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Modeling of Power System Component: (One-Two-Three-Winding) Transformer Model for Utilization of Voltage Levels

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ABSTRACT

The control of power flow along transmission line is becoming interesting in the power systems. Regulating transformer is used as a convenient and practical way of controlling power flow in systems, network. The presence of regulating transformer in electrical system enhance the load flow solution; also redistributes, control the bulk power flowing along the transmission line. The purpose of this proposed model is to incorporate regulating transformer and phase shifter in order to normalize and control voltage stability margin. A properly designed and operated power system should, therefore, meet the following fundamental requirements: The system must be able to meet the continually changing load demand for active and reactive power. Unlike other types of energy, electricity cannot be conveniently stored in sufficient quantities. Therefore, adequate "spinning" reserve of active and reactive power should be maintained and appropriately controlled at all times. The system should supply energy at minimum cost and with minimum ecological impact. The "quality" of power supply must meet certain minimum standards with regard to the following factors; Constancy of frequency; Constancy of voltage; and level of reliability.[1]

Keywords: *Transformer component, Regulating Transformer, Delta Star Transformation, Impedance Model, Utilization Level of Voltage*.

1. INTRODUCTION

Transformers enable utilization of different voltage levels across the system. From the viewpoints of efficiency and power-transfer capability, the transmission voltages have to be high, but it is not practically feasible to generate and consume power at these voltages. In modern electric power systems, the transmitted power undergoes four to five voltage transformations between the generators and the ultimate consumers. Consequently, the total MVA rating of all the transformers in a power system is about five times the total MVA rating of all the generators. [2]

In addition to voltage transformation, transformers are often used for control of voltage and reactive power flow. Therefore, practically all transformers used for bulk power transmission and many distribution transformers have taps in one or more windings for changing the turns ratio. From the power system viewpoint, changing the ratio of transformation is required to compensate for variations in system voltages. Two types of tap-changing facilities are provided: off-load tap changing and under-load tap changing (ULTC). The off-load tap-changing facilities require the transformer to be de-energized for tap changing; they are used when the ratio will need to be changed only to meet long-term variations due to load growth, system expansion, or seasonal changes. The ULTC is used when the changes in ratio need to be frequent; for example, to take care of daily variations in system conditions. The taps normally allow the ratio to vary in the range of ±10% to ±15%. [3]

Transformers may be either three-phase units or three single-phase units. The latter type of construction is normally used for large EHV transformers and for distribution transformers.
Large EHV transformers are of single-phase design due to the cost of spare, insulation requirements, and shipping considerations. The distribution systems serve single-phase loads and are supplied by single-transformers.

Transformer is a device for changing the voltage of a.c. supply; it consists of two coil, called primary and secondary coils, wound round a soft iron core that is more of sheets of soft iron insulated from each other to reduce heat losses, such soft iron core is found to be laminated. [4]

An alternating current applied at the terminals of the primary coil sets up an alternating magnetic flux in the core. This induces an e.m.f in the secondary coil. The induced e.m.f of the secondary coils depends on the e.m.f at the primary coil and the number of turns in both coil such that:

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} \qquad (1)$$

Where:

$$\frac{E_s}{E_p} = \frac{Secondary\text{ e.m.f}}{primary\text{ e.m.f}} \qquad (2)$$

$$\frac{N_s}{N_p} = \frac{number\text{ of turns in the secondary coil}}{number\text{ of turns in the primary coil}} \qquad (3)$$

- Necessary condition for step up and step down transformer.

For step-up transformer: When the number of turn in the secondary coil (N_s) is greater than the number of turns in the primary coil (N_p). (i.e. $N_s > N_p$). this means that the transformer will provide a higher emf at secondary coil than the primary coil, that is step up transformer are therefore used in power station, to increase the emf before it is feed into the power transmission lines. [5]

For step down transformer: This is when the number of turns in the secondary (N_s) is less than the number of turns in the primary (N_p). That is, ($N_s < N_p$) this means that the transformer will provide a lower e.m.f of the primary coil than the secondary coil. This is a step them transformer which is used to reduce the high voltages to the lower voltage that is usable in the home.

That is transformers are designed so that energy loses are reduced to a minimum level. This is achieved by:

- Making the coils with wire of low resistance
- Using a soft iron core
- Laminating the core to reduce energy losses due to eddy currents, that is unwanted induced currents.
- Designing an efficient core

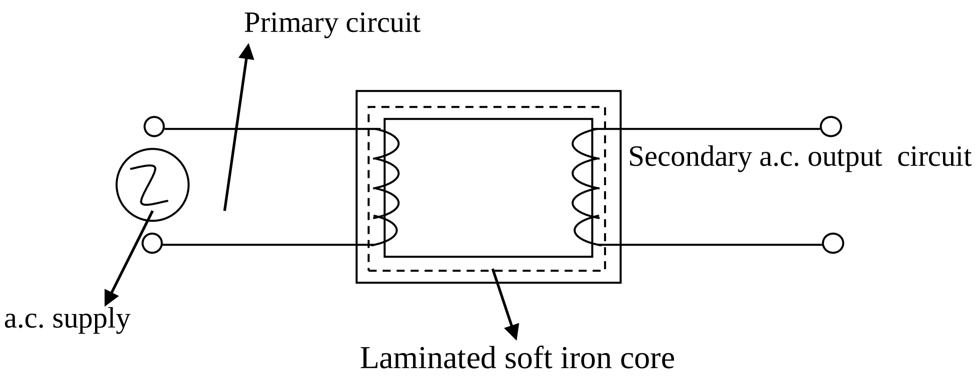


Fig. 1.1: Simple principle of transformer diagram.

Hence electromagnetic force (Emf) a function of voltage.

- Analysis of step-down, step-up, turn ratio of a transformer.

2. GENERAL ANALYSIS

Analysis 1:

- Suppose the emf of the primary coil is 240V a.c and the there are 1000 turns in primary coil, and 500 turns in secondary. Then the emf at the secondary is obtained from equation (1) above:

$$\frac{E_s}{E_p} = \frac{N_s}{N_p}$$

For first insight, since

($N_p > N_s$ or $N_s < N_p$) then it is a step-down transformer and the emf is a function of voltage.

Thus, $E_s = \frac{N_s}{N_p}\, x\, Es\, \frac{500}{1000}\, x\, 240 = 120V$ - - (4)

E_s = 120V.

That is the voltage at primary was 240 voltage are now reduce or step-down to 120 voltage, through over voltage and turn ratio (transformation relationship). **Analysis 2:** Suppose 240 voltage a.c. is applied at the primary coil of a step-down transformer. The ratio of secondary turns to primary turns, if the voltage available at the secondary coil is 60 voltage.

Similarly, from our transformation relationship:

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = K$$

For K > 1, then the transformer is a step up transformer because $N_2 > N_1$, the reverse is the case when K is < 1.

$$\frac{60}{240} = \frac{N_s}{N_p}$$

$$\frac{N_s}{N_p} = \text{turn ratio} = 1:4 \qquad (5)$$

Analysis 3:

Suppose the input emf of a transformer is 240 voltage a.c. the output emf is 960 voltage a.c. If there are 720 turns in the secondary coil, the number of turn in the primary coil is? From our relationship;

$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = K$$

$$\frac{960}{240} = \frac{720}{N_p} \quad - \qquad (6)$$

N_p = 80 turns in primary.

