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A Novel Λ-Based Genetic Algorithm Solution for Economic Dispatch Problem in Large-Scale Systems

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ABSTRACT

This paper presents a new genetic approach for solving the economic dispatch problem in large-scale systems. A new encoding technique is developed. The chromosome contains only an encoding of the normalized system incremental cost in this coding technique. Therefore, the total number of bits of chromosome is entirely independent of the number of units. This salient feature makes the proposed genetic approach attractive in large and complex systems, which other methodologies may fail to achieve. Moreover, the approach can take network losses and generation limits into account because of genetic algorithm's flexibility. Numerical results on system up to 40 units show that the proposed approach is more robust than the well-known lambda-iteration method in large-scale systems.

Keywords: *Economic dispatch, Genetic algorithm, Network losses, Lambda-iteration.*

1. INTRODUCTION

Electrical energy cannot be stored, but is generated from natural sources and delivered as demand arises. Since the source of energy are so diverse (coal, oil or gas, river water, marine tide, a radioactive matter, sun power), the choice of one or the other is made on economic, technical or geographic basis. As there are few facilities to store electrical energy, the net production of a utility (generation plus the inflows over its ties) must clearly track its total load. For an interconnection system, the fundamental problem is one of minimizing the source expenses. The economic dispatching problem is to define the production level of each plant so that the total cost of generation and transmission is minimum for a prescribed schedule of loads [1, 2, 3].

The majority of generators in extant systems are of three types-nuclear, hydro, and fossil (coal, oil or gases). Nuclear plants tend to be operated at constant output levels and hydro plants have essentially no variable operating cost. Therefore, the components of cost that fall under the category of dispatching procedures are the costs of the fuel burnt in the fossil plants. The total cost of operation includes the fuel cost, costs of labour, supplies and maintenance. Generally, costs of labour, supplies and maintenance are fixed percentages of incoming fuel costs. For dispatching purposes, the operating cost is usually approximated by one or more quadratic segments [4, 5, 6].

The fuel cost curve may have a number of discontinuities. The discontinuities occur when the output power has to be extended by using additional boilers steam condensers, or other equipment. Discontinuities also appear if the cost represents the operation of an entire power station, so that the cost has discontinuities on paralleling of generators. Within the continuity range the incremental fuel cost may be expressed by a number of short line segments or piece-wise linearization [4, 5, 6] subject to equality and inequality constraints.

The economic dispatch is optimization problem, usually this constrained optimization problem is converted to an unconstrained optimization problem through Lagrange multiplier and the equality constraints. However, the solution of the augmented function must observe the inequality constraints.

It is, also, unrealistic to neglect transmission losses particularly when long distance transmission of power is involved. A modern electric utility serves over a vast area of relatively low load density. The transmission losses may vary from 5 to 15 percent of the total load. Therefore, it is essential to account for transmission losses while developing an economic load dispatch policy. One of the more important, simple but approximate method of expressing transmission losses as a function of generator powers is through *B*-coefficients [7, 8].

The calculus based method applied to the augmented objective function uses an indirect search technique to seek local extrema by solving the usually nonlinear set of equations resulting from setting the gradient of the objective function equal to zero. While this calculus-based method has been improved, extended, hashed, and re-hashed [1, 2, 3, 9, 10, 11, 12], some simple reasoning shows their lack of robustness.

First, the method is local in scope; the optima it seeks are the best in a neighborhood of the current point. Second, calculus-based methods depend upon the existence of derivatives. The real world of search is fraught with discontinuities and vast multimodal, noisy search spaces. It comes as no surprise that methods depending upon the restrictive requirements of continuity and derivative existence are unsuitable for all but a very limited problem domain. For this reason and because of their inherently local scope of search, we must seek other method free from this unrobustness. Here Genetic Algorithms (GAs) as one of

random search methods come into picture. We shall soon see how GAs help fill this robustness gap [13].
In order for GAs to surpass their more traditional cousins in the quest of robustness, GAs must differ in some very fundamental ways. Genetic algorithms are different from more normal optimization and search procedures in four ways:

1. GAs work with a coding of the parameters set not the para- -meters themselves.
2. GAs search from a population of points, not a single point.
3. GAs use payoff (objective function) information, not derivatives or other auxiliary knowledge.
4. GAs are probabilistic transition rules, not deterministic rules.

