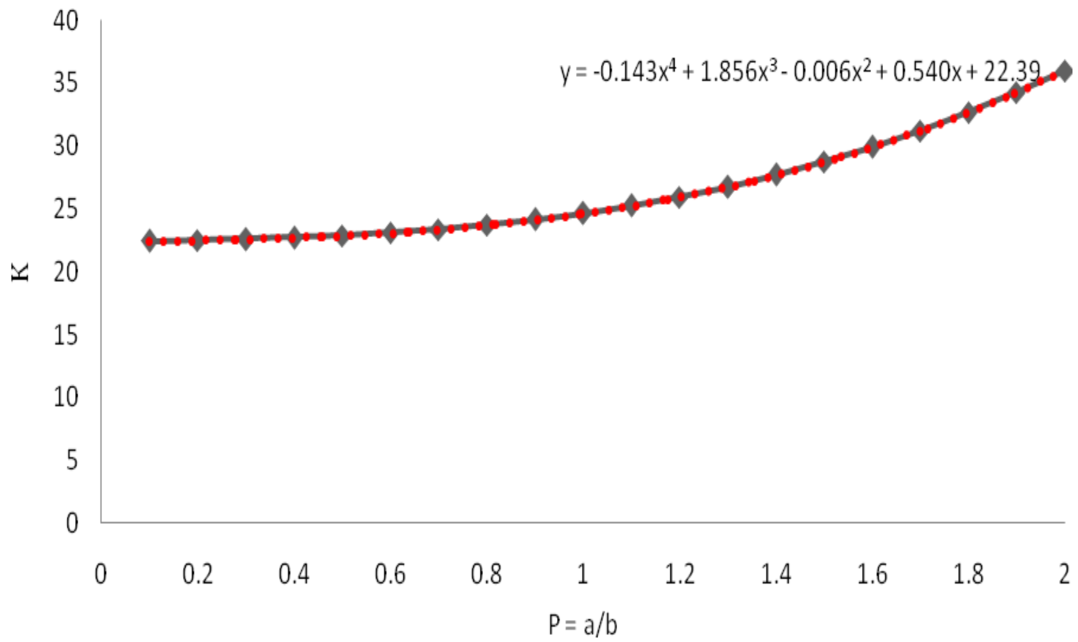


Figure 5: Graph of CCFC Plate P = a/b (note: y = k and x = aspect ratio, p)

The free edge of the plate is on Y – Y axis. With respect to length (a), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y = -0.143x^4 + 1.856x^3 - 0.006x^2 + 0.540x + 22.39$