- Modeling two-Winding Transformer for Utilization of Voltage Level

However, when energy (electricity) is generated at a power station, it has to be distributed to other areas for use, since electric power produced (P) is the product of current and the voltage (V) that is (P = IV), we can transmit the same power with high current and low voltage or else with low current and high voltage (V). since the electricity is conveyed through copper cables; some power is loss in the form of heat in the cables. [6]

The smaller the currents, the less the power loss (since $P = I^2R$) and the bigger the currents the greater the power loss. Therefore in order minimize loss of electric power during transmission, electricity is distributed at low current and high voltage.

This means that, at the point of use the high voltage (HV) is brought-down to a low and less dangerous value by the use of transformers.

Voltage Transformation Ratio (K)

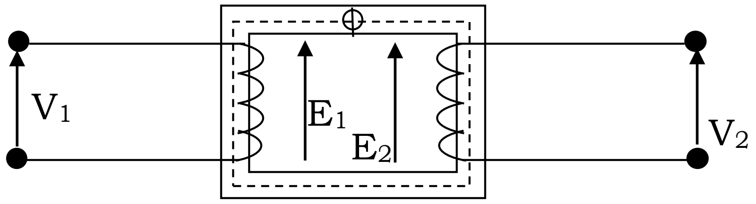


Fig. 1.2: Transformer with Voltage Transformation Relationship

if N_1 = No of turns = N_p in primary
N_2 = No of turns in secondary N_s
Φ_m = maximum flux in core in Weber
= B_m x A (7)
F = frequency of a.c input in H_z.

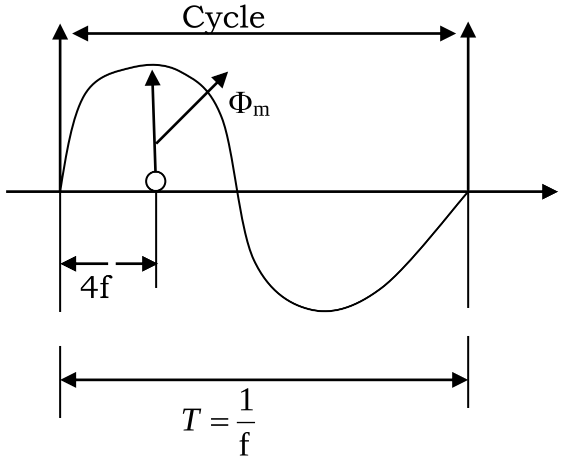


Fig: 1.2a: The electromotive force (emf) profile

That is flux increases from its zero value to maximum value Φ_m in one quarter of the cycle i.e. in ¼f second.

:. Average rate of change of flux $= \frac{\Phi_m}{1/4 f}$ (8)

$= 4f\,\Phi_m \frac{wb}{s}$ or volts (9)

Now rate of change of flux per turn means reduced emf in volts.

That is change emf/turn = $4f\Phi_m$ volts.

If flux Φ_m varies sinusoidally, then rms value of induced emf is obtained by multiplying the average value with form factor

$= \frac{rms\,value}{average\,value} = 1.11$ (10)

That is r.m.s, value of emf/turn = 1.11 x 4f Φ_m = 4.44f Φ_m volts (11)

The r.m.s value of the induced e.m.f in the whole primary winding = (induced e.m.f/turn) x No of primary

$E_1 =$ 4.44f N, Φ_m = 4.4f N_1 Bm A (12)

Similarly r.m.s value of the e.m.f in secondary is;

E_2 = 4.44f Φ_m = 4.44f N_2 B_m A (13)

It is seen from (12) and (13) that $\frac{E_1}{N_1} = \frac{E_2}{N_2} = 4.44f\,\Phi_m$ it means that e.m.f/turn is the same in both the primary and secondary windings. But in an ideal transformer on no load $V_1 = E_1$ and $E_2 = V_2$

(15)

Transformer ratio (K) from (12) and (13) are get

$\frac{E_2}{E_1} = \frac{N_2}{N_1} = K,$ this constant is the voltage transformation ratio.That is if $N_2 > N_1$ or $N_s > N_p$ then K > 1. then transformer is a step- up transformer. Similarly, if $N_2 < N_1$, or $N_s < N_p$ then K < 1 then the transformer is a step-down transformer. [7]

- For ideal transformer;

Input V_A = output V_A

$V_1 I_1 = V_2 I_2$ or $\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{1}{K}$ (17)

That is the currents are therefore, inverse ratio of the (voltage) transformation ratio.

- Why transformer rating are in KVA or MVA?

Copper (copper – loss of a transformer depends on currents and iron loss on voltage.

Hence, total transformer loss depends on volt – ampere (V_A) and not on phase angle between voltage and currents, that is it, is independents of load power factor. That is why rating of a transformer is in KVA or MVA and not in KW.

- Auto Transformer Representation (One-winding Transformer)

This is a transformer with one winding only part of this being common to both primary and secondary. In this transformer, the primary and secondary are not electrically Isolated from each other, as in the case of two-winding transformer, but its theory and operation are similar in that of two-windings transformer, because of one-winding, it uses less copper and hence cheaper. It is used when transformation ratio (K) differ little from unity. [8]

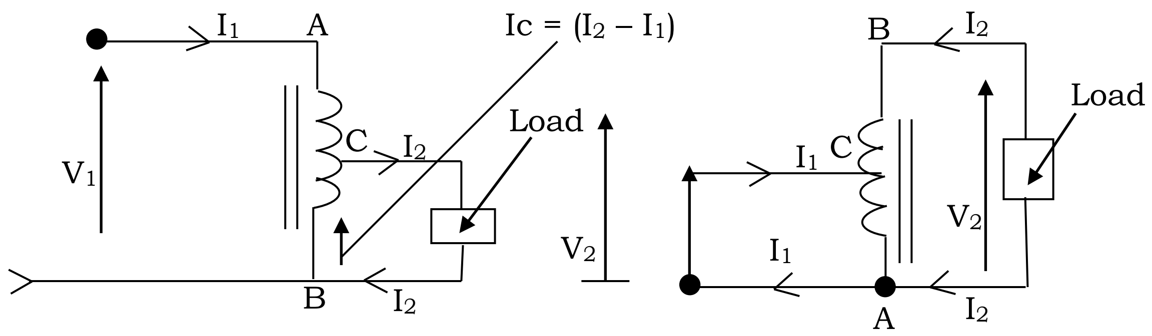


Fig. 1.3: Auto Transformer one-winding Representation

- AB is primary winding having N_1 turns, BC is secondary winding having N_2 turns. Neglecting iron loss and no load currents;

then

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2} = K \qquad (18)$$

The current in section CB is vector difference of I_2 and I_1 but as the two current are practically in phase opposition, the resultant current ($I_2 – I_1$) where I_2 is greater than I_1.

- As compared to copper conducting 2-windings transformer of same output, but an auto-transformer which has higher efficiency but small size. Moreover its, voltage regulation is also superior.