Genetic algorithm is a stochastic searching algorithm. It combines an artificial, i.e. the Darwinian Survival of the Fittest principle with genetic operation, abstracted from nature to form a robust mechanism that is very effective at finding optimal solutions to complex-real world problems. Evolutionary computing is an adaptive search technique based on the principles of genetics and natural selection. They operate on sting structures. The string is a combination of binary digits representing a coding of the control parameters for a given problem. Many such string structures are considered simultaneously, with the most fit of these structures receiving exponentially increasing opportunities to pass on genetically important material to successive generation of string structures. In this way, genetic algorithms search for many points in the search space at once, and yet continually narrow the focus of the search to the area of the observed best performance. The basic elements of genetic algorithms are reproduction, crossover, and mutation.

The first step is the coding of control variables as strings in binary numbers. In reproduction, the individuals are selected based on their fitness values relative to those of the population. In the crossover operation, two individual strings are selected at random from the mating pool and a crossover site is selected at random along the string length. The binary digits are interchanged between two strings at the crossover site. In mutation, an occasional random alteration of a binary digit is done.

2. ECONOMIC DISPATCH PROBLEM [14]

From the unit commitment table of a given plant, the fuel cost curve of the plant can be determined in the form of a polynomial of suitable degree by the method of least squares fit. If the transmission losses are neglected, the total system load can be optimally divided among the various generating plants using equal incremental cost criteria. It is, however, unrealistic to neglect transmission losses particularly when long distance transmission of power is involved. A modern electric utility serves over a vast area of relatively low load density. The transmission losses may vary from 5 to 15 % of the total load. Therefore, it is essential to account for losses while developing an economic load dispatch policy.

The economic dispatch problem is defined so as to minimize the total operating cost of a power system while meeting the total load plus transmission losses within generator limits.

Mathematically, the problem is defined as

Minimize $$F(P_{gi}) = \sum_{i=1}^{NG} \left(a_i P_{gi}^2 + b_i P_{gi} + c_i \right) \quad \$/h \quad (1a)$$

subject to (i) the energy balanced equation

$$\sum_{i=1}^{NG} P_{gi} = P_D + P_L \qquad (1b)$$

(ii) the inequality constraints

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max} \qquad (\, i = 1,2,\ldots\ldots, NG) \qquad (1c)$$

where

a_i, b_i and c_i are cost coefficients
P_D is load demand
P_{gi} is real power generation and will act as decision variable
P_L is power transmission loss
NG is the number of generation buses.

One of the most important, simple but approximate methods of expressing transmission loss as a function of generator powers is through *B*-coefficients. This method uses the fact that under normal operating condition, the transmission loss is quadratic in the injected bus real powers. The general form of the loss formula using *B*-coefficients is

$$P_L = B_{00} + \sum_{i=1}^{NG} B_{i0} P_{gi} + \sum_{i=1}^{NG} \sum_{j=1}^{NG} P_{gi} B_{ij} P_{gj} \qquad \text{MW} \qquad (2)$$

where

P_{gi}, P_{gj} are real power injections at the *i*th, *j*th buses, respectively.
B_{00}, B_{i0}, and B_{ij} are loss coefficients which are constant under certain assumed conditions.
NG is number of generation buses.

The above loss formula is known as the George's formula. The above constrained optimization problem is converted into an unconstrained one. Lagrange multiplier method is used in which a function is minimized subject to side conditions in the form of equality constraints. Using Lagrange multipliers, an augmented function is defined as

$$L(P_{gi}, \lambda) = F(P_{gi}) + \lambda \left(P_D + P_L - \sum_{i=1}^{NG} P_{gi} \right) \qquad (3)$$

where

λ is the Lagrange multiplier.

