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EM and MAP Methods for Joint Path Delay and Complex Gain Estimation of a Slowly Varying Fading Channel for CPM Signals

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ABSTRACT

This paper addresses the joint path delay and time-varying complex gain estimation for continuous phase modulation (CPM), over a time-selective slowly varying Rayleigh flat fading channel. We propose two estimation methods: an expectation-maximization (EM) algorithm for path delay estimation in a Kalman framework, and a Maximum a Posteriori (MAP) method for joint path delay and complex gain estimation. The time-varying complex gains are modeled by a first order autoregressive (AR) process. Such a modeling yields to the representation of the problem by a dynamic Bayesian system in a state-space model, which allows the use of the EM algorithm in the context of unobserved data for obtaining an estimate of the path delay, coupled with a Kalman smoother for the complex gain estimation. Also, the direct joint estimation using a MAP estimator over the observation block is given. We derive analytically a closed-form expression of the modified hybrid Cramér-Rao bound (MHCRB) for the path delay and complex gain estimation problem. Some numerical examples are presented to illustrate the performance of the proposed methods compared to both the conventional generalized correlation method and the MHCRB.

Keywords-*Continuous phase modulation (CPM), EM algorithm, hybrid Cram\'er-Rao bound, path delay estimation, maximum-likelihood (ML) estimation, MAP estimator, Kalman smoother filter, fading channels.*

I. INTRODUCTION

Continuous phase modulation (CPM) is preferred in numerous wireless communications and mobile applications for its constant envelope property and high spectral efficiency [1]. Binary CPM systems, such as minimum-shift keying (MSK) and Gaussian MSK (GMSK), containing non-circular (or improper) process [2], [3], have been widely employed in many applications.

Due to the importance of CPM signals, many frequency and timing synchronization algorithms have been developed for such signals [4], [5]. These algorithms, typically categorized in Data-Aided (DA) (see, e.g. [6]) and Non-Data-Aided (NDA) (see, e.g. [7], [8], [9]) methods, have been designed under the assumption of additive white Gaussian noise channels (AWGN). However, few research works address CPM time synchronization over a time-varying channel. We can cite the recent works proposed in [13], [18] for flat-fading channels. A maximum likelihood (ML) approach was employed in [13] for estimating the time delay for CPM signals in the special case of using a MSK signal. However, [13], [18] do not consider the Bayesian approach which takes into account the a priori information of the unknown time-varying complex gains of the channel. In practice, we have to estimate the prior distribution of the time-varying complex gains.

Assuming that the time-varying complex gains can be modeled by a first order autoregressive (AR1) process (e.g. [22]), the problem of time synchronization over Rayleigh flat-fading channels can be formulated as a dynamic state-space Bayesian system with unknown (hidden) complex gains. So, we are facing a problem of state estimation in a nonlinear dynamical system with unknown parameters, which is a problem of practical interest in numerous applications. In many cases, the parameters of the dynamic model for a real system are not known exactly and need to be estimated. In reference [14], an EM algorithm [12] combined with a Kalman smoother [23] was proposed to compute the ML estimates of the speech recognition system parameters while also providing the state estimates. Recently, the EM algorithm has been applied to a lot of problems for parameter estimation and learning (see e.g. [15], [17], [16]).

In order to assess an estimator performance, lower bounds on the Mean Square Error (MSE) are needed. In communications systems, we usually choose a bound within the Cramér-Rao Bound (CRB) family [29]. Depending on the prior knowledge available on parameters, the CRB has different expressions. Among them, the Hybrid CRB (HCRB) is considered in the case of hybrid vectors that contains deterministic and random parameters (see e.g., [26], [27]). We note that the HCRB generalizes the classical CRB (see e.g., [29], [28]) and the Bayesian CRB (BCRB) [24]. However, the true expression of the CRB is sometimes difficult to derive analytically. To overcome this difficulty, other CRBs have been considered in the literature such as the Modified CRB (MCRB) (see, e.g., [4]), which is in general looser (i.e., lower) than the CRB.

In this paper, we describe the EM algorithm and a MAP method for jointly estimating path delay and complex gains over a slow Rayleigh flat-fading channel in the general case of CPM signals. First, we model the time-varying complex gains using a state-space model. Then, we present the EM algorithm to estimate the path delay coupling this method with a Kalman smoother for complex gain estimation. We also present a MAP method for joint path delay and complex gain estimation. Due to the presence of random parameters, we derive an analytical closed-form expression of the modified HCRB (MHCRB) for path delay and complex gains. This bound is used to evaluate the performance of the proposed methods.

The paper is organized as follows. Section 2 describes the CPM signal model, the AR model and the state-space representation of the problem. The EM and MAP methods are presented in sections 3 and 4. The MHCRB is derived in section 5 and finally some simulations are presented in section 6.

The following notations are used throughout the paper. Matrices and vectors are represented by bold upper case and bold lower case characters, respectively. I is the identity matrix. Vectors are by default in column orientation, while T, H and ∗ stand for transpose, conjugate transpose and conjugate, respectively. E(.), Tr(.), ||.||, $\Re(.)$ and $\Im(.)$ are the expectation, trace, norm, real and imaginary part operators respectively.

II. PROBLEM STATEMENT AND ESTIMATION OBJECTIVES

Following [1], the complex envelope $s(t,a)$ of a CPM signal can be written as $s(t,a)=e^{i\phi(t,a)}$ where the phase $\phi(t,a)$ of $s(t,a)$ is given by

$$\phi(t,a)=2\pi h\sum_{j\in Z} a_j q(t-jT), \qquad (1)$$

where T is the symbol period and $a \overset{\text{def}}{=} (a_j)_{j\in Z}$ is the independent identically distributed (i.i.d.) binary data sequence, with each element taking values on $\{\pm 1\}$.

$q(t)=\int_0^t g(u)du$ corresponds to the phase pulse shapi-g function that describes how the underlying phase chan-nge $\pm\pi h$ evolves with time, where $g(t)$, the frequency shaping filter, is positive and non-zero on the interval $[0,LT]$, and L is the correlation length. The modulatio-n index h determines the rate of change of frequency in the signalling interval. Finally, we note that if $nT\le t\le (n+1)T$ the phase $\phi(t,a)$ given by (1) can be written as

$$\phi(t,a)=\pi h\sum_{j=-\infty}^{n-L} a_j + 2\pi h\sum_{j=0}^{L-1} a_{n-j} q(t-(n-j)T). \qquad (2)$$

Transmitting $s(t,a)$ over a frequency-flat, slow fading Rayleigh channel results in the following received waveform

$$y(t)=\alpha(t)s(t-\tau,a)+b(t) \qquad (3)$$

where $\alpha(t)$ is a zero-mean Gaussian complex circular m-ultiplicative gain of variance σ_α^2, introduced by the flat f-ading channel with autocorrelation function $R_\alpha(\Delta t)\overset{\text{def}}{=}\sigma_\alpha^2 \mathrm{E}(\alpha(t)\ \alpha^*(t-\Delta t))$, τ is the fixed un-known path delay, and $b(t)$ is an additive white Gaussia-n complex circular noise with bilateral spectral power density N_0.

The equivalent discrete-time signal model observed during N signaling intervals, after low-pass filtering and sampling at rate $T_s=T/M$ is given by

$$y(kT_s)=\alpha(kT_s)s_k(\tau,a)+b(k), k=0,\ldots,MN-1$$

(4)

where $s_k(\tau,a)\overset{\text{def}}{=}s(kT_s-\tau,a)=e^{i\phi_k(\tau,a)}$ and $\phi_k(\tau,a)\overset{\text{def}}{=}\phi(kT_s-\tau,a)$. Note that, after antialiasing filtering (with cut-off frequency M/T) and sampling, the noise term $b(k)$ is assumed white with a known variance $\sigma^2=\dfrac{N_0}{T_s}$.

