International Journal of Engineering and Technology Volume 4 No. 6, June, 2014

Profit Analysis of Computer Numerical Control (Cnc) System with Redundant Component and Expert Repairman

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ABSTRACT

In this paper we discussed the model based on Computer Numerical Control (CNC) system in which there is a redundant component of Mechanical (M) part. The distribution of repair time of unit is taken to be negative exponential with different parameter. The expert repairman will repair the failed unit; we assume that the expert repairman can take decision to replace the failed unit of the Central Processing Unit (CPU) if he is not able to repair the failed unit. The system is analyzed by making use of semi-Markov process and regenerative point technique.

Key Words: *Computer Numerical Control (CNC) System, Redundant, Semi-Markov Process, Regenerative Point Technique Etc.*

1. INTRODUCTION

In order to improve the profit of an industry, researchers are always interested in analyzing the real existing industrial system models. Arora et al. (2000), analysis the system and maintenance management for coal handling system in paper plant, Gupta , R. and Varshney, G.et al. (2004) studied Reliability analysis of gas leakage detection system in an industrial workshop with the application of Boolen function technique, Goyal, R et al. (2005) analyzed Stochastic analysis of reliability models with three types of repair policy.
In this paper, as we can understand by the name computer numerical control system is the combination of three different units. The first unit is Mechanical (M) part the second is software (S) and control processing unit (H). The expert repairman will repair the failed unit, in case he is not able to repair the failed unit, then he can take decision to replace the failed unit of the Central Processing Unit (CPU). Also assume that there is redundant component of Mechanical (M) part. The distribution of repair time of unit is taken to be negative exponential with different parameter.

Application of CNC technology has raised the level of automation in logical control systems. It has made it much more productive, flexible, expandable and convenient to operate. The need of numerical control machines was felt for machining complex-shaped small batch components as those belonging to an aircraft. The first demonstration of this prototype was held in 1952. Later on, several commercial CNC units were introduced into the market machine builders serving a variety of application.

The system is analyzed by making use of semi-Markov process and regenerative point technique. The following measures of the system effectiveness have been obtained:

- State transition probabilities and mean sojourn times & Mean time to system failure
- Availability of the system & Expected busy period of repairman
- Expected busy period of repairman (Replacement time only)
- Expected number of visits by the repairman

The graphical behaviour for different parameters is also studied.

2. NOTATIONS

E	:	Set of regenerative state $\{S_i, i = 0, 1, 2, 3, 7\}$
$\overline{E}$	:	Set of non-regenerative state $\{S_j; j = 4, 5, 6\}$
α	:	Failure rate of mechanical unit
β	:	Failure rate of control processing unit
γ	:	Failure rate of software
$g_1(t)$, $G_1(t)$	:	p.d.f. and c.d.f. of repair time of repairman for mechanical part
$g_2(t)$, $G_2(t)$	:	p.d.f. and c.d.f. of repair time of repairman for control processing unit
$g_3(t)$, $G_3(t)$	:	p.d.f. and c.d.f. of repair time of repairman for software part
P	:	Probability of success
Q	:	Probability of failure