When the voltage transformation ratio is small autotransformers are normally used. The primary and secondary of autotransformer are interconnected so 'that the power to be transformed by magnetic coupling is only a portion of the total power transmitted through the transformer. There is thus inherent metallic connection between the primary side and secondary side circuits; this is unlike the conventional two-winding transformer which isolates the two circuits.

Autotransformers are usually Y connected, with neutrals solidly grounded to minimize the propagation of disturbances occurring on one side into the other side. [9]

It is a common practice to add a low-capacity delta-connected tertiary winding. The tertiary winding provides a path for third harmonic current, thereby reducing their flow on the network. It also assists in stabilizing the neutral. Reactive compensation is often provided through use of switched reactors and capacitors on a tertiary bus.

- As compared to the conventional two-winding transformer, the transformer has advantages of lower cost, higher efficiency, and better regulation. These advantages become less significant as the transformation ratio increases; hence, autotransformers are used for low transformation ratios (for example, 500/230 kV).

- In interconnected systems, it sometimes becomes necessary to make electrical connections that form loop circuits through one or more power systems. To control the circulation of power and prevent overloading of certain lines, it is usually necessary in such situations to use *phase-angle transformers*. Oftenly, it is necessary to vary the extent of phase shift to suit changing system conditions; this requires provision of on- load phase-shifting capability. Voltage transformation may also be required in addition to phase shift.

- Conversion of 2-Windiflg Transformer into Auto-transformer

Any two-winding transformer can be converted into an auto-transformer either step-down or step-up. Fig. 1.4 (a) shows such a transformer with its polarity markings. Suppose it is a 20-kVA, 2400/240V transformer. If we employ additive polarity between the high voltage and low-voltage sides,' we get a step-up auto-transformer. If we however, we use the subtractive polarity we get a step-down auto-transformer. [10]

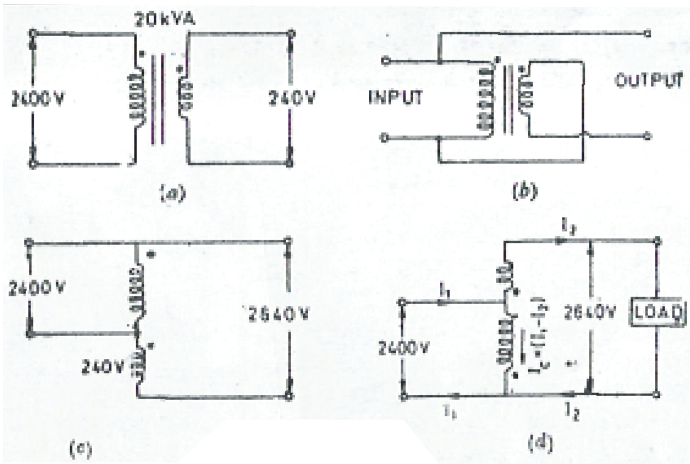


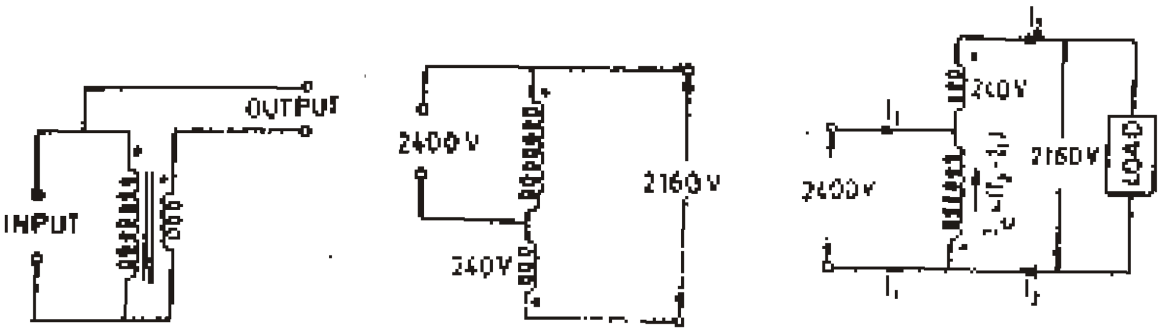
Fig. 1.4: Additive Polarity of Auto-transformer

a. Additive Polarity

Connections for such a polarity are shown in Fig. 1.4 (b). The circuit is redrawn in Fig. 1.4 (c) showing common terminal of the transformer at the top whereas Fig 1.4 (d) shows the same circuit with common terminal at the bottom. Because of additive polarity V_2 = 2400 + 240 = 2640 V and V_1 is 2400 V. There is a marked increase in the KVA of the auto-transformer. In figure 1.4 (d) the common current flows towards the common terminal. The transformer acts as a step-up transformer.

b. Subtractive Polarity:

Such a connection is shown in fig. 1.5 (a) The circuit has been redrawn with common polarity at top in Fig. 1.5 (b) and at bottom in Fig. 1.5 (c). In this case, the transformer act as a transformer step-down transformer.



(a) (b) (c)

Fig. 1.5: Subtractive Polarity of Auto-Transformer

The common current flows away from the common terminal. In this case also, there is a very large increase in KVA rating of the auto-transformer though not as marked as in the previous case. Here, V_2 = 2400 – 240 = 216V.

- Representation of Two-Winding Transformers: Basic equivalent circuit in physical units.

The basic equivalent circuit of a two-winding transformer with all quantities in physical units is shown in Figure fig. 1.6 subscribes *P* and *S* refer to primary and secondary quantities, respectively.

The magnetizing reactance X_{mp} is very large and is usually neglected. For special studies requiring representation of transformer saturation, the magnetizing reactance representation may be approximated by moving it to the primary or secondary terminal and treating it as a voltage-dependent variable shunt reactance.

- Per Unit Equivalent Circuit

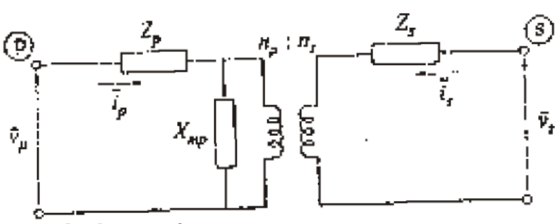
With appropriate choice of primary and secondary side base quantities, the equivalent circuit can be simplified by eliminating the ideal transformer. However, this is not always possible and the base quantities often have to be chosen independent of the actual turns ratio. It is therefore necessary to consider an off-nominal turns ratio.