From (2), we obtain after some easy manipulations of indices that the phase term $\phi_k(\tau,a)$ can be expressed as

$$\phi_{nM+m}(\tau,a)\overset{\text{def}}{=}\phi((nM+m)T_s-\tau,a) \qquad (5)$$

$$=\pi h\sum_{j=-\infty}^{n-L} a_j + 2\pi h\sum_{j=0}^{L-1} a_{n-j} q_{j,m}(\tau),$$

(5)

for each $n\ge 0$ and for each m such as $0\le m\le M-1$, the τ-dependent coefficients $q_{j,m}(\tau)$ are defined by $q_{j,m}(\tau)\overset{\text{def}}{=}q(mT_s+jT-\tau)$. The complex gain of the channel $\alpha(.)$ does not change during symbol period but varies from symbol to symbol because the gain is assumed to be slowly time-varying. This implies that the coefficients $\alpha((nM+m)T_s)$ for $m=0,\ldots,M-1$, are all equal to the same value denoted by α_n. Then the discrete-time version of (4) can be written as follows:

$$y((nM+m)T_s)=\alpha_n s_{nM+m}(\tau,a)+b((nM+m)T_s).$$

(6)

Collecting the samples of the received signal within one s-lot to form a vector

$$y_n \overset{\text{def}}{=} (y((nM)T_s),\ldots,y((nM+M-1)T_s))^T,$$

yields the following model

$$y_n=\alpha_n g_n(\tau,a)+b_n, \qquad (7)$$

where $g_n(\tau,a)\overset{\text{def}}{=}\left(e^{i\phi_{nM}(\tau,a)},\ldots,e^{i\phi_{nM+M-1}(\tau,a)}\right)^T$, and

$$b_n \overset{\text{def}}{=} \left(b((nM)T_s),\ldots,b((nM+M-1)T_s)\right)^T,$$

is a $M\times 1$ noise vector with covariance matrix $\sigma^2 I$.

Among various channel models, the information theoretic results in [10] show that the first-order AR model provides a sufficiently accurate model for time-selective fading channels and therefore, will be adapted henceforth. Specifically, α_n varies according to

$$\alpha_n = \gamma\alpha_{n-1} + e_n, \qquad (8)$$

where the noise e_n is zero-mean Gaussian complex circular with a known variance σ_e^2 and is statistically independent of α_{n-1}. Using (8), simple manipulations lead to

$$\sigma_e^2 = \sigma_\alpha^2(1-\gamma^2) and \gamma = \mathrm{E}(\alpha_n\alpha_{n-1}^*) \qquad (9)$$

According to Jakes' model [21], we have $\gamma = J_0(2\pi f_d T)$, where $J_0(.)$ is the first kind 0th-order Bessel function and f_d denotes the maximum Doppler shift. From eqs. (7,8), we can obtain the following state space representation of the problem

$$\begin{cases} \alpha_n = \gamma\alpha_{n-1} + e_n \\ y_n = \alpha_n g_n(\tau, a) + b_n. \end{cases} \qquad (10)$$

The initial state α_0 is assumed to be Gaussian complex circular with a known variance σ_0^2.

In general, the objective is to jointly estimate the path delay parameter τ and the state $\boldsymbol{\alpha} \overset{\text{def}}{=} (\alpha_1, \ldots, \alpha_{N-1})$ using the set of received signals $y \overset{\text{def}}{=} (y_0^T, \ldots, y_{N-1}^T)^T$. In this paper, we assume that the transmit symbol sequence a is known at the receiver and we note $g_n(\tau) \overset{\text{def}}{=} g_n(\tau, a)$ (except in section 5, where the bound is computed for a more general NDA case).

If τ is known, the state parameters α_n can be inferred using a Kalman smoother [11]. Due to the presence of unobserved data $\boldsymbol{\alpha}$, the maximum likelihood (ML) method can't be used because the computation of the likelihood function $f(y \mid a; \tau) = \mathrm{E}(f(y \mid \boldsymbol{\alpha}, a; \tau))$ in a closed-form and its maximization w.r.t. τ seems to be an intractable problem. In the following section, we describe the Expectation-Maximization (EM) algorithm to find the ML estimates.

III. THE EM ALGORITHM

The EM algorithm [12] is an iterative method to find the ML estimate of parameters in the presence of unobserved data. The idea behind the algorithm is to augment the observed data with latent data, which can be either missing data or parameter values. We now describe an EM algorithm for our model. We are interested in the ML estimation of the desired parameter τ. Then, the state vector $\boldsymbol{\alpha}$ is regarded as "unobserved latent variables", or "hidden data" for the estimation of τ. Following the procedure given in [14, Sec. B], we consider the received data y as incomplete data, and define the complete data as $z \overset{\text{def}}{=} (\mathrm{y}^T, \boldsymbol{\alpha}^T)^T$. Since the state is Markov, the likelihood function of the complete data is given by

$$P(z \mid a; \tau) = P(\alpha_0)\prod_{n=1}^{N-1} P(\alpha_n \mid \alpha_{n-1}) \prod_{n=0}^{N-1} P(y_n \mid \boldsymbol{\alpha}_n, a; \tau)$$

(11)

Due to the Gaussian noise assumption, we have

$$\ln P(z;\tau) = C - \frac{1}{\sigma^2}\sum_{n=0}^{N-1} \| y_n - \alpha_n g_n(\tau) \|^2 - \frac{1}{\sigma_e^2}\sum_{n=1}^{N-1} |\alpha_n - \gamma\alpha_{n-1}|^2 - \frac{1}{\sigma_0^2}\|\alpha_0\|^2, \qquad (12)$$

where C is a constant that only depends on the state noise variances. Each iterative process $p = 0, 1, 2, \ldots$, in the EM algorithm for estimating τ from y consists of the following two steps:

E-step: Given the measurements y and an estimate of the model parameters from the previous iteration $\tau^{(p)}$, we calculate:

$$\mathrm{Q}(\tau, \tau^{(p)}) \overset{\text{def}}{=} \mathrm{E}(\ln P(z;\tau) \mid y, a; \tau^{(p)}),$$

where the expectation is taken with respect to $\boldsymbol{\alpha}$ conditioned on y and the latest estimate of τ, $\tau^{(p)}$. M-**M-step:** This step finds $\boldsymbol{\tau}^{(p+1)}$, the value of τ that maximizes $\mathrm{Q}(\tau, \tau^{(p)})$ over all possible values of τ:

$$\boldsymbol{\tau}^{(p+1)} = argmax_\tau Q(\tau, \tau^{(p)})$$

This procedure is repeated until the sequence $\tau^{(0)}$, $\tau^{(1)}$, ... converges.