Transition Diagram

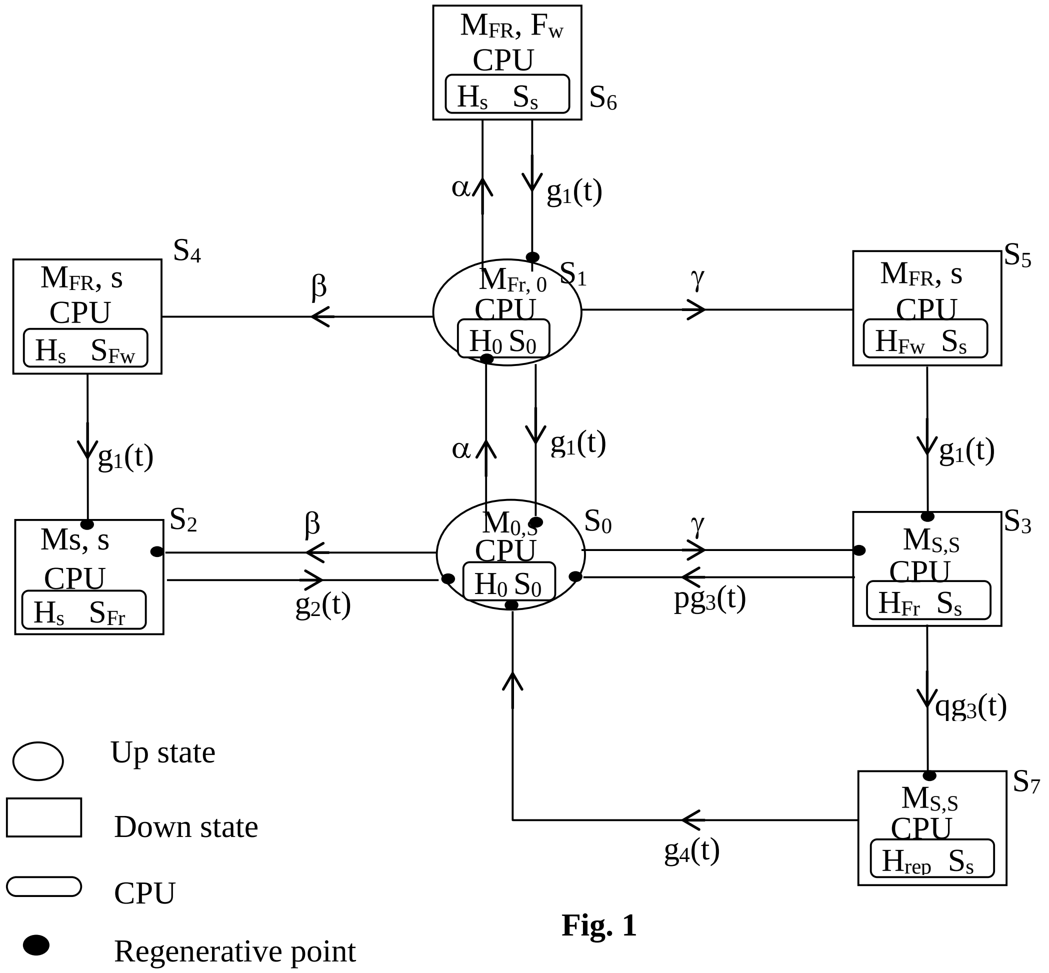

Fig. 1

3. MEAN TIME TO SYSTEM FAILURE

By probabilistic arguments, we obtain the following recursive relation for $\phi_1(t)$:

$\phi_0(t) = Q_{0,1}(t)$ Ⓢ $\phi_1(t) + Q_{0,2}(t) + Q_{0,3}(t)$;

$\phi_1(t) = Q_{1,0}(t)$ Ⓢ $\phi_0(t) + Q_{1,4}(t) + Q_{1,5}(t) + Q_{1,6}(t)$

Taking Laplace-Stieltjes Transform (L.S.T) of above relation and solving for $\phi_0^{**}(s)$, we mean time to system failure when the system starts from the state '0' given by

$\phi_6 = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s}$; As $\phi_0^{**}(s) = 1$, therefore using L'Hospital rule, we get: $\phi_6 = \frac{N}{D}$

$\Rightarrow D = 1 - P_{0,1}.\ P_{1,0}$ and

$N = \mu_0 + P_{0,1}.\ \mu_1$

4. AVAILABILITY ANALYSIS

Let $A_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state I at t = 0. Using the arguments of the theory of regenerative, process, the availability $A_i(t)$ is given to satisfy the following recursive relations:

$A_0(t) = M_0(t) + q_{0,1}(t)© A_1(t) + q_{0,2}(t)© A_2(t) + q_{0,3}(t)© A_3(t)$

$A_1(t) = M_1(t) + q_{1,0}(t)© A_0(t) + q_{1,2}^{(4)}(t)© A_2(t) + q_{1,3}^{(5)}(t)© A_3(t) + q_{1,1}^{(6)}(t)$ © $A_1(t)$