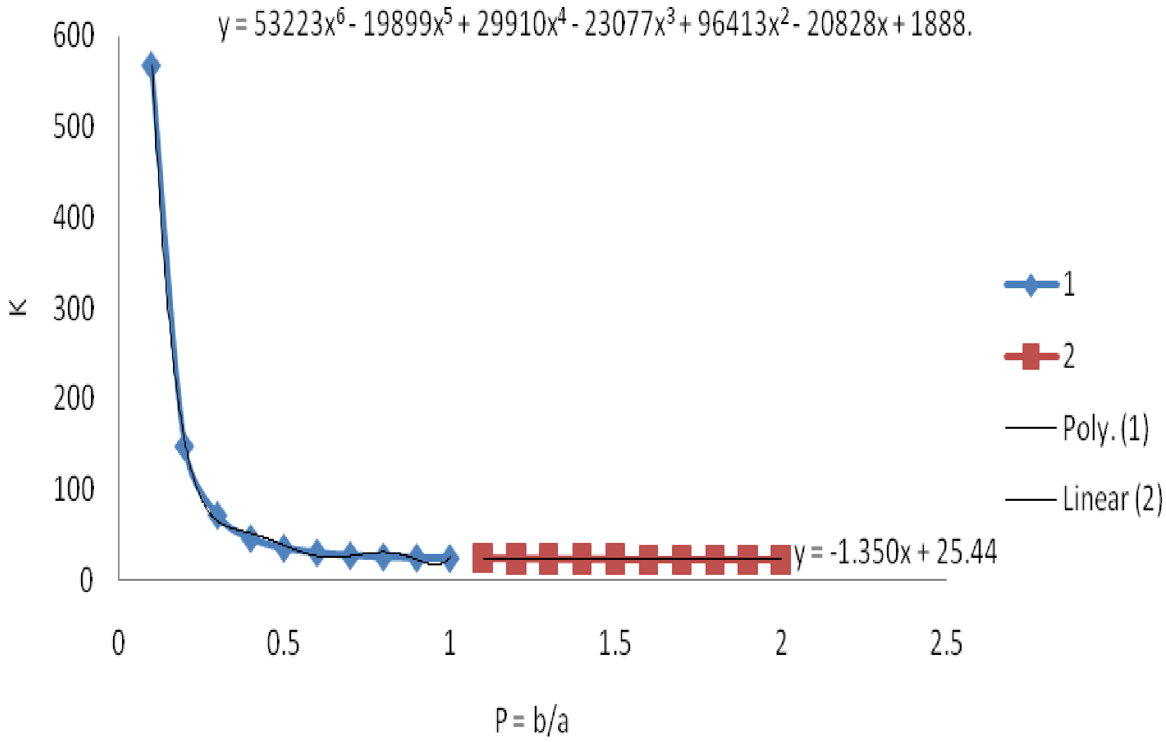


Figure 6: Graph of CCFC Plate P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the plate is on Y-Y axis. With respect to length (a), the natural frequency decreases as the aspect ratio increases,

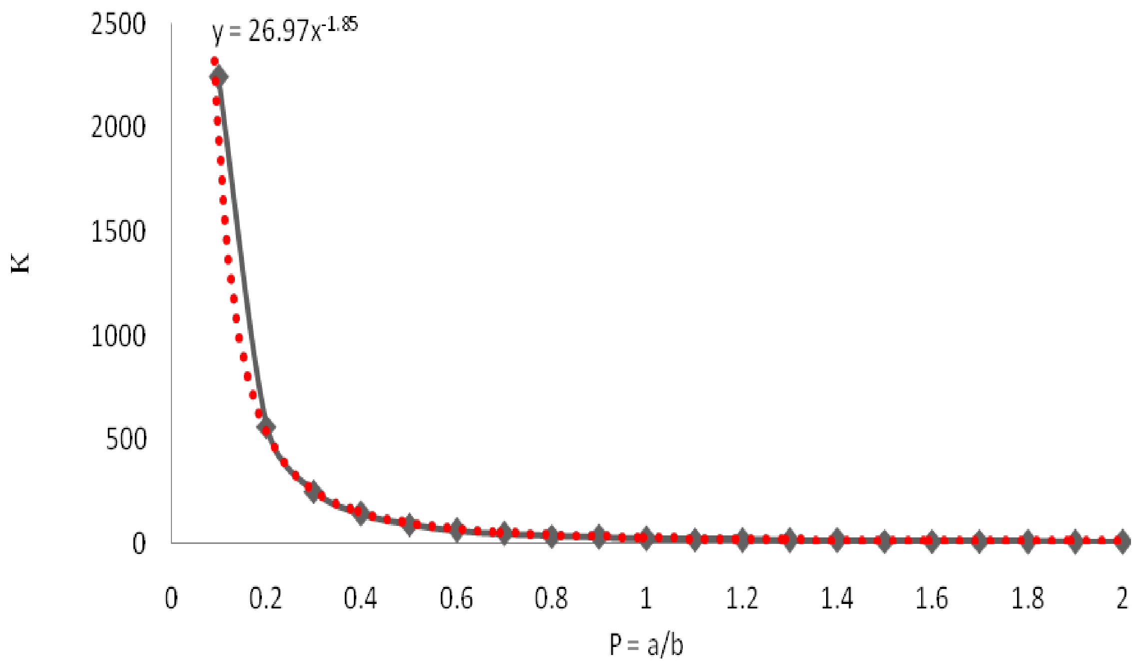


Figure 7: Graph of CCFC Plate P = a/b (note: y = k and x = aspect ratio, p)