Due to the Gaussian nature of the problem, the computation of the function Q can be expressed as (see Appendix A) :

$$\mathrm{Q}(\tau, \tau^{(p)}) = -\frac{1}{\sigma^2}\sum_{n=1}^{N}\Big(\mathrm{Tr}\Big(S_{n|N}^{(p)} g_n(\tau) g_n^H(\tau) + \big(y_n - \hat{\alpha}_{n|N}^{(p)} g_n(\tau)\big)\big(y_n - \hat{\alpha}_{n|N}^{(p)} g_n(\tau)\big)^H\Big)\Big) \qquad (13)$$

$S_{n|N}^{(p)} \overset{\text{def}}{=} \mathrm{E}((\alpha_n - \hat{\alpha}_{n|N}^{(p)})\ (\alpha_n - \hat{\alpha}_{n|N}^{(p)})^H \mid y, a; \tau^{(p)})$ and $\hat{\alpha}_{n|N}^{(p)} \overset{\text{def}}{=} \mathrm{E}(\alpha_n \mid y, a; \tau^{(p)})$ can be computed for all $n = 0, \ldots, N-1$ from the fixed interval Kalman smoother [14, 23], using the parameter estimates obtained at iteration p

The E-step is thus in first place an *Estimation-step* of the state vector α. And it can be obtained by a Kalman smoother, since our model (10) becomes a so called Gaussian linear model for the estimation of α_n assuming previous knowledge of τ.

The smoother consists of a Backward pass that follows the standard Kalman filter Forward recursions given as:

Forward recursion:

$$\hat{\alpha}_{n+1|n}^{(p)} = \gamma\hat{\alpha}_n^{(p)}$$

$$S_{n+1|n}^{(p)} = \gamma^2 S_n^{(p)} + \sigma_e^2$$

$$K_{n+1} = S_{n+1|n}^{(p)} g_n^H(\tau^{(p)})(\sigma^2 I + S_{n+1|n}^{(p)} g_n(\tau^{(p)}) g_n^H(\tau^{(p)}))$$

$$\hat{\alpha}_{n+1}^{(p)} = \hat{\alpha}_{n+1|n}^{(p)} + K_{n+1}(y_n - g_n(\tau^{(p)})\hat{\alpha}_{n+1|n}^{(p)})$$

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$$S_{n+1}^{(p)} = S_{n+1|n}^{(p)} - S_{n+1|n}^{(p)} K_{n+1} g_n(\tau^{(p)}) \tag{14}$$

Backward recursion:

$$J_{n-1} = \gamma S_{n-1}^{(p)} S_{n|n-1}^{(p)-1} \tag{15}$$

$$\hat{\alpha}_{n-1|N}^{(p)} = \hat{\alpha}_{n-1}^{(p)} J_{n-1}(\hat{\alpha}_{n|N}^{(p)} - \gamma \hat{\alpha}_{n-1}^{(p)})$$

$$S_{n-1|N}^{(p)} = S_{n-1}^{(p)} + J_{n-1}(S_{n|N}^{(p)} - S_{n|n-1}^{(p)}) J_{n-1}^{*}$$

$$S_{n,n-1|N}^{(p)} = S_{n}^{(p)} J_{n-1}^{H} + J_n (S_{n+1,n|N}^{(p)} - \gamma S_n^{(p)}) J_{n-1}^{*}$$

Remark 1

The steps of the EM algorithm can be extended in the case where the parameters $(\sigma^2, \sigma_e^2, \ \sigma_0^2, \gamma)$ are assumed unknown. By adapting the same steps of the approach proposed in [15], the estimates of these parameters can be obtained in E-step by maximizing the function Q given by (24).

We note that in the M-step, we maximize only w.r.t. τ. This simplifies the algorithm compared to the joint estimation of both the time-delay and the complex gains, but this step can be problematic if the function $\mathrm{Q}(\tau, \tau^{(p)})$ have local maxima. We can use a gradient algorithm to find the minima of the function $-\mathrm{Q}(\tau, \tau^{(p)})$.

Fig. 1 shows the values of the negative of the cost function (13) for each iteration of the algorithm, initialized at $\tau/T = 0.58$. We can see that the cost function has only one minimum for each iteration, and that the algorithm converges to the neighborhood of $\tau/T = 0.4$ after 15 iterations.

Fig.2 shows the diagram of the proposed iterative algorithm operating from a block of received samples y, and an initial value for the delay. It combines the Kalman smoother algorithm for complex gain estimation and EM algorithm for delay estimation. We note that each estimation block for one parameter operates under the assumption that the other parameter is known, but in using the estimates instead of the actual missing value.

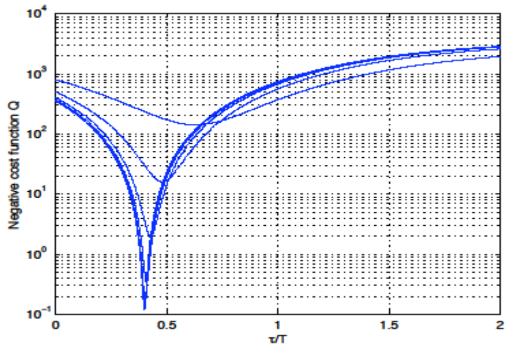

Fig. 1. Negative of the Cost function $-\mathrm{Q}(\tau, \tau^{(p)})$, GMSK signal with $BT = 0.3$ and $L = 4$, N BPSK i.i.d transmitted symbols.

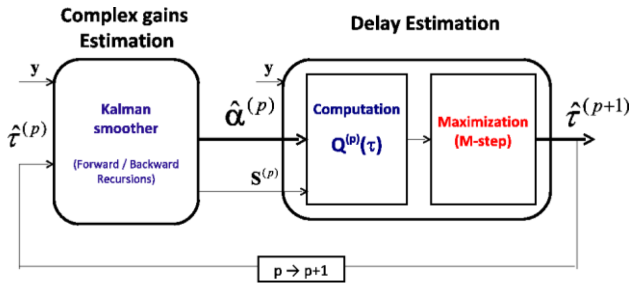

Fig. 2. Diagram of the EM-Kalman smoother proposed algorithm for joint estimation of path delay and complex gains.

On the other hand, this scheme is somewhat generic, and could be used with other optimization functions than the $\mathrm{Q}(.)$ function of the EM algorithm.

IV. MAXIMUM A POSTERORI (MAP) ESTIMATI-ON

In this section, we assume that we know the statistical properties of the channel (i.e., we have available to us the channel correlation matrix R_α defined below). The complex gains $\boldsymbol{\alpha}$ generated with the Jakes's model or its AR(1) approximation, are circular, complex and Gaussian, with correlation matrix

$$R_\alpha(n, n-k) \overset{def}{=} \begin{cases} \sigma_\alpha^2 J_0(2\pi f_d T k), & \textit{Jakes' Model} \\ \sigma_\alpha^2 \gamma^{|k|}, & AR(1)\ \textit{Model} \end{cases}$$

Consequently, the probability density function (pdf) of $\boldsymbol{\alpha}$ is given by:

$$P(\boldsymbol{\alpha}) = \frac{1}{\pi^N \det(R_\alpha)} e^{-\boldsymbol{\alpha}^H R_\alpha^{-1} \boldsymbol{\alpha}}. \tag{16}$$

We can rewrite the observation vector y over the whole observation block as

$$y = G(\boldsymbol{\tau})\boldsymbol{\alpha} + b$$

Where

$G(\tau) \overset{def}{=} Diag(g_1(\tau),..., g_N(\tau))$ and $b = \left(b_0^T, \dots, b_{N-1}^T\right)^T$ is a $NM \times 1$ noise vector with covariance matrix $\sigma^2 I$.