$A_2(t) = q_{2,0}(t)© A_0(t)$; $A_3(t) = q_{3,0}(t)© A_0(t) + q_{3,7}(t)© A_7(t)$; $A_7(t) = q_{7,0}(t)\ A_0(t)$

where, $M_0(t) = e^{-(\alpha+\beta+\gamma)t}$; $M_1(t) = e^{-(\alpha+\beta+\gamma)t}\ \overline{G}_1(t)$

In steady state, $A_0 = \lim_{s\to 0} sA_0^*(s) = \frac{N_1}{D_1}$ where $N_1 = M_0\ (1-P_{1,6}^{(6)}) + M_1\ P_{0,1}$

and $D_1 = (P_{1,1}^{(6)} - 1)\ \mu_0 + (1-P_{0,1})\ \mu_1 + (P_{0,2}.\ P_{1,1}^{(6)} - P_{0,2} - P_{0,1}.P_{1,2}^{(4)})\ \mu_2$

$$+(P_{0,3}.P_{1,1}^{(6)} - P_{0,3} - P_{0,1}.P_{1,3}^{(5)})\mu_3 + (P_{0,3}.P_{1,1}^{(6)}.P_{3,7} + P_{0,1}.P_{1,3}^{(5)} P_{3,7} - P_{0,3}.P_{3,7}).\mu_7$$

where $m_{0,1} + m_{0,2} + m_{0,3} = \mu_0$; $m_{1,0} + m_{1,1}^{(6)} + m_{1,2}^{(4)} + m_{1,3}^{(5)} = \mu_1$; $m_{2,0} = \mu_2$; $m_{3,0} + m_{3,7} = \mu_3$; $m_{7,0} = \mu_7$

5. BUSY PERIOD ANALYSIS OF REPAIRMAN

Let $B_i(t)$ be the probability that the system is in up-state at instant t given that the system entered regenerative state i at t = 0. Using the arguments of the theory of regenerative process, the busy period $B_i(t)$ is given to satisfy the following recursive relations:

$B_0(t) = q_{0,1}(t)© B_1(t) + q_{0,2}(t)© B_2(t) + q_{0,3}(t)© B_3(t)$

$B_1(t) = W_1(t) + q_{1,0}(t)© B_0(t) + q_{1,2}^{(4)}(t) © B_2(t) + q_{1,3}^{(5)}(t)© B_3(t) + q_{1,1}^{(6)}(t)© B_1(t)$

$B_2(t) = W_2(t) + q_{2,0}(t)© B_0(t)$; $B_3(t) = W_3(t) + q_{3,0}(t)© B_0(t) + q_{3,7}(t)© B_7(t)$; $B_7(t) = q_{7,0}(t)© B_0(t)$

where, $W_1(t) = \overline{G}_1(t);\ W_2(t) = \overline{G}_2(t);\ W_3(t) = \overline{G}_3(t)$

In steady-state, the total fraction of time which the system is under repair of the repairman, is given by

$B_0 = \lim_{s\to 0} (s B_0^*(s)) = \frac{N_2}{D_1}$ where, $N_2 = W_1.P_{0,1} + W_2(P_{0,1}.P_{1,2}^{(4)} + P_{0,2} - P_{0,2}.P_{1,1}^{(6)})$

and D_1 is already specified.

6. BUSY PERIOD ANALYSIS OF REPAIRMAN (REPLACEMENT TIME ONLY)

By probabilistic argument, we have the following recursive relation for $B_{iR}(t)$

$B_{0R}(t) = q_{0,1}(t)© B_{1R}(t) + q_{0,2}(t)© B_{2R}(t) + q_{0,3}(t)© B_{3R}(t)$

$B_{1R}(t) = q_{1,0}(t)© B_{0R}(t) + q_{1,1}^{(6)}(t)© B_{1R}(t) + q_{1,2}^{(4)}(t)© B_{2R}(t) + q_{1,3}^{(5)}(t)© B_{3R}(t)$