From the equivalent circuit of figure 1.6 with X_{mp} neglected, we have

$$\bar{v}_p = Z_p \bar{i}_p + \frac{n_p}{n_s}\bar{v}_s - \frac{n_p}{n_s} Z_s \bar{i}_s \qquad (20)$$

$$\bar{v}_s = \frac{n_s}{n_p}\bar{v}_p - \frac{n_s}{n_p} Z_p \bar{i}_p + Z_s \bar{i}_s$$

- Consider an Ideal Transformers



Ideal Transformer

Figure 1.6: Basic equivalent circuit of a two-winding transformer

$Z_p = R_p + jX_p$; $Z_s = R_s + jX_s$

R_p, R_s = Primary and secondary winding resistances ; $Z_{po} = Z_p$ at nominal side tap position

X_p, X_s = Primary and secondary winding leakage reactances ; $Z_{so} = Z_s$ at normal secondary side tap position

n_p, n_s = number of turns of primary and secondary winding ; n_{po} = primary side normal number of turns

X_{mp} = magnetizing reactance referred to the primary side ; n_{so} = secondary side normal number of turns

Expressing Equations 20 and 21 in terms of the above nominal values,

$$\bar{v}_p = \left(\frac{n_p}{n_{po}}\right)^2 Z_{po}\bar{i}_p + \frac{n_p}{n_s}\bar{v}_s - \frac{n_p}{n_s}\left(\frac{n_p}{n_s}\right)^2 Z_{so}\bar{i}_s$$

(22)

$$\bar{v}_s = \frac{n_s}{n_p}\bar{v}_p - \frac{n_s}{n_p}\left(\frac{n_p}{n_{po}}\right)^2 Z_{po}\bar{i}_p + \left(\frac{n_s}{n_{so}}\right)^2 Z_{so}\bar{i}_s$$

(23)

- Here, we have assumed that both leakage reactance and resistance of a transformer winding are proportional to the square of the number of turns. This assumption is generally valid for the leakage reactance, but not for the resistance. Since the resistance is much smaller than the leakage reactance and since the deviation of the actual turns ratio from the nominal turns ratio is not very large, the resulting approximation is acceptable. For convenience, we will assume that both primary and secondary windings are connected so as to form a Y – Y connected three-phase bank.

$$\frac{n_{po}}{n_{so}} = \frac{v_{pbase}}{v_{sbase}}$$

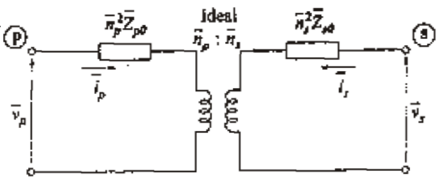


Fig. 1.7: Per unit equivalent circuit

The equivalent circuit of figure 1.7 can be reduced to the standard form shown in figure 1.8, where $\bar{n}$ is the per unit ratio.

and $v_{pbase} = Z_{pbase}\, i_{pbase}$, $v_{sbase} = Z_{sbase}\, i_{sbase}$

Equations 22 and 23 in per unit form become:

$$\bar{v}_p = \bar{n}_p^2 \bar{Z}_{po}\bar{i}_p + \frac{\bar{n}_p}{\bar{n}_s}\bar{v}_s - \bar{n}_s^2\frac{\bar{n}_p}{\bar{n}_s}\bar{Z}_{so}\bar{i}_s \quad (24)$$

$$\bar{v}_s = \frac{\bar{n}_s}{\bar{n}_p}\bar{v}_p - \bar{n}_p^2\frac{\bar{n}_s}{\bar{n}_p}\bar{z}_{po}\bar{i}_p + \bar{n}_s^2\bar{z}_{so}\bar{i}_s \quad (25)$$

Where the superbars denote per unit values, with $\bar{v}_p$, $\bar{v}_s$, $\bar{i}_p$, $\bar{i}_s$ equal to per unit values of *phasor* voltage and currents, and

$$\bar{n}_p = \frac{n_p}{n_{po}} \quad (26)$$

$$\bar{n}_s = \frac{n_s}{n_{so}} \quad (27)$$

The per unit equivalent circuit representing Equations 24 and 25 is shown

$$\bar{n} = \frac{n_p}{n_s} = \frac{n_p n_{so}}{n_{po} n_s} \quad (28)$$

$$\bar{z}_e = \bar{n}_s^2\left(\bar{z}_{po} + \bar{z}_{so}\right) \quad (29)$$

$$= \left(\frac{n_s}{n_{so}}\right)\left(\bar{z}_{po} + \bar{z}_{so}\right)$$

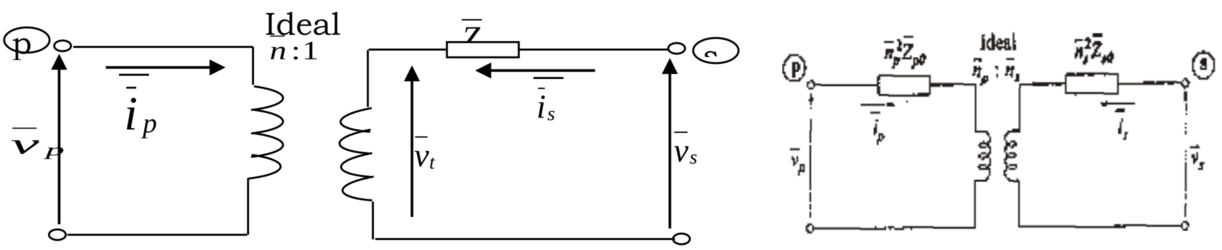


Fig. 1.8: Standard equivalent circuit for a transformer

- The equivalent circuit of figure 1.8 are widely used for representation of two-winding transformers in power flow and stability studies.

We see from equation 29 that $\bar{z}_e$ does not change with $\bar{n}_p$. therefore, if the tap is on the primary side, only $\bar{n}$ changes.

If the actual turns ratio is equal to n_{po}/n_{so}, then $\bar{n}$ = 1.0, and the ideal transformer vanishes. When the actual turns ratio is not equal to the nominal turns ratio, $\bar{n}$ represents the off-nominal ratio (ONR).

- The equivalent circuit of figure 1.8 can be used to represent a transformer with a fixed (or off-load) tap on one side and an under-load tap changer (ULTC) on the other side. The off-nominal turns ratio is assigned to the with ULTC and $\bar{z}_e$ has a value corresponding to the fixed-tap position of the other side, as given by equation 29.

- In digital computer analysis of power flow, it is not convenient to present an ideal transformer. We will therefore reduce the equivalent circuit of figure 1.8 to the form of a π network of figure 1.7.

The terminal current at bus *p* is

$$\bar{i}_p = \left(\bar{v}_t - \bar{v}_s\right)\frac{\bar{Y}_e}{\bar{n}} \qquad (30)$$

$$= \left(\frac{\bar{v}_p}{\bar{n}} - \bar{v}_s\right)\frac{\bar{Y}_e}{\bar{n}}$$

$$= \left(\bar{v}_p - \bar{n}\bar{v}_s\right)\frac{\bar{Y}_e}{\bar{n}^2}$$

Where $\bar{Y}_e = 1/\bar{Z}_e$. similarly, the terminal current at bus *s* is

$$\bar{i}_s\left(\bar{n}\bar{v}_p - \bar{v}_p\right)\frac{\bar{Y}_e}{\bar{n}} \qquad (31)$$

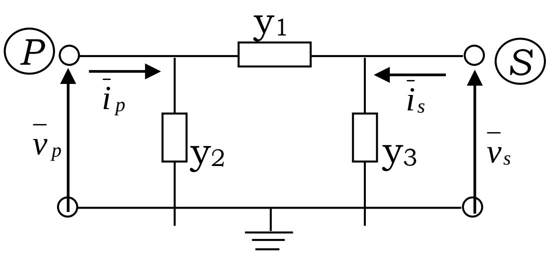
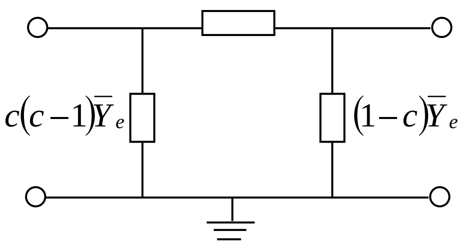