As we consider the DA case, y is zero-mean, Gaussian, complex and circular, with covariance matrix given by:

$$R_y \overset{def}{=} G(\tau)R_\alpha G^H(\tau) + \sigma^2 I.$$

In the following, we consider the Bayesian approach based on the MAP estimator for joint time-delay τ and channel complex gains estimation,

$$\begin{aligned}\{\hat{\tau}, \hat{\boldsymbol{\alpha}}\} &= Arg\ max_{\tau,\boldsymbol{\alpha}} P(\boldsymbol{\alpha} \mid y, a; \tau) \\ &= arg\ max_{\tau,\boldsymbol{\alpha}} P(y \mid \boldsymbol{\alpha}, a; \boldsymbol{\tau}) P(\boldsymbol{\alpha}) \\ &= Arg\ min_{\boldsymbol{\tau},\boldsymbol{\alpha}} \frac{1}{\sigma^2} \| y - G(\boldsymbol{\tau})\boldsymbol{\alpha} \|^2 + \boldsymbol{\alpha}^H R_\alpha \boldsymbol{\alpha}. \quad (17)\end{aligned}$$

For a fixed $\boldsymbol{\tau}$ and using the fact that for CPM signals, $G^H(\tau)G(\tau) = MI$, the channel estimate $\hat{\boldsymbol{\alpha}}$ that minimizes the cost function (17) is

$$\begin{aligned}\hat{\boldsymbol{\alpha}}(\tau) &= \left(G^H(\tau)G(\tau) + \sigma^2 R_\alpha^{-1}\right)^{-1} G^H(\boldsymbol{\tau})y \\ &= \left(MI + \sigma^2 R_\alpha^{-1}\right)^{-1} G^H(\tau)y \quad (18)\end{aligned}$$

which also corresponds to the minimum mean square error (MMSE) estimate of $\boldsymbol{\alpha}$. Replacing (18) in (17), and after some manipulations, we obtain the following cost function, which only depends on $\boldsymbol{\tau}$,

$$\hat{\tau} = Arg\ max_\tau\, p(\tau) \quad (19)$$

where

$$p(\tau) \overset{def}{=} y^H G(\tau)\left(MI + \sigma^2 R_\alpha^{-1}\right)^{-1} G^H(\tau)y \quad (20)$$

We can maximize the function (20) using iterative optimization techniques to find $\hat{\tau}$, and consequently the channel estimate is given by $\hat{\boldsymbol{\alpha}} \overset{def}{=} \hat{\boldsymbol{\alpha}}(\hat{\tau})$.

From (18) and (19), it is clear that the estimates of $\hat{\tau}$ and $\hat{\boldsymbol{\alpha}}$ are decoupled. In consequence, instead of a 2-D search, the proposed scheme requires 1-D search.

Remark 2

1. There is no identifiability issue due to regularity of the matrix $MI + \sigma^2 R_\alpha^{-1}$, that $g(.)$ depends on.
2. Because[1]

$$R_y^{-1} = \frac{1}{\sigma^2} I - \frac{1}{\sigma^2} G(\tau)(MI + \sigma^2 R_\alpha^{-1})^{-1} G^H(\tau), \quad (21)$$

we can conclude that maximizing the function $p(.)$ is equivalent to minimize $y^H R_y^{-1} y$, which maximizes the likelihood function $P(y \mid a; \boldsymbol{\tau})$. From this, we can see that using the Bayesian approach proposed, the estimated parameters $\hat{\boldsymbol{\tau}}$ and $\hat{\boldsymbol{\alpha}}$ are obtained separately, and so their estimation can be decoupled. We can estimate $\hat{\boldsymbol{\tau}}$ using an ML method and then estimate $\hat{\boldsymbol{\alpha}}$ via MAP or MMSE.

V. HYBRID CRAMÉR RAO BOUND

The CRB is an important criterion to evaluate how good any unbiased estimator can be, since it provides the MSE bound among all unbiased estimators. In this section, we assume that the symbol $\{a_n\}$ are i.i.d. and equiprobable with each element taking values on $\{\pm 1\}$. Since the parameters of interest are the deterministic parameter τ and the random parameter state $\boldsymbol{\alpha}$, we derive an analytical expression of the MHCRB using well known properties of the gaussian distribution and Markov state evolution of AR parameters. In Section 6, we will show the performance of the proposed EM and MAP algorithms, and compare it to the MHCRB, which is summarized by on the following (proof in Appendix B).

Result 1

The state and time-delay parameters are decoupled in the modified hybrid Fisher information matrix (MHFIM) in the case of CPM signals as follows:

$$I = \begin{pmatrix} I_{(\tau,\tau)} & 0 \\ 0 & I_{(\alpha,\alpha)} + B \end{pmatrix}$$

where $I_{(\tau,\tau)} = 8\pi^2 h^2 N\rho\xi(\tau)$, $I_{(\alpha,\alpha)} = \frac{M}{\sigma^2} I$ and the matrix B has the following non-zeros elements

$$B(1,1) = \frac{1+\gamma^2}{\sigma_e^2} - \mathrm{E}_{\alpha_0}\left(\frac{\partial^2 \ln P(\alpha_0)}{\partial \alpha_0 \partial \alpha_0^*}\right)$$

$$B(k,k-1) = B(k-1,k) = -\frac{\gamma}{\sigma_e^2}, \text{ for } k = 2,\dots,N$$

$B(k,k) = \frac{1+\gamma^2}{\sigma_e^2}$ and where $\rho \overset{def}{=} \frac{\sigma_\alpha^2}{\sigma^2}$ is the SNR and

$$\xi(\tau) \overset{def}{=} \sum_{m=0}^{M-1} \sum_{j=0}^{L-1} g^2(mT_e + jT - \tau).$$

Consequently,

$$MHCRB(\tau) = \frac{1}{8\pi^2 h^2 N\xi(\tau)} \frac{1}{\rho} \quad (22)$$

$$MHCRB(\boldsymbol{\alpha}) = \left(\frac{M}{\sigma^2} I + B\right)^{-1} \quad (23)$$

[1] The equality (21) is established using the following matrix inversion Lemm $(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1}$ for invertible matrices A and C and arbitrary matrices B and D .

We assume a classical non-informative prior on α_0 (see, e.g., [19]). As a consequence, $\mathrm{E}_{\alpha_0}\left(\dfrac{\partial^2 \ln P(\alpha_0)}{\partial\alpha_0\partial\alpha_0^*}\right)=0$.

We remark that the $MHCRB(\tau)$ is inversely proportional to ρ and depends on the modulation index h, the shaping filter g and the correlation length. (22) remains valid for all CPM signal. This expression also is similar to the MCRB derived in [4, rel. (2.4.54)]. Finally, we remark that $MHCRB(\boldsymbol{\alpha})$ does not depend on τ and on the CPM signals parameters thanks to the constant envelope property of CPM signals (see Appendix B).

VI. SIMULATION RESULTS

In this section, we present numerical examples to illustrate the performance of the proposed algorithms to jointly estimate the path delay and complex gains in the special case of a binary GMSK signal with a bandwidth-bit time product $BT=0.3$, a modulation index $h=1/2$ and a $4T$ -wide approximation of the Gaussian shaping filter, *i.e.* $L=4$ (see *e.g.* [4, rel. (4.2.8)]). These parameters are those of GSM systems. The channel is simulated according to the Jakes model [21, 22] with doppler-time product of $f_dT=0.000738$ corresponding to a carrier frequency of 1.8 GHz, a mobile speed of 120 km/h, and a transmission rate of 270 kb/s. A first order AR process, with a known coefficient $\gamma=0.99999$ (which corresponds to a slow fading channel) is chosen to approximate the time variation of the complex gains. The symbols $\{a_n\}$ are assumed known at the receiver, the number N of signaling intervals is set to $N=200$, the oversampling ratio is equal to $M=8$, and a fixed value $(\tau=0.4)$ is used as the normalized unknown delay τ/T. In the simulations, each value of the MSE is obtained by averaging over 1000 independent runs.
The initial estimate of the unknown parameter τ is chosen randomly in the interval $[0, LT)$ or in the neighborhood of $0.4T$, and the channel is initialized to the known state α_0 assumed to be a trial of a complex Gaussian random variable with a known variance $\sigma_0^2=1$.