The free edge of the plate is on Y-Y axis. With respect to width (b), the natural frequency decreases as the aspect ratio increases, the power equation curve is represented by $y = 26.97x^{-1.85}$

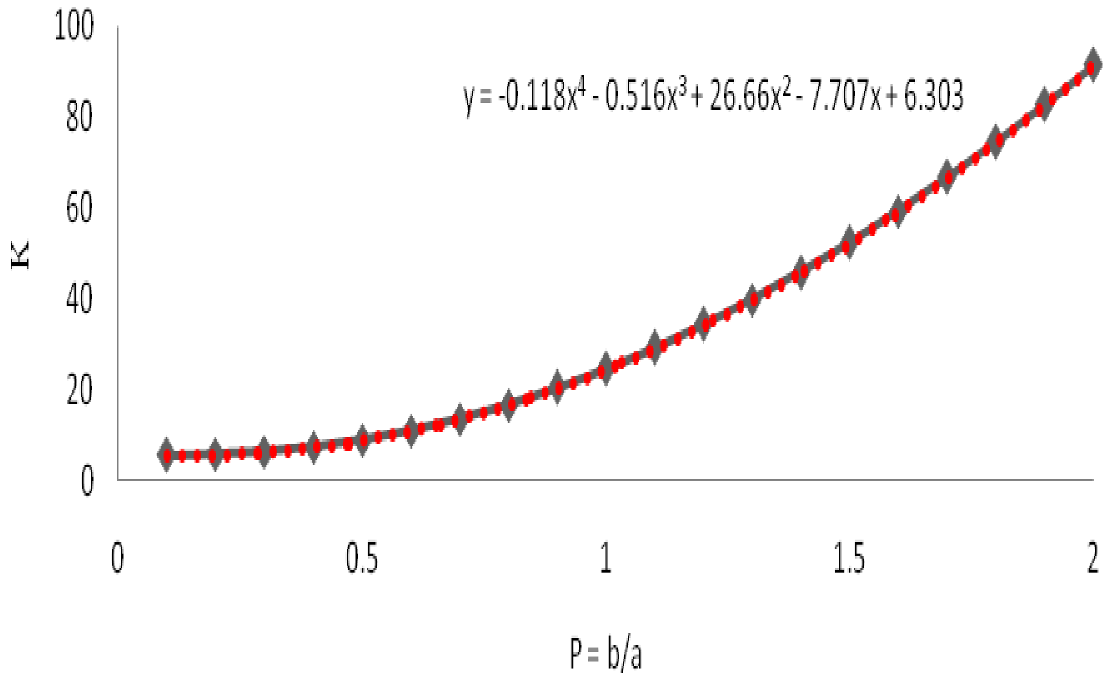


Figure 8: Graph of CCFC Plate P = b/a (note: y = k and x = aspect ratio, p)

The free edge of the plate is on Y-Y axis. With respect to width(b), the natural frequency increases as the aspect ratio increases, the polynomial equation curve is represented by $y = -0.118x^4 - 0.516x^3 + 26.66x^2 - 7.707x + 6.303$

Comparison of CCFC Plate

Comparing the results of CCFC plate from previous researchers with difference in percentage. In table 1, the difference between Lessia [1] and the present result ranges from 0.64% to 7.48% given an average difference of 2.79%,

Table 1: CCFC Plate

$\lambda = \frac{K}{a^2}\sqrt{\frac{D}{\rho h}}$			
$P = \frac{a}{b}$	**Present**	**Leissa**	**% diff**
0.4	22.7228	22.577	0.64
0.6	23.1085	23.015	0.41
1.0	24.6475	24.020	2.61
1.5	28.7310	26.731	7.48

S. Chakraverty [6] worked on aspect ratio P = 1, with λ = 23.960, giving a difference of 2.87% to the present value

4. CONCLUSION

The study obtained new energy functional based on Ritz's total potential energy and Taylor series deflection equation for panels of various support conditions.

The study came up with a new relationship between fundamental natural frequency and aspect ratios.

The study came up with graphical models which can be used in place of the primary equations.

The study has created a new data base of fundamental natural frequencies for different panel and aspect ratio for designers.

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