Fig. 1.8a: General π network



$$\bar{Y}_e = 1/\bar{Z}_e$$
$$c = 1/\bar{n}$$

Fig. 1.8b: Equivalent π circuit

Figure 1.8 (a and b): Transformer representation with ONR

The corresponding terminal currents for the network shown in figure 1.8a are

$$\bar{i}_p = y_1(\bar{v}_p - \bar{v}_s) + y_2\bar{v}_p \qquad (32)$$

$$\bar{i}_s = y_1(\bar{v}_s - \bar{v}_p) + y_3\bar{v}_s \qquad (33)$$

Equating the corresponding admittance terms in Equations 30 and 31, we have

$$y_1 = \frac{1}{\bar{n}}\bar{Y}_e = c\bar{Y}_e \qquad (34)$$

and

$$y_2 = \left(\frac{1}{\bar{n}^2} - \frac{1}{\bar{n}}\right)\bar{Y}_e = \left(c^2 - c\right)\bar{Y}_e \qquad (35)$$

Where c = $\frac{1}{\bar{n}}$, similarly, from Equations 31 and 33,

$$y_3 = (1-c)\overline{Y}_e \qquad (36)$$

The equivalent π circuit with parameters expressed in terms of the ONR and transformer leakage impedance is shown in figure 1.8b.

- Consideration of three-phase transformer connections .The standard equivalent circuit of Figure 1.7 represents the single-phase equivalent of a three-phase transformer. In establishing the ONR, the nominal turns ratio (n_{po}/n_{so}) is taken to be equal to the ratio of line-to-line base voltages on both side of the transformer irrespective of the winding connections (Y-Y, Δ-Δ, or Y-Δ). For Y-Y and Δ-Δ connected transformers, this makes the ratios of the base voltages to the ratios of the nominal turns of the primary and secondary windings of transformer phase. For a Y-Δ connected transformer, this in addition accounts each transformer phase. For a Y-Δ connected transformer, this in additions for the factor $\sqrt{3}$ due to the winding connection.

- In the case of a Y-Δ connected transformer, a 30^0 phase shift is introduced between line-to-line voltages on the two sides of the transformer. The line-to-neutral voltage and line currents are similarly shifted in phase due to the winding connections. It is usually not necessary to take this phase shift into consideration in system studies. Thus, the single-phase equivalent of a Y-Δ transformer does not account for the phase shift, except as the phase shift of voltages are due to the impedance of the transformer.

- Modeling of two-winding transformer test data:

Consider a 60 Hz, two-winding, three-phase transformer with the following data:

MVA rating : : 42.00 MVA
Primary (HV) nominal voltage : 110.00KV
Secondary (LV) nominal voltage : 28.40KV
Winding connections (HV/LV) : Y/Δ
Resistance:

$\overline{R}_{po} + \overline{R}_{so}$ = 0.00411 pu on rating/phase

Leakage reactance:

$\overline{X}_{po} + \overline{X}_{so}$ = 0.1153 pu on rating/phase

Off-load tap changer on HV side:
4 steps, 2.75 KV/step
Under-load tap changer on LV side:
±2.84 KV in 16 steps

- Let us examine the condition when the LV winding is initially at its nominal position, and the HV winding is manually set two steps above its nominal position, i.e., at 115.5 KV. The parameters of the standard equivalent circuit (figure 1.7) with the ONR on the LV (ULTC) side and values expressed in per unit of the transformer rated values are as follows:

- Initial off-nominal turns ratio:

$$\overline{n} = \frac{28.4}{28.4}\frac{110}{115.5} = 0.95238$$

- Per unit equivalent impedance:

$$\overline{Z}_e = \left(\frac{115.5}{110}\right)^2 (0.00411 + j0.1153)$$

= 0.00453 + j0.12712 pu

- Maximum per unit turns ratio:

$$\overline{n}_{max} = \frac{31.24}{28.4}\frac{110}{115.5} = 1.04762$$

- Minimum per unit turns ratio:

$$\overline{n}_{min} = \frac{25.56}{28.4}\frac{110}{115.5} = 0.85714$$

- Per unit turns ratio step:

$$\Delta\overline{n} = \frac{2.84}{16\times 28.4}\frac{110}{115.5}$$

= 0.0059524

Now, if the common system voltage and MVA base values are:

Primary system voltage base: 115.0KV
Secondary system voltage base: 28.4KV
System MVA base: 100 MVA

The corresponding per unit parameters of the equivalent circuit are as follows:

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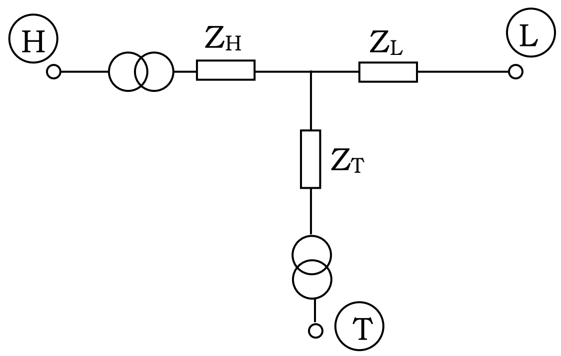


- Initial off-nominal turns ratio:

$$\bar{n} = 0.95238 \frac{28.4}{28.4} \frac{115}{110}$$

= 0.99567

- Per unit equivalent impedance:

$$\overline{Z_e} = (0.00453 + j0.12712)\left(\frac{110}{115}\right)^2 \frac{100}{42}$$

= 0.009868 + j0.27692

- Maximum per unit turns ratio:

$$\bar{n}_{man} = 1.04762 \frac{28.4}{28.4} \frac{115}{110} =$$

1.09524

- Minimum per unit turns ratio:

$$\bar{n}_{\min} = 0.85714 \frac{28.4}{28.4} \frac{115}{110} =$$

0.89610

- Per unit turns ratio step:

$$\Delta\bar{n} = 0.005924 \frac{2.84}{28.4} \frac{115}{110} =$$

0.006193

The equivalent π circuit (figure 1.8) parameters representing the initial tap position are as follows:

$$y_1 = \frac{1}{\bar{n}Z_e} = \frac{1}{0.99567(0.009868 + j0.27692)}$$

= 0.12908 – j3.62226

$$y_2 = \left(\frac{1}{\bar{n}} - 1\right) y_1$$

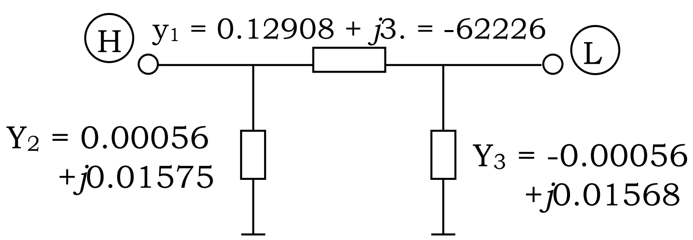
$$= \left(\frac{1}{0.99567}\right) \frac{1}{0.99567(0.009868 + j0.27692)}$$

= 0.00056-j0.01575

$$y_3 = \left(1 - \frac{1}{\bar{n}}\right)\frac{1}{Z_e} = \left(1 - \frac{1}{0.99567}\right)\frac{1}{0.009868 + j0.27692}$$

= -0.00056 + j0.01568

- Using Delta Star or Delta-star Impedance Transformation Relationship



- Representation of Three-Winding Transformers

Figure 1.9 shows the single-phase equivalent of a three-winding transformer under balanced conditions. The effect of the magnetizing reactance has been neglected, and the transformer is represented by three impedances connected to form a star. The common star point is fictitious and unrelated to the neutral.