Fig.3 shows one realization of the recursive estimates of the normalized path delay obtained with the EM algorithm versus the number of iterations for a SNR of 30 dB. This figure shows the estimated normalized path delay parameter converges to the true value fairly quickly in the case of slow flat-fading channel. We note that the EM algorithm still gives a valid estimate of τ when $f_dT>0.001$; however, in this case, the EM algorithm converges after about 30 iterations as shown in this figure with $f_dT=0.02$.

Fig.4 compares the $MHCRB(\tau)$ normalized to T^2 (given by (22)) to the MSE of the normalized path delay (*i.e.* $\mathrm{E}(\hat{\tau}-\tau)^2/T^2$) given by the EM algorithm initialized by the estimate given by the correlation method (see, *e.g.* [20]), the MAP method and the correlation method, as a function of the SNR. For comparison purpose, we have computed the MSE associated to the ML method with a perfect knowledge of the complex
gains. As seen from the simulation results, performance of the EM and MAP methods are very close to the ML method in the case of perfect knowledge of the complex gains. We observe also that both proposed methods significantly outperforms the correlation method based on the maximum of the delay-Doppler ambiguity function. On the other hand, the performance of both methods are close to the MHCRB, what is not the case of the correlation method.

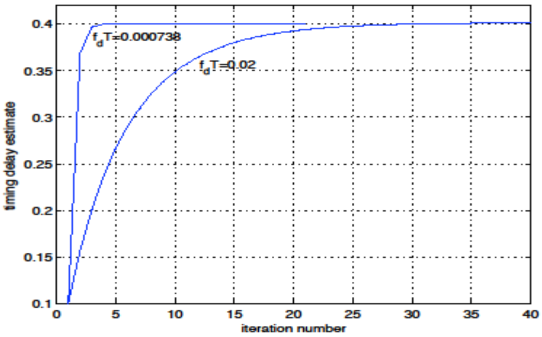

Fig. 3. An EM trajectory for two values of f_dT $SNR=30$ dB.

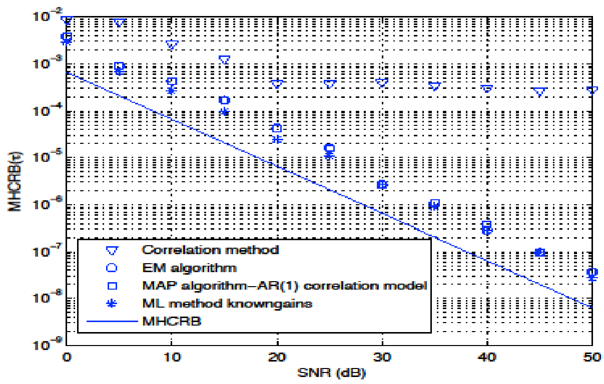

Fig. 4. Normalized $MHCRB(\tau)/T^2$, and estimated MSE $\mathrm{E}(\hat{\tau}-\tau)^2/T^2$ given by the EM algorithm (ten iterations), the MAP method and the correlation method for $f_dT=0.000738$, versus SNR.

Fig.5 compares the MSE for the complex gains $\mathrm{E}(\|\hat{\boldsymbol{\alpha}}-\boldsymbol{\alpha}\|^2)$ given by the EM algorithm, the MAP method, and the Kalman smoother with a perfect knowledge of the delay path to the $\mathrm{Tr}(MHCRB(\boldsymbol{\alpha}))$ given by (23), as a function of SNR. We see that the performance of the EM algorithm, the MAP method and the Kalman smoother are very close, and the associated estimates reach the MHCRB when the SNR increases. At low SNR, we recall that the MHCRB is, in general, looser than the true CRB.

Fig. 6 compares the normalized MSE $\mathrm{E}(\hat{\tau}-\tau)^2/T^2$ given by the EM algorithm, the MAP method and the correlation method (see, *e.g.* [20]), as a function of $f_d T$ with $SNR = 30$ dB.

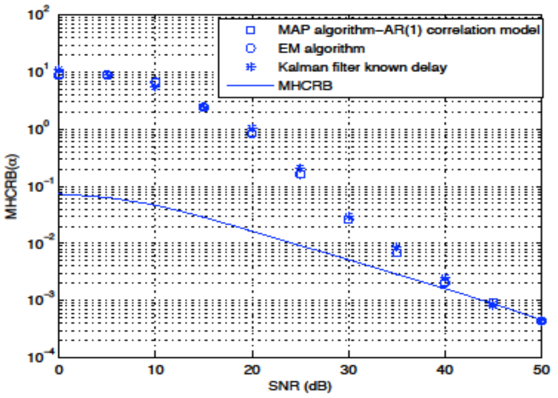

Fig. 5. Normalized $MHCRB(\boldsymbol{\alpha})$, and estimated normalized MSE $\frac{1}{N}\mathrm{E}(\|\hat{\boldsymbol{\alpha}}-\boldsymbol{\alpha}\|^2)$ given by the EM algorithm (ten iterations) and the MAP method for $f_d T = 0.000738$, versus SNR.

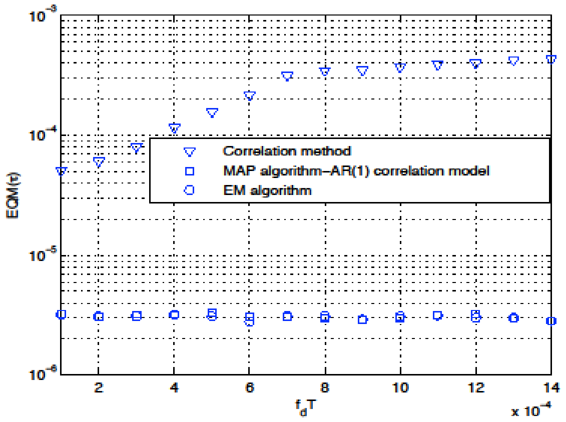

Fig. 6. The normalized MSE $\mathrm{E}(\hat{\tau}-\tau)^2/T^2$ given by the EM algorithm (ten iterations), the MAP method and the correlation method, for $SNR = 30$ dB, versus $f_d T$.

We observe from this figure that as $f_d T$ increases, the MSE of the delay given by both proposed methods, the EM algorithm and the MAP method, remains almost constant contrarily to the MSE given by the correlation method which increases when $f_d T$ increases.

VII. CONCLUSIONS

We have presented two methods, an EM algorithm and a MAP estimation method, for joint path delay and time-varying complex gains estimation for CPM signals over a time-selective slowly varying flat Rayleigh fading channel. We have modeled the flat fading channel as a first order autoregressive process. The EM algorithm has been combined with a Kalman smoother to yield time-varying complex gains estimation and ML estimate of the path delay. This algorithm was reduced to a single-parameter search over the path delay only. We have also derived a closed-form expression of the MHCRB for path delay and time-varying complex gains parameters. The performance of the proposed methods have been evaluated in terms of the MSE and the MHCRB. Finally, the simulation results have shown that we obtain the same performance with both the EM and the MAP methods, and that both provide better estimation of the delay and complex gains parameters compared to the conventional correlation method. Moreover, the performance of the proposed algorithms in term of delay estimation are very close to the performance of the ML method in the case of perfect knowledge of complex gains.