$B_{2R}(t) = q_{2,0}(t)© B_{0R}(t)$; $B_{3R}(t) = q_{3,0}(t)© B_{0R}(t) + q_{3,7}(t)© B_{7R}(t)$; $B_{7R}(t) = W_7(t) + q_{7,0}(t)© B_{0R}(t)$

where, $W_7(t) = \overline{G}_4(t)$

In steady-state, the total fraction of time which the system is under repair of the repairman, is given by

$B_{0R} = \lim_{s\to 0} [s B_{0R}^*(s)] = \frac{N_3}{D_1}$ where, $N_3 = W_1(p_{0,3}.P_{1,1}^{(6)}.P_{3,7} - P_{0,3}.P_{3,7} - P_{0,1}.P_{1,3}^{(5)}P_{3,7})$

and D_1 is already specified.

7. EXPECTED NUMBER OF VISITS BY THE REPAIRMAN

By probabilistic arguments, we have the following recursive relations for $V_i(t)$

$V_0(t) = Q_{0,1}(t)$ Ⓢ $[1 + V_1(t)] + Q_{0,2}(t)$ Ⓢ $[1 + V_2(t)] + Q_{0,3}(t)$ Ⓢ $[1 + V_3(t)]$

$V_1(t) = Q_{1,0}(t)$ Ⓢ $V_0(t) + Q_{1,1}^{(6)}(t)$ Ⓢ $V_1(t) + Q_{1,2}^{(4)}(t)$ Ⓢ $V_2(t) + Q_{1,3}^{(5)}(t)$ Ⓢ $V_3(t)$

$V_2(t) = Q_{2,0}(t)$ Ⓢ $V_0(t)$; $V_3(t) = Q_{3,0}(t)$ Ⓢ $V_0(t) + Q_{3,7}(t)$ Ⓢ $V_7(t)$; $V_7(t) = Q_{7,0}(t)$ Ⓢ $V_0(t)$

$N_4(s) = [Q_{0,1}^{**}(s) + Q_{0,2}^{**}(s) + Q_{0,3}^{**}(s)](1 - Q_{1,1}^{(6)**}(s))$

In steady-state, the total number of visits by the repairman per time is given by

$V_0 = \lim_{s\to 0}(s\ V_0{**}(s)) = \frac{N_4}{D_1}$ where, $N_4 = 1 - P_{1,1}^{(6)}$

and D_1 is already specified.

8. PROFIT ANALYSIS

The expected total profit earned by the system in steady-state is given by

$P = C_0A_0 - C_1B_0 - C_2B_{0R} - C_3V_0$

where, C_0 = revenue per unit up time of the system, C_1 = cost per unit time for which repairman is busy

C_2 = cost per unit time for which repairman is busy for replacing the unit

C_3 = cost per unit visit for the repairman

9. PARTICULAR CASE

For the graphical interpretation, the following case is considered.

$g_1(t) = \delta e^{-\delta t}$; $g_2(t) = \eta\, e^{-\eta t}$; $g_3(t) = \lambda\, e^{-\lambda t}$; $g_4(t) = \phi\, e^{-\phi t}$ Thus, we can easily obtain the following

$$P_{0,1} = \frac{\alpha}{\alpha+\beta+\gamma}; \quad P_{0,2} = \frac{\beta}{\alpha+\beta+\gamma}; \quad P_{0,3} = \frac{\gamma}{\alpha+\beta+\gamma};$$

$$P_{1,0} = \frac{\delta}{\alpha+\beta+\gamma+\delta}; \ P_{1,4} = P_{1,2}^{(4)} = \frac{\beta}{\alpha+\beta+\gamma+\delta}; \ P_{1,6} = P_{1,1}^{(6)} = \frac{\alpha}{\alpha+\beta+\gamma+\delta}; {}_{1,5} = P_{1,3}^{(5)} = \frac{\gamma}{\alpha+\beta+\gamma+\delta}; \ P_{3,0} = p;$$

$P_{3,7}=q$; $P_{2,0} = P_{7,0}= 1$ (where, $p + q = 1$); $\mu_0 = \frac{1}{\alpha+\beta+\gamma}$; $\mu_1 = \frac{1}{\alpha+\beta+\gamma+\delta}$; $\mu_2 = \frac{1}{\eta}$; $\mu_3 = \frac{1}{\lambda}$; $\mu_7 = \frac{1}{\phi}$

Using the above equations we can have the expression for MTSF, availability, busy period and expected number of visits etc. for this particular case.