- Using Delta Star or Delta-star Impedance Transformation Relationship

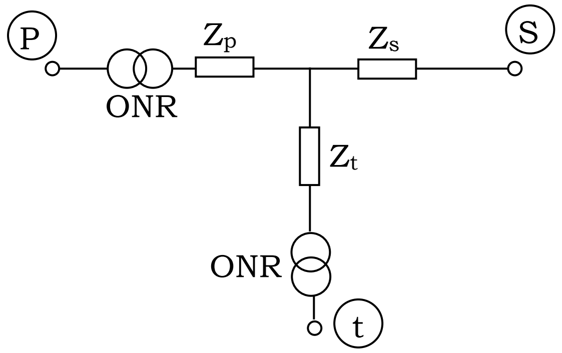


Figure 1.9: Equivalent circuit of a three-winding transformer

The three windings of the transformer may have different MVA ratings. However, the per unit impedances must be expressed on the same MVA base. As in the case of the two-winding transformer equivalent circuit developed, in the off-nominal turns ratios are used to account for the differences between the ratios of actual turns and the base voltages. The values of the equivalent impedances Z_p, Z_s and Z_t may be obtained by standard short-circuit tests as follows:

Z_{ps} = Leakage impedance measured in primary with secondary shorted and tertiary open

Z_{pt} = Leakage impedance measured in primary with tertiary shorted and secondary open

Z_{st} = Leakage impedance measured in the secondary with tertiary shorted and primary open.

With the above impedances in ohms referred to the same voltage base, we have:

$$Z_{ps} = Z_p + Z_s$$
$$Z_{pt} = Z_p + Z_1$$
$$Z_{st} = Z_s + Z_t \qquad (37)$$

Hence,

$$Z_p = \frac{1}{2}(Z_{ps} + Z_{pt} - Z_{st})$$
$$Z_s = \frac{1}{2}(Z_{ps} + Z_{st} - Z_{pt}) \qquad (38)$$
$$Z_t = \frac{1}{2}(Z_{pt} + Z_{st} - Z_{ps})$$

In large transformers, Z_s is small and may even be negative.

- Consider of modeling three-winding transformers:

We will consider a 60 Hz, three-winding, three-phase transformer with the following data:

MVA rating : 750 MVA

High/low/tertiary nominal voltages: 500/240/28 KV

Winding connections (H/L/T) : Y/Y/Δ

Measured positive-sequence impedances in pu on transformer MVA rating and nominal voltages at nominal tap position:

Z_{H-L} = $0.0015 + j0.1339$

Z_{L-T} = $j0.1895$

Z_{T-H} = $0 + j0.3335$

ULTC at high voltage side: 500 ± 50 KV in 20 steps.

Neglecting the magnetizing reactance, the equivalent star circuit with ULTC at nominal tap position is:

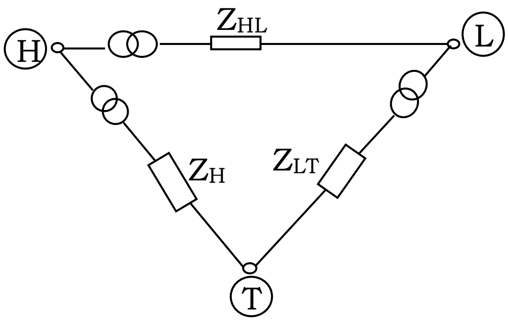
$$Z_H = \frac{Z_{H-L} + Z_{T-H} - Z_{L-T}}{2} = 0.00075 + j0.13895$$

$$Z_L = \frac{Z_{H-L} + Z_{L-T} - Z_{T-H}}{2} = 0.00075 + j0.00505$$

$$Z_T = \frac{Z_{L-T} + Z_{T-H} - Z_{H-L}}{2} = -0.00075 + j0.19455$$

Equivalent Delta circuit with parameters in pu on transformer MVA rating and nominal voltages.

- Using Delta Star or Delta-star Impedance Transformation Relationship



$\Sigma = Z_H Z_L + Z_H Z_T + Z_L Z_T$

$= -0.02535 + jo.0002914$

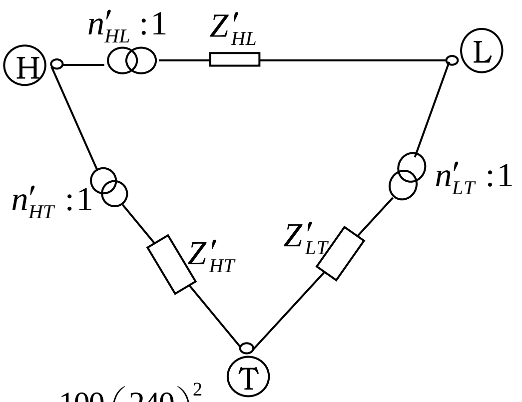
$Z_{HL} = \dfrac{\Sigma}{Z_T} = 0.0020 + j0.1303$

$Z_{LT} = \dfrac{\Sigma}{Z_H} = 0.0011 + j0.1824$

$Z_{HT} = \dfrac{\Sigma}{Z_L} = 0.7859 - j4.9029$

Equivalent delta circuit with parameters in pu on system MVA base of 100 MVA and voltage bases (H/L/T) of 500/220/27.6 KV:

- Using Delta Star or Delta-star Impedance Transformation Relationship



$$Z'_{HL} = Z_{HL}\frac{100}{750}\left(\frac{240}{220}\right)^2 = 0.00032 + j0.02067$$

$$Z'_{LT} = Z_{LT}\frac{100}{750}\left(\frac{28.0}{27.6}\right)^2 = 0.00015 + j0.02504$$

$$Z'_{TH} = Z_{TH}\frac{100}{750}\left(\frac{28.0}{27.6}\right)^2 = -0.10784 - j0.6728$$

ULTC data:

$$n'_{HL\max} = \frac{550}{500}\frac{500}{500}\frac{220}{240} = 1.00833$$

$$n'_{HL} = \frac{500}{500}\frac{220}{240} = 0.91667$$

$$n'_{LT} = \frac{240}{220}\frac{27.6}{28.0} = 1.07532$$

$$n'_{HL} = \frac{500}{500}\frac{27.6}{28.0} = 0.98571$$

$$n'_{HL\min} = \frac{450}{500}\frac{500}{500}\frac{220}{240} = 0.8250$$

$$\Delta n'_{HL} = \frac{1.00833 - 0.825}{20} = 0.00917$$

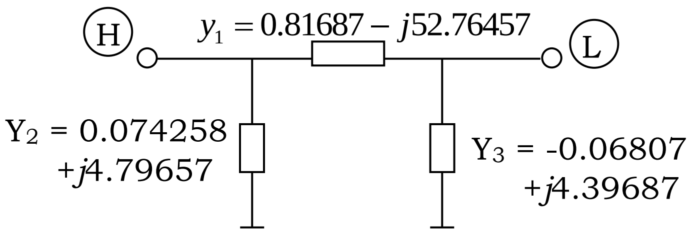
$$n'_{HL\max} = \frac{550}{500}\frac{500}{500}\frac{27.6}{28.0} = 1.08429$$

$$n'_{HL\min} = \frac{450}{500}\frac{500}{500}\frac{27.6}{28.0} = 0.88714$$

$$\Delta n'_{HT} = \frac{1.08429 - 0.8814}{20} = 0.01014$$

We should recognize that the ULTC action at the high-voltage side changes the ONRs n^1_{HL} and n^1_{HT} these two ONRs cannot be adjusted independently.