APPENDIX A

Proof of Eq. (13)

Taking the expectation with respect to $\boldsymbol{\alpha}$ conditioned on y, given the current parameter estimate $\tau^{(p)}$, we obtain from (12) the expectation of the log-likelihood function of the complete data which can be expressed as

$$\mathrm{Q}(\tau,\tau^{(p)}) = \mathrm{E}(\ln(P(z;\tau) \mid y,a;\tau^{(p)}) \quad (24)$$

$$= c(\sigma^2,\sigma_e^2,\sigma_0^2) - \frac{1}{\sigma^2}\sum_{n=0}^{N-1}\mathrm{Tr}\Big(y_n y_n^H + P_{n|N}^{(p)} g_n(\tau) g_n^H(\tau) - \hat{\alpha}_{n|N}^{(p)*} y_n g_n^H(\tau) - \hat{\alpha}_{n|N}^{(p)} g_n(\tau) y_n^H\Big)$$

$$-\frac{1}{\sigma_e^2}\sum_{n=0}^{N-1}\Big(P_{n|N}^{(p)} + \gamma^2 P_{n-1|N}^{(p)} - \gamma(P_{n,n-1|N}^{(p)} + P_{n-1,n|N}^{(p)})\Big) \quad (25)$$

$$-\frac{1}{\sigma_0^2}\,|\,P_{0|N}^{(p)}\,|^2, \quad (26)$$

with $P_{n|N}^{(p)} \overset{\mathrm{def}}{=} \mathrm{E}(\alpha_n \alpha_n^* \mid y,a;\tau^{(p)})$, $P_{n,n-1|N}^{(p)} \overset{\mathrm{def}}{=} \mathrm{E}(\alpha_n \alpha_{n-1}^* \mid y,\ a;\tau^{(p)})$, $P_{n-1,n|N}^{(p)} \overset{\mathrm{def}}{=} \mathrm{E}(\alpha_{n-1}\alpha_n^* \mid y,a;\tau^{(p)})$ and $\hat{\alpha}_{n|N}^{(p)} \overset{\mathrm{def}}{=} \mathrm{E}(\alpha_n \mid\ y,a;\tau^{(p)})$. We remark that the terms given by (26), (25) and the variance-dependent constant C do not depend on τ, then these terms can be removed from (24).

Since

$$S_{n|N}^{(p)} \overset{\mathrm{def}}{=} \mathrm{E}\Big(|\,\alpha(n) - \hat{\alpha}_{n|N}^{(p)}\,|^2 \mid y,a;\tau^{(p)}\Big)$$

$$= P_{n|N}^{(p)} - \hat{\alpha}_{n|N}^{(p)}\hat{\alpha}_{n|N}^{(p)*} \quad (27)$$

By deducing the value of $\hat{P}_{n|N}^{(p)}$ from the relation (27) and replacing it into (24), we obtain (13).

APPENDIX B

Proof of Result 1

First, we remind the considered state-space model:

$$\begin{cases} \alpha_n = \gamma\alpha_{n-1} + e_n \\ y_n = \alpha_n g_n(\tau) + b_n \end{cases} \quad (28)$$

The general expression of the HCRB for the parameter vector $\boldsymbol{\theta} \overset{def}{=} (\tau, \boldsymbol{\alpha}^T)^T$ is given by the inverse of the Hybrid Information Matrix (HFIM) [26, rel. (3)] :

$$HCRB(\boldsymbol{\theta}) = I^{-1} \tag{29}$$

$$I \overset{def}{=} \mathrm{E}_{\boldsymbol{\alpha}} \mathrm{E}_{y|\boldsymbol{\alpha}} \left(-\frac{\partial^2 \ln f(y|\boldsymbol{\alpha})}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^H} \right) + \begin{pmatrix} 0 & 0^T \\ 0 & \mathrm{E}_{\boldsymbol{\alpha}} \left(-\frac{\partial^2 \ln g(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha} \partial \boldsymbol{\alpha}^H} \right) \end{pmatrix} \tag{30}$$

This bound is difficult to obtain because the function $f(y|\boldsymbol{\alpha}) = \mathrm{E}_a f(y|\boldsymbol{\alpha}, a)$ doesn't have an easy analytical form. We ressort to the Modified HCRB (MHCRB), which is given by the inverse of the Modified HFIM (MHFIM), and is easier to obtain,

$$MHCRB(\boldsymbol{\theta}) = I_h^{-1} \tag{31}$$

$$I_h \overset{def}{=} \mathrm{E}_a \mathrm{E}_{\boldsymbol{\alpha}} \mathrm{E}_{y|\boldsymbol{\alpha}} \left(-\frac{\partial^2 \ln f(y|\boldsymbol{\alpha}, a)}{\partial \boldsymbol{\theta} \partial \boldsymbol{\theta}^H} \right) + \begin{pmatrix} 0 & 0^T \\ 0 & \mathrm{E}_{\boldsymbol{\alpha}} \left(-\frac{\partial^2 \ln g(\boldsymbol{\alpha})}{\partial \boldsymbol{\alpha} \partial \boldsymbol{\alpha}^H} \right) \end{pmatrix} \tag{32}$$

where

$$g(\boldsymbol{\alpha}) \overset{def}{=} P(\alpha_0) \prod_{n=1}^{N} g(\alpha_n | \alpha_{n-1})$$

$$f(y|\boldsymbol{\alpha}, a) \overset{def}{=} \prod_{n=1}^{N} f(y_n | \alpha_n, a)$$

As we assumed that the process and measurement noises, b_n and e_n, are both Gaussian, we can write that

$$f(y_n | \alpha_n, a) = \frac{1}{\pi^M \sigma^{2M}} e^{-\frac{\|y_n - \alpha_n g_n(\tau)\|^2}{\sigma^2}} \tag{33}$$

$$g(\alpha_n | \alpha_{n-1}) = \frac{1}{\pi \sigma_e^2} e^{-\frac{\|\alpha_n - \gamma \alpha_{n-1}\|^2}{\sigma_e^2}} \tag{34}$$

First, we compute the first matrix of the MHFIM given by relation relation (32). After suppression of the constant term, the log-pdf $f(y|\boldsymbol{\alpha}, a)$ is,

$$\ln(f(y|\boldsymbol{\alpha}, a)) = -\frac{1}{\sigma^2} \sum_{n=1}^{N} \| y_n - g_n(\tau) \alpha_n \|^2 \tag{35}$$

We compute the partial derivatives of the expression (35) with respect to τ and α_n,

$$\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \tau \partial \tau} = -\frac{2}{\sigma^2} \sum_{n=1}^{N} \{ \Re(\alpha_n^* g_n^{'H}(\tau) g_n^{'}(\tau) \alpha_n) - \Re(\alpha_n^* g_n^H(\tau)(y_n - g_n(\tau)\alpha_n)) \}$$

$$\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \alpha_n \partial \alpha_m^*} = -\frac{1}{\sigma^2} g_n^H(\tau) g_n(\tau) \delta_{nm}$$

$$\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \tau \partial \alpha_n} = -\frac{1}{\sigma^2} \{ \alpha_n^* g_n^{'H}(\tau) g_n(\tau) - (y_n - g_n(\tau)\alpha_n)^H g_n^{'}(\tau) \},$$

where $g_n^{'}(\tau) \overset{def}{=} \frac{\partial g_n(\tau)}{\partial \tau}$. An after the conditional expectation with respect to the pdf of $y | \boldsymbol{\alpha}$, we directly obtain

$$\mathrm{E}_{y|\boldsymbol{\alpha}} \left(\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \tau \partial \tau} \right) = -\frac{2}{\sigma^2} \sum_{1}^{N} \Re(\alpha_n^* g_n^{'H}(\tau) g_n^{'}(\tau) \alpha_n)$$
$$= -\frac{2}{\sigma^2} \Re(Tr(G^H(\tau) G(\tau) \boldsymbol{\alpha}\boldsymbol{\alpha}^H)).$$