Necessary conditions for the optimization problem are

$$\frac{\partial L(P_{gi}\lambda)}{\partial P_{gi}} = \frac{\partial F(P_{gi})}{\partial P_{gi}} + \lambda \left(\frac{\partial P_L}{\partial P_{gi}} - 1 \right) = 0 \qquad (i = 1,2,\ldots, NG)$$

Rearranging the above equation

$$\frac{\partial F(P_{gi})}{\partial P_{gi}} = \lambda \left(1 - \frac{\partial P_L}{\partial P_{gi}} \right) \qquad (i = 1,2,\ldots, NG\,) \qquad (4)$$

Where

$\frac{\partial F(P_{gi})}{\partial P_{gi}}$ is the incremental cost of the *i*th generator ($/MWh).

$\frac{\partial P_L}{\partial P_{gi}}$ represents the incremental transmission losses.

Equation (4) is known as the exact coordination equation and

$$\frac{\partial L(P_{gi},\lambda)}{\partial \lambda} = P_D + P_L - \sum_{i=1}^{NG} P_{gi} = 0 \qquad (5)$$

Equation (4), numbering NG is solved simultaneously with Eq.(5) to yield a solution for Lagrange multiplier λ and the optimal generation of NG generators.

By differentiating the transmission loss equation, Eq.(2), with respect to P_{gi} the incremental transmission loss can be obtained as

$$\frac{\partial P_L}{\partial P_{gi}} = B_{i0} \sum_{j=i}^{NG} 2B_{ij} P_{gi} \qquad (i=1,2,\ldots, NG) \qquad (6)$$

and by differentiating the cost function, Eq.(1a), with respect to, P_{gi} the incremental cost can be obtained as

$$\frac{\partial F(P_{gi})}{\partial P_{gi}} = 2a_i P_{gi} + b_i \qquad (i=1,2,\ldots, NG) \qquad (7)$$

To find the solution, substitute Eqs. (6) and (7) into Eq. (4) to obtain

$$2a_i P_{gi} + b_i = \lambda \left(1 - B_{i0} \sum_{j=1}^{NG} 2B_{ij} P_{gj} \right) \qquad (i=1,2,\ldots, NG)$$

Rearranging the above equation to get P_{gi}, i.e.

$$2(a_i + \lambda B_{ii}) P_{gi} = \lambda \left(1 - B_{i0} - \sum_{\substack{j=1 \\ j\neq i}}^{NG} 2B_{ij} P_{gi} \right) - b_i \qquad (i=1,2,\ldots, NG)$$

The value of P_{gi} can be obtained as

$$P_{gi} = \frac{\lambda \left(1 - B_{i0} - \sum_{\substack{j=1 \\ j\neq i}}^{NG} 2B_{ij} P_{gi} \right) - b_i}{2(a_i + \lambda B_{ii})} \qquad (i=1,2,\ldots, NG) \qquad (8)$$

If the initial values of P_{gi} (i = 1,2,……, NG) and λ are known, the above equation can be solved iteratively until Eq. (5) is satisfied by modifying λ. The optimum solution thus obtained depends on the proper choice of the initial values. To enhanced selection of initial values of the control parameters, genetic algorithms may be used. In this study λ, securing optimum solution is obtained using genetic algorithms [13].

3. Genetic Algorithm Solution Methodology

The detailed solution methodology includes: the encoding and decoding technique, constrained generation output calculation, the fitness function, parent selection, and parameter selection.

3.1 Encoding and Decoding

Decoding a binary string into an unsigned integer can play very important roles in genetic algorithm implementation. The inequality power limit constraint is performed in such a way that the individual string is normalized over the unit's operating region. The inequality constraints are handled in the manner, which efficiently reduces the searching space, and thus enhanced the performance of the system. Binary coded strings having 1s and 0s are used. The equivalent decimal integer of binary string λ is obtained as

$$y^j = \sum_{i=1}^{l} 2^{i-1} b_i^j \qquad (j=1, 2,\ldots, L) \qquad (9)$$

where

b_i^j is the ith binary digit of the jth string
l is the length of the string
L is the number of strings or population size

The continuous variables λ can be obtained to represent a point in the search space according to a fixed mapping rule, i.e.