The three branches of the delta equivalent circuit can each be represented by an equivalent circuit as shown in figure 1.8.



- L-T Branch:

$$y_1 = \frac{1}{n'_{LT}Z'_{LT}} = \frac{1}{1.07532(0.00015 + j0.02504)}$$

$$= 0.22247 - j37.13748$$

$$y_2 = \left(\frac{1}{n'_{LT}} - 1\right)y_1 = \left(\frac{1}{1.07532} - 1\right)(0.22247 - j37.13748)$$

$$= -0.01558 - j2.60127$$

$$y_3 = \left(1 - \frac{1}{n'_{LT}}\right)\frac{1}{Z'_{LT}} = \left(1 - \frac{1}{1.07532}\right)\frac{1}{0.00015 + j0.02504}$$

$$= 0.01676 - j2.79719$$

The equivalent π circuit representing the initial ULTC tap position are as follows:

- H-L Branch:

$$y_1 = \frac{1}{n'_{HL}Z'_{HL}} = \frac{1}{0.91667(0.00032 + j0.02067)}$$

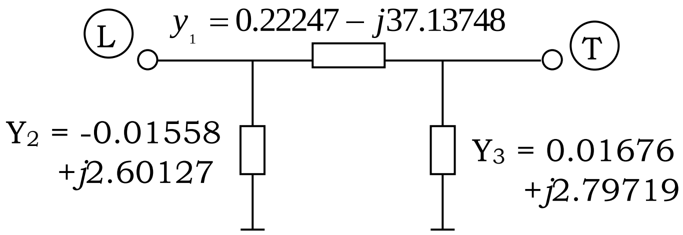
$$= 0.81687 - j52.76457$$

$$y_2 = \left(\frac{1}{n'_{HL}} - 1\right)y_1 = \left(\frac{1}{0.91667} - 1\right)(0.81687 - j52.26457)$$

$$= 0.074258 - j4.79657$$

$$y_3 = \left(1 - \frac{1}{n'_{HL}}\right)\frac{1}{Z'_{HL}} = \left(1 - \frac{1}{0.91667}\right)\frac{1}{0.00032 + j0.02067}$$

$$= -0.06807 - j4.39687$$



- H-T Branch:

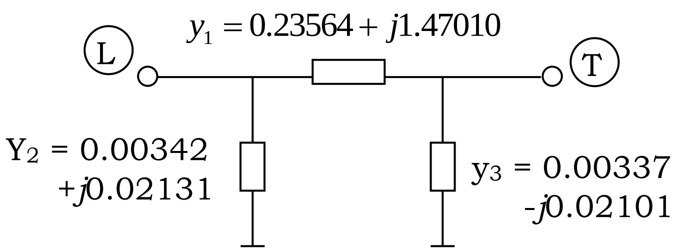
$$y_1 = \frac{1}{n'_{HT} Z'_{HT}} \frac{1}{0.98571(-0.10784 - j0.67280)}$$

$$= -0.23564 - j1.47010$$

$$y_2 = \left(\frac{1}{n'_{HT}} - 1\right) y_1 = \left(\frac{1}{0.98571}\right)\left(-0.23564 - j1.47010\right)$$
$$= -0.00342 - j0.02131$$

$$y_3 = \left(1 - \frac{1}{n'_{HT}}\right)\frac{1}{Z'_{HT}} = \left(1 - \frac{1}{0.98571}\right)\frac{1}{-0.10784 + j0.67280}$$

$$= -0.00337 - j0.02101$$



- Phase-Shifting Transformers

A phase-shifting transformer can be represented by the equivalent circuit shown in figure 1.9, it consists of an admittance in series with an ideal transformer having a complex turns ratio, $\bar{n}$ =n<α. the phase angle step size may not be equal at different tap positions. However, equal step is used in power flow and transient stability programs.

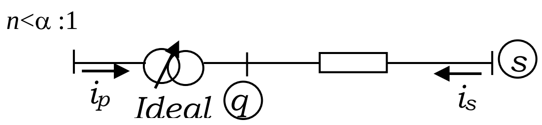


Figure 1.9 phase-shifting transformer representation

By definition:

$$\frac{\bar{v}_p}{\bar{v}_q} = n < \alpha = n(\cos\alpha + j\sin\alpha) \qquad (39)$$

$$= \quad \alpha_s + j_{bS}$$

Where α is the phase shift from bus p to bus q; it is positive when $\bar{v}_p$ leads $\bar{v}_q$. Since there is no power loss in an ideal transformer,

$$\bar{v}_p \bar{i}_p = \bar{v}_q \bar{i}_s \qquad (40)$$

Therefore, the transformer current at bus p is

$$\bar{i}_p = \frac{1}{a_s - j_{bs}} \bar{i}_s$$

$$= \frac{Y_e}{a_b - j_{bs}} \left(\bar{v}_q - \bar{v}_s \right) \qquad (41)$$

Substituting for $\bar{v}_q$ from equation 39, we have

$$\bar{i}_p = \frac{Y_e}{a_b - j_{bs}} \left[\frac{1}{a_s + j_{bs}} \bar{v}_p - \bar{v}_s \right] \qquad (42)$$

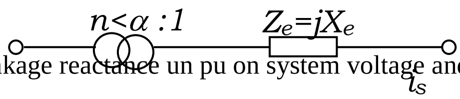
$$= \frac{Y_e}{a_s^2 + b_s^2} \left[\bar{v}_p - (a_s + j_{bs}) \bar{v}_s \right]$$

From equation 41

$$i_s = (a_s - j_{bs}) \bar{i}_p$$

Substituting for $\bar{i}_p$ from equation 42 gives

$$\bar{i}_s = \frac{Y_e}{a_s + j_{bs}} \left[(a_s + j_{bs}) \bar{v}_s - \bar{v}_p \right] \qquad (43)$$



- Leakage reactance un pu on system voltage and MVA base:

$$X_e = 0.145 \times \frac{100}{300} \times \left(\frac{240}{230} \right)^2$$

$$= \quad 0.05263 \text{ pu}$$

- Off-nominal turns ratio:

$$n = \frac{240}{220} \times \frac{230}{240} \quad = 1.04545$$

Phase-shift angle limits:

$$a_{max} \quad = \quad 40^0 \; ; \; a_{min} \quad = \quad -40^0$$

The impedance of the transformer changes with the phase-shift angle.