On the basis of the numerical values taken as: $\alpha = 0.004$, $\beta = 0.001$; $\gamma = 0.002$; $\delta = 0.01$, $\eta = 0.03$, $\phi = 0.02$.

The values of various measures of the system effectiveness are obtained as:

Mean time to system failure (MTSF) = 205.4512

Availability (A_0) = 0.62968389

Busy period of repairman (B_0) = 0.221123

Busy period of repairman (Replacement time) (B_{0R}) = 0.001198

Expected number of visits by the repairman (V_0) = 0.004094

For the graphical interpretation, the mentioned particular case is considered.

Fig. 2 depicts that availability (A_0) increase with increase in the value of repair rate (γ) for different value of failure rate (β).

Fig. 3 shows the behaivour of MTSF with respect to failure rate (β). From the figure, it is clear that MTSF get decrease as increase in the value of failure rate (β).

Fig. 4 depicts the profit increase with increase in the values of revenue (C_1) for different values of cost (C_0) and it is lower for higher value of cost (C_0).

We can make the following conclusion:

(i) For $\gamma = 0.2$, P > or = or < 0 according as failure rate α < or = or > 0.41652. Thus the system is profitable if $\alpha < 0.41652$.

(ii) For $\gamma = 0.4$, P > or = or < 0 according as failure rate α < or = or > 0.44584. Thus the system is not profitable if $\alpha \geq 0.44584$.

(iii) For $\gamma = 0.6$, P > or = or < 0 according as failure rate α < or = or > 0.47865. Thus the system is profitable if $\alpha < 0.47865$.

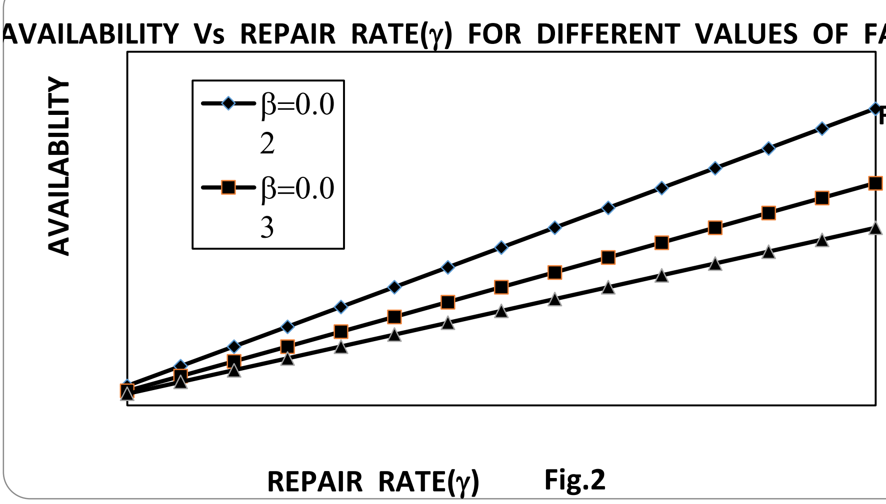

Fig.2

Fig. 2 shows the behaviour of AVAILABILITY with respect to repair rate (γ). It is clear from the graph that AVAILABILITY gets increase with increase