$$\lambda^j = \lambda^{\min} + \frac{\lambda^{\max} - \lambda^{\min}}{2^l - 1} y^j \qquad (j=1, 2,\ldots, L) \qquad (10)$$

where

λ^{min} is the minimum value of variable, λ
λ^{max} is the maximum value of variable, λ
y^j is the decimal coded value of the string

The number of binary digits needed to represent a continuous variation in accuracy of $\Delta\lambda$ can be computed from the relation

$$l \geq \log_2 \left(\frac{\lambda^{\max} - \lambda^{\min}}{\Delta\lambda} \right) + 1 \qquad (11)$$

Simple GA, starts with random generation of a population. A population consists of a set of chromosomes (strings). Usually, the string size ranges from 10-100. The population may be of any size according to the accuracy required. The population size remains constant throughout the whole process. A chromosome should contain, in some way, information about solution that it represents. A string then could look like this (in the binary case):

String 1	1101100100110110
String 2	1101111000011110

In some cases, a string in GAs may be divided into a number of sub-strings. The number of sub-strings, usually, equals to the number of the problem variables. Therefore, the above strings could be a representation of one parameter (16 bit) or 2 parameters (8 bits) or any other combination.

3.2 Calculation of Generation and Transmission Losses

When the incremental cost λ^j is known for whole population, then the generation can be obtained from Eq.(8), i.e. (B_{i0} may be neglected as it is very small).

$$2(a_i + \lambda^j B_{ii}) P_j^i + \lambda^j \sum_{\substack{k=1 \\ k\neq i}}^{NG} 2B_{ik} P_{gk}^i = \lambda^j - b_i \qquad (i=1,2,\ldots, NG;\ j=1,2,\ldots, L) \qquad (12)$$

The above equation can be rewritten as

$$\sum_{k=1}^{NG} A_{ik}^j P_{gk}^j = C_t^j \qquad (i=1,2,\ldots, NG;\ j=1,2,\ldots, L) \qquad (13)$$

where

$$A_{ii}^j = 2(a_i + \lambda^j B_{ii})$$

$$A_{ik}^{j} = 2\lambda^{j} B_{ik} \qquad (i \neq k)$$
$$C_{t}^{j} = \lambda^{j} - b_{i}$$

Transmission loss for whole population can be obtained as

$$P_L^j = \sum_{i=1}^{NG} \sum_{k=1}^{NG} P_{gi}^j B_{ik} P_{gk}^j \qquad (j=1,2,\ldots, L) \qquad (14)$$

3.3 Fitness Function and Parent Selection

Implementation of a problem in a genetic algorithm is realized within the fitness function. Since the proposed approach uses the equal incremental cost criterion as its basis, the constraint Eq.(5) can be rewritten as

$$\varepsilon^j = \left| p_D + p_L^j - \sum_{i=1}^{NG} p_i^j \right| \qquad (15)$$

Then the converging rule is when ε decreases to within a specific tolerance.

In order to emphasize the 'best' chromosomes and speed up convergence of the iteration procedure, fitness is normalized into range between 0 and 1. The fitness function adopted is

$$f^j = 1 \div \left(1 + \alpha \frac{\varepsilon^j}{p_D} \right) \qquad (j=1,2,\ldots, L) \qquad (16)$$

where α is the scaling constant.

When the fitness of each chromosome is calculated, the "stochastic remainder roulette wheel selection" technique is used to select the best parents according to their fitness.

Selection

Inspired by the role of nature selection in evolution, GAs performs a selection process in which the "most fit" members of the population survive, and the "least fit" members are eliminated. In a constrained optimization problem, the notion of "fitness" depends partly on whether a solution is feasible

(i.e. whether it satisfies all of the constraints), and partly on its objective function value.

The selection process is the step that guides the evolutionary algorithm towards ever-better solutions. The parent selection can carried out in a variety of ways. In this study, the process is carried out using roulette wheel technique [13].

Crossover

After reproduction, we can proceed to crossover operation. Crossover operates on selected genes from parent chromosomes and creates new offspring.