$$\mathrm{E}_{y|\boldsymbol{\alpha}} \left(\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \boldsymbol{\alpha} \partial \boldsymbol{\alpha}^H} \right) = -\frac{1}{\sigma^2} G^H(\tau) G(\tau) \delta_{nm}$$

$$\mathrm{E}_{y|\boldsymbol{\alpha}} \left(\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \tau \partial \boldsymbol{\alpha}^T} \right) = -\frac{1}{\sigma^2} \alpha_n^* g_n^{'H}(\tau) g_n(\tau)$$
$$= -\frac{1}{\sigma^2} \boldsymbol{\alpha}^H G^{'H}(\tau) G(\tau),$$

where $G(\tau)$ is the matrix defined as $G(\tau) \overset{def}{=} Diag(g_1(\tau), \ldots, g_N(\tau))$ and $G^{'}(\tau) \overset{def}{=} \frac{\partial G(\tau)}{\partial \tau} = Diag(g_1^{'}(\tau), \ldots, g_N^{'}(\tau))$. After the expectation with respect to the pdf of $\boldsymbol{\alpha}$, we obtain

$$\mathrm{E}_{\boldsymbol{\alpha}} \mathrm{E}_{y|\boldsymbol{\alpha}} \left(\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \tau \partial \tau} \right) = -\frac{2}{\sigma^2} \Re(Tr(G^H(\tau) G(\tau) R_\alpha))$$

$$\mathrm{E}_{\boldsymbol{\alpha}} \mathrm{E}_{y|\boldsymbol{\alpha}} \left(\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \boldsymbol{\alpha} \partial \boldsymbol{\alpha}^H} \right) = -\frac{1}{\sigma^2} G^H(\tau) G(\tau)$$

$$\mathrm{E}_{\boldsymbol{\alpha}} \mathrm{E}_{y|\boldsymbol{\alpha}} \left(\frac{\partial^2 \ln(f(y|\boldsymbol{\alpha}, a))}{\partial \tau \partial \boldsymbol{\alpha}^T} \right) = 0,$$

where we used

$$\mathrm{E}_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}) = 0 \tag{36}$$

$$R_\alpha \overset{def}{=} \mathrm{E}_{\boldsymbol{\alpha}}(\boldsymbol{\alpha}\boldsymbol{\alpha}^H)$$
$$= \sigma_\alpha^2 \begin{bmatrix} 1 & \gamma & \gamma^2 & \ldots & \gamma^{N-1} \\ \gamma & 1 & \gamma & \ldots & \gamma^{N-2} \\ \gamma^2 & \gamma & 1 & \ldots & \gamma^{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma^{N-1} & \gamma^{N-2} & \gamma^{N-3} & \ldots & 1 \end{bmatrix} \tag{37}$$

We note that $g_n^H(\tau)g_n(\tau) = M$, and so

$$G^H(\tau)G(\tau) = \mathrm{Diag}(g_n^H(\tau)g_n(\tau),\ldots,g_N^H(\tau)g_N(\tau)) = MI$$

Using the definition of $g_n(\tau)$ and the relation (5), we obtain

$$g'_n(\tau) = i\left(\phi'_{nM}(\tau,a)e^{i\phi_{nM}(\tau,a)},\ldots,\phi'_{nM+M-1}(\tau,a)e^{i\phi_{nM+M-1}(\tau,a)}\right)$$

where $\phi'_{nM+m}(\tau,a) = 2\pi h\sum_{j=0}^{L-1} a_{n-j}\, g(mT_e + jT - \tau)$. As the symbols a_n are i.i.d. and equiprobables, whe have that

$$\begin{aligned}
\xi(\tau) &\overset{\text{def}}{=} \mathrm{E}_a(g_n'^H(\tau)g'_n(\tau)) \\
&= \sum_{m=0}^{M-1}\mathrm{E}_a((\phi'_{nM+m}(\tau,a)^2) \\
&= 4\pi^2h^2\sum_{m=0}^{M-1}\sum_{j=0}^{L-1} g^2(mT_e + jT - \tau)
\end{aligned}$$

And as a consequence,

$$\begin{aligned}
&\mathrm{E}_a(G'^H(\tau)G'(\tau)) \\
&= \mathrm{Diag}(\mathrm{E}_a(g'H_1(\tau)g'_1(\tau)),\ldots,\mathrm{E}_a(g'H_N(\tau)g'_N(\tau))) \\
&= \xi(\tau)I
\end{aligned}$$

Using relation (37), we can say that

$$\mathrm{Tr}(\mathrm{E}_a(G'^H(\tau)G'(\tau))R_\alpha) = \xi(\tau)\mathrm{Tr}(R_\alpha) = N\sigma_\alpha^2\xi(\tau)$$

And from this we obtain that

$$\begin{aligned}
I_{(\tau,\tau)} &= \mathrm{E}_a\mathrm{E}_{\boldsymbol{\alpha}}\mathrm{E}_{y|\boldsymbol{\alpha}}\left(-\frac{\partial^2 \ln f(y\,|\,\boldsymbol{\alpha})}{\partial\tau\partial\tau}\right) \\
&= \frac{2}{\sigma^2}Tr(\mathrm{E}_a(G'^H(\tau)G'(\tau))R_\alpha) \\
&= \frac{2N\xi(\tau)\sigma_\alpha^2}{\sigma^2} \qquad (38)
\end{aligned}$$

$$I_{(\tau,\alpha)} = \mathrm{E}_{\boldsymbol{\alpha}}\mathrm{E}_{y|\boldsymbol{\alpha}}\left(-\frac{\partial^2 \ln f(y\,|\,\boldsymbol{\alpha})}{\partial\tau\partial\boldsymbol{\alpha}}\right) = 0 \qquad (39)$$

$$I_{(\alpha,\alpha)} = \mathrm{E}_a\mathrm{E}_{\boldsymbol{\alpha}}\mathrm{E}_{y|\boldsymbol{\alpha}}\left(-\frac{\partial^2 \ln f(y\,|\,\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^H}\right) = \frac{M}{\sigma^2}I.$$

From expression (39), we deduct that both parameters, time-delay and complex gains, are decoupled in the MHFIM.

We compute now the second matrix of the MHFIM, given by relation (32). From relation (34), we obtain the following expression for the *a priori* log-PDF of $\boldsymbol{\alpha}$

$$\ln g(\boldsymbol{\alpha}) = \ln P(\alpha_0) + \Lambda(\boldsymbol{\alpha}) \qquad (40)$$

where $\Lambda(\boldsymbol{\alpha}) \overset{\text{def}}{=} \ln(\pi\sigma_e^2) - \frac{1}{\sigma_e^2}\sum_{n=1}^{N}|\alpha_n - \gamma\alpha_{n-1}|^2$

After some manipulations, we obtain the second partial derivatives of $\ln g(\boldsymbol{\alpha})$. The only non-nul elements of the matrix are

$$\left(\frac{\partial^2 \ln \Lambda(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^*}\right)_{1,1} = -\frac{1+\gamma^2}{\sigma_e^2} + \left(\frac{\partial^2 \ln P(\alpha_0)}{\partial\alpha_0\partial\alpha_0^*}\right) \qquad (41)$$

$$\left(\frac{\partial^2 \ln \Lambda(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^*}\right)_{k,k} = \left(\frac{\partial^2 \ln \Lambda(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^*}\right)_{k-1,k-1} = -\frac{1+\gamma^2}{\sigma_e^2} \quad 42)$$

$$\left(\frac{\partial^2 \ln \Lambda(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^*}\right)_{k,k-1} = \left(\frac{\partial^2 \ln \Lambda(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^*}\right)_{k-1,k} = \frac{\gamma}{\sigma_e^2} \qquad (43)$$