Combining equations 42 and 43, we obtained the following matrix equation relating the phase-shifter terminal voltages and currents.

$$\begin{bmatrix} \bar{i}_p \\ \bar{i}_s \end{bmatrix} = \begin{bmatrix} \dfrac{Y_e}{a_s^2 + b_s^2} & \dfrac{-Y_e}{a_s - jb_s} \\ \dfrac{-Y_e}{a_s + jb_s} & Y_e \end{bmatrix} \begin{bmatrix} \bar{v}_p \\ \bar{v}_s \end{bmatrix} \qquad (44)$$

We see that the admittance matrix in the above equation is not symmetrical, that is, the transfer admittance from p to s is not equal to the transfer admittance s to p, therefore, a π equivalent circuit is not possible.

- If the turns ratio is real (i.e., $a_s = \bar{n}$ and $b_s = 0$), the model reduces to the equivalent π circuit shown in figure 1.8b.

- Modeling a Phase-shifting Transformer for Test Data Analysis

Let us consider a three-phase, two-winding phase shifter with the following data:

MVA rating : 300 MVA
Primary/secondary base voltages : 240/240 KV
Resistance per phase : 0
Leakage reactance per phase : 0.145 pu
Phase-shift range and steps : ±40^0, 36 steps
System voltage base
(primary/secondary) : 220/230 KV
System MVA base : 100 MVA

Table 1.1: The following table (provided by the manufacturer) gives values of the impedance multiplier as a function of the angle.

Angle in degree	±40	±29.5	±25.1	±20.6	0
Impedance multiplier	1.660	1.331	1.228	1.144	1.0

The admittance matrix of equation 44 representing the phase shifter is

$$Y_s = \begin{bmatrix} \dfrac{Y_e}{a_s^2 + b_s^2} & \dfrac{-Y_e}{a_s - jb_s} \\ \dfrac{-Y_e}{a_s + jb_s} & Y_e \end{bmatrix}$$

As an illustration, we will determine the elements of the admittance matrix for two values of α.

- Case 1: when α = 0:

$$Y_e = \frac{1}{jX_e} = \frac{1}{j0.05263} = -j19.0006 \quad pu$$

The turns ratio of the ideal phase shifter is

$a_s + jb_s = n(\cos a + j\sin a$

= 1.0455(cos0 +*j*sin0) = 1.0455 + *j*0

Case 2: when α corresponding to the 10^{th} step:

$$a = \frac{40}{36} \times 10 = 11.11^0$$

The turns ratio is

$a_s + jb_s = n(\cos 11.11^0 + j\sin 11.11^0)$

= 1.02585 + *j*0.20147

The phase-shifter leakage reactance at this value of *a* by interpolation is:

$$X_e = \left[1.0 + \frac{11.11(1.144 - 1.0)}{20.6}\right] \times 0.05263$$

= 0.05672

Hence,

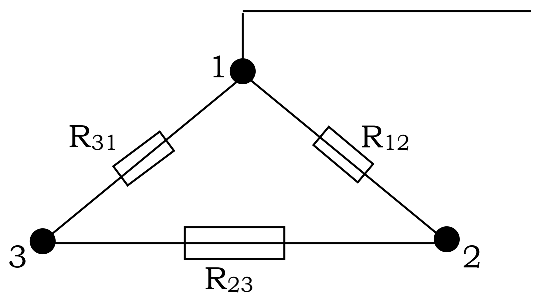
$$Y_e = \frac{1}{jX_e} = j17.6305$$

The admittance matrix $Y_{s'}$ with $a_s + jb_s$ = 1.02585 +*j*0.20147 and Y_e = *j*17.6305 is:

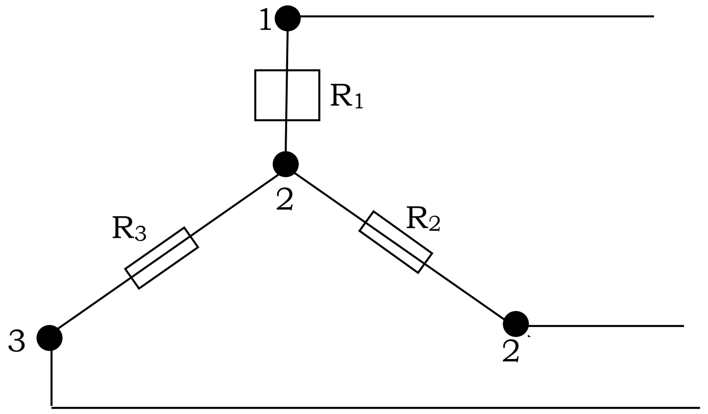
$$Y_e = \begin{bmatrix} -j16.1310 & (-3.2499 + j16.5479 \\ (3.2499 + j16.5479) & j17.6305 \end{bmatrix}$$

Delta-star transformation (in terms of resistance).

In solving network (having considerable number of branches) by the application of kirchoff's law one could experienced great difficulty due to a large number of simultaneous equations that have to be solved. However such complicated equations network can be simplified by successively replacing delta meshes by equivalent star systems and replacing star meshes by an equivalent delta system.



a



b

- Suppose we are given three resistance R_{12}, R_{23}, and R_{31} connected in delta pattern between terminal 1, 2 and 3 as strains fig 1a above. Now these three resistance are be replaced by three resistance R_1, R_2 and R_3 connected in star as show in (fig 1b).
- These two arrangements will be electrically equivalent if the resistance is measured between any pair of terminals is the same in the both arrangements.
- Now consider Delta connection (Δ) between terminal 1 and 2, there are two parallel paths, are having resistance of (R_{12}) and the other having a resistance of (R_{23} + R_{31}) resistance between terminals 1 and 2 is:

$$= \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \qquad (45)$$

- Now take star connection in (Y)

The resistance between the same terminal 1 and 2 is (R_1 + R_2).

- As terminal resistance have to be the same:

Thus, $R_1 + R_2 = \frac{R_{12} \times (R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$ (46)

- Similarly, for terminals 2 and 3 and terminals 3 and 1, we have;

$$R_2 + R_3 = \frac{R_{23} \times (R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \qquad (47)$$

$$R_3 + R_1 = \frac{R_{31} \times (R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \qquad (48)$$

- Now subtracting (47) from (46) and adding the result to (48), we have;

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \qquad (49)$$

$$R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} \qquad (50)$$

$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \qquad (51)$$

Similarly, the transformation can be done by using equations (46), (47) and (48), give above. Multiplying (46), and (47) and (47) and (46) respectively and adding them together are then multiplying them; we have:

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \qquad (52)$$

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \qquad (53)$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = R_1 + R_3 + \frac{R_3 R_1}{R_2} \qquad (54)$$

3. CONCLUSION

The control of electric power-flow along transmission line is becoming of good interest and concern to the power industry. A properly designed, modeled and operating power system should therefore meet the fundamental requirements and constraints existing in electric power system. The incorporation of regulating transformer in the model formulation of an electric power system has enhanced the output performance of the system and simplified the load-flow solutions. The analysis has shown the usefulness of regulating transformers in controlling power flows in system networks.

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