The simplest way to do that is to choose, randomly, some crossover point, copy everything before this points from the first parent, and then copy everything after the crossover point from the other parent. Crossover can be illustrated as follows: (| is the crossover point):

Chromosome 1	11011 | 00100110110
Chromosome 2	11011 | 11000011110
Offspring 1	11011 | 11000011110
Offspring 2	11011 | 00100110110

There are other ways to implement crossover, for example, we can choose more crossover points. Crossover can be quite complicated and depends mainly on the encoding of chromosomes. Specific crossover made for a specific problem can improve performance of the GAs.

Mutation

After performing crossover, mutation takes place. Mutation is used to prevent falling of all solutions in the population into a local optimum of the solved problem.

Mutation operation randomly changes the offspring resulted from crossover. In case of binary encoding, we can switch a few randomly chosen bits from 1 to 0 or from 0 to 1. Mutation can be illustrated as follows:

Original offspring 1	110**1**111000011110
Original offspring 2	1101100**0**100110110
Mutated offspring 1	110**0**111000011110
Mutated offspring 2	1101101**1**00110110

4. Numerical Results

Three example cases are studied in this section to illustrate the performance of the proposed approach in practical applications. The software was written in Matlab language and executed on a personal computer.

Taking network losses, and generation limits into account does not affect solution time because of GAs flexibility in canceling constraints.

Throughout the study, the lambda-iteration method is used as the main benchmark of comparison for the proposed approach.

Example 1

In this example a simple system with three thermal units [15] is used to demonstrate how the proposed approach works. The units characteristic are given in Table 1.

Table 1 Generating unit's capacity and coefficients of Example 1

Unit	p_{gi}^{min} (MW)	p_{gi}^{max} (MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)
1	50	250	0.00525	8.663	328.13
2	5	150	0.00609	10.04	136.91
3	10	100	0.00529	9.76	59.16

The demand is 300 MW.

The loss formula coefficients are:

$$B_{ij} = \begin{bmatrix} 0.000136 & 0.0000175 & 0.000184 \\ 0.0000175 & 0.000154 & 0.000283 \\ 0.000184 & 0.000283 & 0.00161 \end{bmatrix}$$

The convergence is obtained in the tenth population generation. Fig.1 shows how the fitness evolves. A summary of results and the corresponding results extracted from [15, 16, 17, 18, 19, 20] are given in Table 2. Results from [15] are obtained using lambda-iteration method and those from [16, 17, 18, 19, 20] by GAs.

Table 2 Results for Example 1

	Proposed Approach	[15]	[16, 17, 18, 19, 20]
Generation (MW)	198.978	200.3913	194.265
	77.240	78.4289	50
	31.688	32.0959	79.627
P_{loss} (MW)	10.709	10.9161	24.011
λ ($/MWh)	11.5720	10.5730	10.7028
Gen. cost ($)	3580.95	3613.33	-

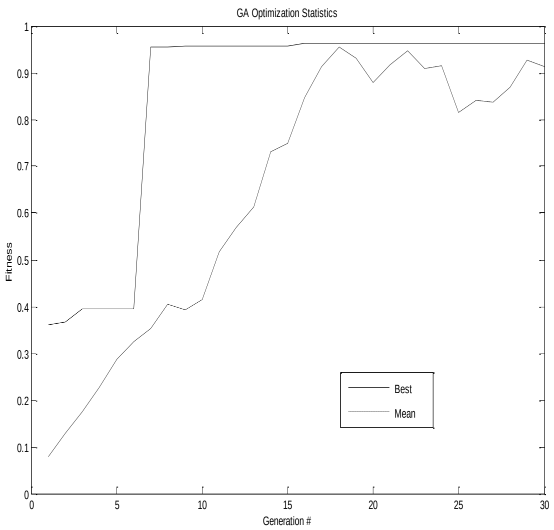

Fig.1 Fitness vs Generation Number

Example 2

In this example a six thermal units system [21] is studied. The units characteristic are given in Table 3.