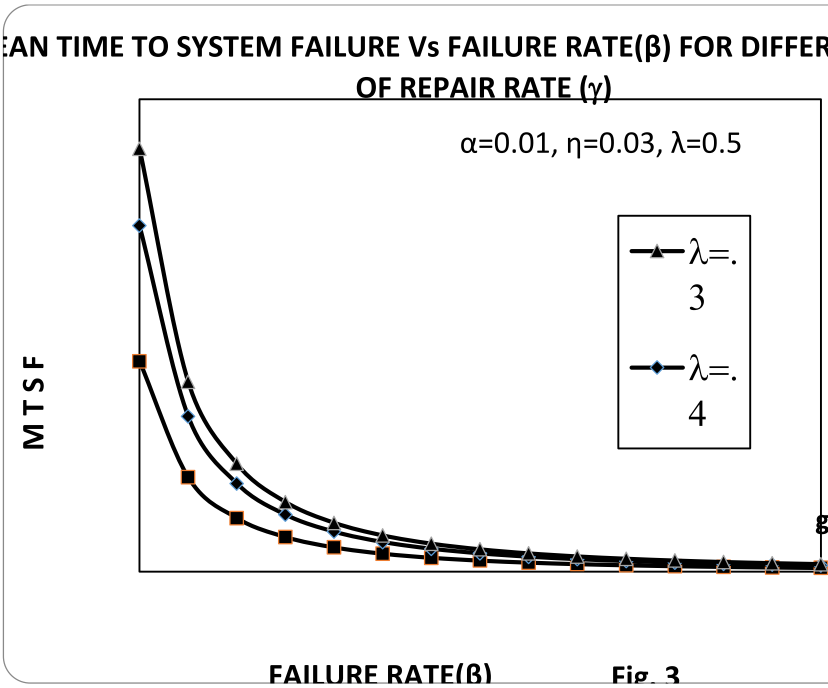

Fig. 3

Fig.3 shows the behaviour of MTSF with respect to failure rate(β). It is clear from the graph that MTSF gets decrease with increase in the value of failure rate.

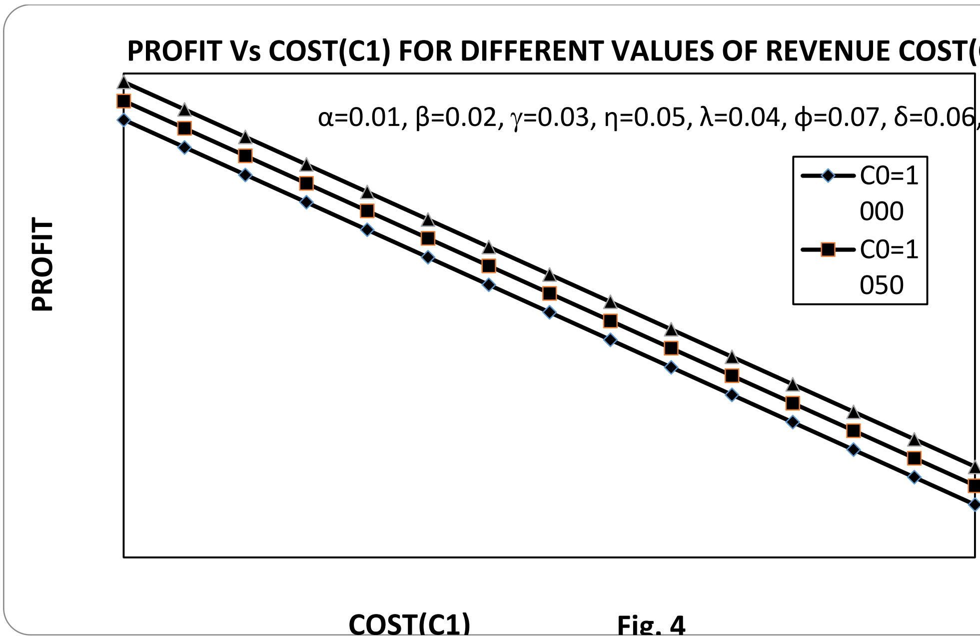

Fig. 4

Fig. 4 shows the behaviour of PROFIT vs revenue cost(C0). It is clear from the graph that PROFIT gets increase with increase in the values of revenue cost(C0) for the different

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