From relations (41), (42) et (43), we obtain

$$B \overset{\text{def}}{=} \mathrm{E}_{\boldsymbol{\alpha}}\left(-\frac{\partial^2 \ln g(\boldsymbol{\alpha})}{\partial\boldsymbol{\alpha}\partial\boldsymbol{\alpha}^H}\right)$$

$$= \begin{pmatrix}
\frac{1+\gamma^2}{\sigma_e^2} - \mathrm{E}_{\alpha_0}\left(\frac{\partial^2 \ln P(\alpha_0)}{\partial\alpha_0\partial\alpha_0^*}\right) & -\frac{\gamma}{\sigma_e^2} & 0 & \ldots & 0 \\
-\frac{\gamma}{\sigma_e^2} & \frac{1+\gamma^2}{\sigma_e^2} & -\frac{\gamma}{\sigma_e^2} & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & 0 \\
\vdots & & -\frac{\gamma}{\sigma_e^2} & \frac{1+\gamma^2}{\sigma_e^2} & -\frac{\gamma}{\sigma_e^2} \\
0 & \ldots & 0 & -\frac{\gamma}{\sigma_e^2} & -\frac{1}{\sigma_e^2}
\end{pmatrix}$$ And

finally,

$$I_h = \begin{pmatrix} I_{(\tau,\tau)} & 0^T \\ 0 & I_{(\alpha,\alpha)} + B \end{pmatrix}.$$

The inverse of this matrix, gives us the expressions (22) and (23).

REFERENCES

[1] T. Aulin and C. E. Sundberg, “Continuous phase modulation-parts I and II,” *IEEE Trans. Commun.*, vol. COM-29, pp. 196-225, Mar. 1981.

[2] P. Chevalier and F. Pipon, “New insights into optimal widely linear array receivers for the demodulation of BPSK, MSK, and GMSK signals corrupted by noncircular interferences-application to SAIC,” *IEEE Trans. Signal Process.*, vol. 54, no. 3, pp. 870-883, 2006.

[3] H. Abeida and J.P. Delmas, “Statistical performance of MUSIC-like algorithms in resolving noncircular sources,” *IEEE Trans. Signal Process.*, vol. 56, no. 9, pp. 4317-4329, September 2008.

[4] U. Mengali and A. N. D’Andrea, *Synchronization techniques for digital receivers*, New York: Plenum Press, 1997.

[5] T. Aulin and C.-E. Sundberg, "Synchronization properties of continuous phase modulation", *in Proc. Globcom*, pp. D7.1.1-D7.1.7., Nov. 1982.

[6] J. Huber and W. Liu, "Data-aided synchronization of coherent CPM receivers," *IEEE Trans. Commun.*, vol. 40, no. 1, pp. 178-188, Jan. 1992.

[7] J. Riba and G. Vazquez, "Non-data-aided frequency offset and symbol timing estimation for binary CPM: performance bounds," *in Proc. ICASSP*, 2000, vol. 5, pp. 2549-2552.

[8] A. N. D'Andrea and U. Mengali, "Symbol timing estimation with CPM modulation," *IEEE Trans. Commun.*, vol. 44, no. 10, pp. 1362-1372, Oct. 1996.

[9] M. Morelli and U. Mengali, "Joint frequency and timing recovery for MSK-type modulation," *IEEE Trans. Commun.*, vol. 47, no. 6, pp. 938-946, Jun. 1999.

[10] H.Wang and P. Chang, "On verifying the first-order Markovian assumption for a Rayleigh fading channel model," *IEEE Trans. Veh. Technol.*, vol. 45, pp. 353-357, May 1996.

[11] R. E. Kalman, "A new approach to linear filtering and prediction problems," *Transactions of the ASME, Journal of Basic Engineering*, vol. 82, pp. 34-45, March 1960.

[12] A. P. Dempster, N. M. Laird and D. B. Rubin, "Maximum likelihood from incompleter data via EM algorithm," *J. Roy. Statist. Soc.*, vol. 39, pp. 1, 1998.

[13] R. Dabora, J. Goldberg, and H. Messer, "Training-Based time-delay estimation for CPM signals over time-selective fading Channels," *IEEE Trans. Commun.*, vol. 52, no. 7, pp. 1169-1177, July 2004.

[14] V. Digalakis, J. Rohlicek and M. Ostendorf, "ML estimation of a stochastic linear system with the EM algorithm and its application to speech recognition," *IEEE Trans. Speech Audio Processing*, vol. 1, no. 4, pp. 431-442, Sept. 1993.

[15] M. E. Khan and D.N. Dutt, "An expectation-maximization algorithm based kalman smoother approach for event-related desynchronization (ERD) estimation from EEG," *IEEE Trans. Biom.*, vol. 54, no. 7, pp. 1191-1198, July 2007.

[16] J. F. G. de Freitas, M. Niranjan and A.H. Gee, "Nonlinear state space learning with EM and neural networks," IEEE International Workshop on Neural Networks in Signal Processing, Cambridge, England. pp. 254-263, Sept. 1998.

[17] H. Chen, R. Perry and K. Buckley, "Direct and EM-based MAP sequence estimation with unknown time-varying channels," *in Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, vol. 4, pp. 2129-2132, May 2001.

[18] F. Patenaude, J. H. Lodge and P.A. Galko, "symbol timing tracking for continous phase modulation over fast flat-fading channels," *IEEE Trans. veh. Techn.*, vol. 40, pp. 615-626, August 1991.

[19] P. Tichavsky , C. H. Muravchik and A. Nehorai, "Posterior Cramér Rao bounds for discrete-time nonlinear filtering," *IEEE Trans. Signal Process.*, vol. 46, pp. 1386, May 1998.

[20] C. H. Knapp and G. C. Carter, "The generalized correlation method for estimation of time delay," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-24, pp. 320-327, Aug. 1976.

[21] W. C. Jakes, *Microwave Mobile Communications*, New York: Wiley, 1974.

[22] K. E. Baddour and N. C. Beaulieu, "Autoregressive modeling for fading channel simulation," *IEEE Trans. Wireless Commun.*, vol. 4, no. 4, pp. 1650-1662, Jul. 2005.

[23] H. E. Rauch, "Soltutions to the linear smoothing problem," *IEEE Trans. on Automatic Control*, vol. 8, pp. 371-372, 1963.

[24] H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Part I*, New York, NY: John Wiley and Sons, 1968.

[25] Y. Rockah and P.M. Schultheiss, "Array Shape Calibration Using Sources in Unknown Locations-Part I: Near-Field Sources," *IEEE Trans. Acoust., Speech, Signal Process*, vol. ASSP-35, pp. 286-299, March 1987.

[26] S. Bay, B. Geller, A. Renaux, J.P. Barbot and J.M. Brossier, "On the Hybrid Cramer-Rao bound and its application to dynamical phase estimation," *IEEE Signal Processing letters*, vol. 15, pp. 453-456, 2008.

[27] H. Abeida and J.P. Delmas, "Cramer-Rao bound for direction estimation of non-circular signals in unknown noise fields," *IEEE Trans. on Signal Processing*, vol. 53, no. 12, pp. 4610-4618, December 2005.

[28] H. Cramér, *mathematical methods of statistics*, Princeton, NJ: Prinston Univetisty press, 1946.