Table 3 Generating units' capacity and coefficients of Example 2

Unit	P_{gi}^{min} (MW)	P_{gi}^{max} (MW)	a_i ($/MW²)	b_i ($/MW)	c_i ($)
1	100	500	0.0070	7.0	240
2	50	200	0.0095	10.0	200
3	80	300	0.0090	8.5	220
4	50	150	0.0090	11.0	200
5	50	200	0.0080	10.5	220
6	50	120	0.0075	12.0	190

The demand is 1263 MW.
The loss formula coefficients are:

$$B_{ij} = \begin{bmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & 0.0008 & 0.0002 & 0.0150 \end{bmatrix}$$

The results obtained after a 300 generations and those obtained from lambda-iteration method using the program codes of [21] are given in Table 4. Fig.2 shows the fitness evolving.

Table 4 Results for Example 2

	Proposed Approach	[11]
Generation (MW)	445.948	447.0688
	172.352	173.1805
	263.051	263.9225
	138.134	139.0512
	164.702	165.5762
	85.726	86.6165
P_{loss} (MW)	12.315	12.4157
λ ($/MWh)	13.5234	13.5402
Gen. cost ($)	15369.56	15442.66

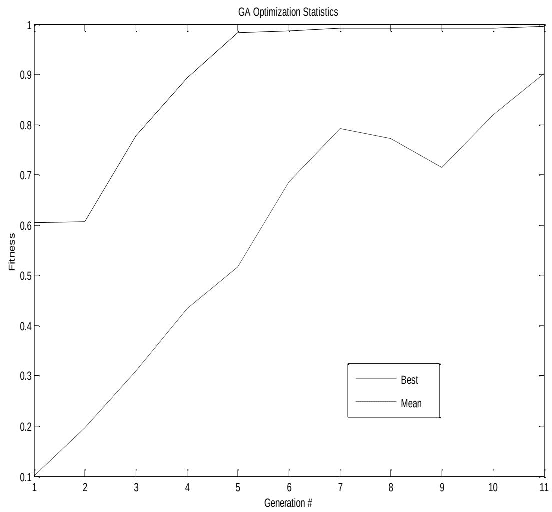

Fig.2 Fitness vs Generation Number

Example 3

This example investigates the applicability of the proposed approach in mixed-generation case. The complete 40 units data are listed in the Appendix. Table 5 shows the convergence results with reference to the proposed approach and the lambda-iteration method using the program code of [21]. Losses are ignored in this example. Fig.3 shows evolving of the fitness.

Table 5 Results for Example 3

Unit	Generation [MW]	
	Proposed Approach	[21]
1	80	80
2	60	60
3	190	190
4	24	24
5	42	42
6	140	140
7	300	300
8	300	300
9	300	300
10	291.844	130
11	335.332	94
12	320.204	94
13	467.805	125
14	500	500
15	500	500
16	500	500
17	500	500
18	500	500
19	500	500
20	550	550
21	550	550
22	550	550
23	550	550
24	550	550
25	550	550
26	550	550
27	550	550
28	12.571	10
29	12.571	10
30	12.571	10
31	20	20
32	20	20
33	20	20
34	20	20
35	18	18
36	18	18
37	20	20
38	25	25
39	25	25
40	25	25
Total gen. (MW)	10499.898	9520
λ ($/MWh)	16.4399	
Gen. cost ($)	7445496.61	7430641.10

The solution based on lambda-iteration methods oscillates between the generation limits ($p_{gi}^{\max}$ and $p_{gi}^{\min}$).

Also the solution is unfeasible since total generation scheduled (9520 MW) is less than the system demand (10500 MW). Therefore, the generation cost obtained by this method is meaningless.

Moreover, the experimental results in this paper show that the lambda-iteration method has oscillatory problems in a large-scale, mixed-generation unit system which results on a shower solution time.

The proposed approach uses the equal system λ (equal system incremental cost) criterion as its basis. The only encoded parameter is the system incremental cost. The advantage of using system λ instead of units output as the encoded parameter is that the number of bits of chromosomes will be entirely independent of the number of units. This is particularly attractive in large-scale systems.

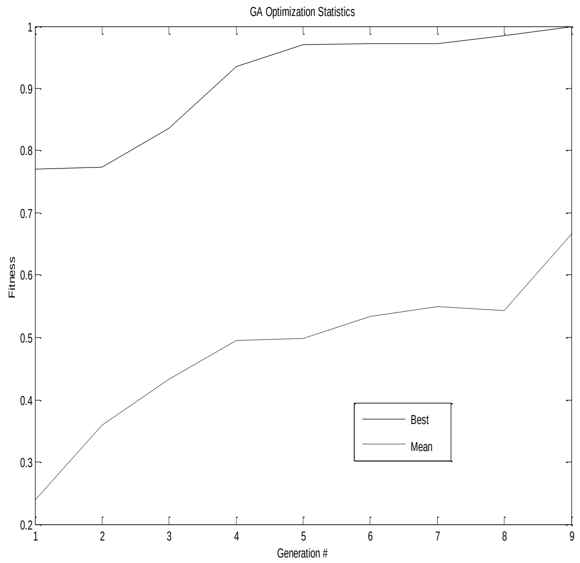

Fig.3 Fitness vs Generation Number

5. Conclusion

This paper suggests a different approach for solving the economic dispatch (ED) problem. In comparison with other ED methods- four differences: direct use of a coding, search of many optimum points in parallel, blindness to auxiliary information, and using probabilistic rules (rather than deterministic rules) impart to the proposed approach and global optimization algorithms. This approach can also take network losses and generation limits into account to make the dispatch more practical. Evaluation results show that the approach always secure feasible solution compared to the well-known lambda-iteration method in large-scale systems. The chromosomes contain only an encoding of the normalized system incremental cost. Therefore, the total number of bits of chromosomes is entirely independent of the number of units. This salient feature makes the proposed genetic approach attractive in large and complex systems with other methodologies may fail to achieve.

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APPENDIX

Generating units capacity and coefficients of Example 3

Unit	p_{gi}^{min} (MW)	p_{gi}^{max} (MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)
1	40	80	0.03073	8.336	170.44
2	60	120	0.02028	7.0706	309.54
3	80	190	0.00942	8.1817	369.03
4	24	42	0.08482	6.9467	135.48
5	26	42	0.09693	6.5595	135.19
6	68	140	0.01142	8.0543	222.33
7	110	300	0.00357	8.0323	287.71
8	135	300	0.00492	6.999	391.98
9	135	300	0.00573	6.602	455.76
10	130	300	0.00605	12.908	722.82
11	94	375	0.00515	12.986	635.2
12	94	375	0.00569	12.796	654.69
13	125	500	0.00421	12.501	913.4
14	125	500	0.00752	8.8412	1760.4
15	125	500	0.00708	9.1575	1728.3
16	125	500	0.00708	9.1575	1728.3
17	125	500	0.00708	9.1575	1728.3
18	220	500	0.00313	7.9691	647.85
19	220	500	0.00313	7.955	649.69
20	242	550	0.00313	7.9691	647.83
21	242	550	0.00313	7.9691	647.81
22	254	550	0.00298	6.6313	785.96
23	254	550	0.00298	6.6313	785.96
24	254	550	0.00284	6.6611	794.53
25	254	550	0.00284	6.6611	794.53
26	254	550	0.00277	7.1032	801.32
27	254	550	0.00277	7.1032	801.32
28	10	150	0.52124	3.3353	1055.1
29	10	150	0.52124	3.3353	1055.1
30	10	150	0.52124	3.3353	1055.1
31	20	70	0.25098	13.052	1207.8
32	20	70	0.16766	21.887	810.79
33	20	70	0.2635	10.244	1247.7
34	20	70	0.30575	8.3707	1219.2
35	18	60	0.18362	26.258	641.43
36	18	60	0.32563	9.6956	1112.8
37	20	60	0.33722	7.1633	1044.4
38	25	60	0.23915	16.339	832.24
39	25	60	0.23915	16.339	834.24
40	25	60	0.23915	16.339	1035.2

The demand is 10500 MW.