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Galerkin's Method of Weighted Residual for a Convective Straight Fin with Temperature-Dependent Conductivity and Internal Heat Generation

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ABSTRACT

In this paper, Galerkin's method of weighted residual was applied to study the heat transfer and thermal stability of a convective straight fin with temperature-dependent thermal conductivity and internal heat generation. The method of weighted residual provides a very powerful and accurate approximate analytical solution to the fin problem. The simple but highly accurate solution was validated by the exact solution for the linear problem. The developed heat transfer models were used to investigate the effects of thermo-geometric parameters, the coefficient of heat transfer and thermal conductivity (non-linear) parameters on the temperature distribution, heat transfer and thermal performance of the longitudinal rectangular fin. From the results, it shows that the fin temperature distribution, the total heat transfer, the fin effectiveness, and the fin efficiency are significantly affected by the thermo-geometric and thermal parameters of the fin. The analysis revealed that the operational parameters must be carefully chosen to ensure that the fin retains its primary purpose of removing heat from the primary surface. Therefore, the results obtained in this analysis serve as the basis for comparison of any other method of analysis of the problem and they also provide a platform for improvement in the design of fin in heat transfer equipment.

Keywords: *Heat Transfer Analysis, Longitudinal Fin, Galerkin's Method of Weighted Residual, Temperature-Dependent Thermal Conductivity And Internal Heat Generation.*

1. INTRODUCTION

Extended surfaces are extensively employed in the design and construction of various types of heat-transfer equipment and components such as air conditioning, refrigeration, superheaters, automobile, power plants, heat exchangers, convectional furnaces, economizers, gas turbines, chemical processing equipment, oil-carrying pipelines, computer processors, electrical chips. These surfaces are used to implement the flow of heat between a source and a sink. In practice, various types of fins with different geometries are used, but due to the simplicity of its design and ease of construction and manufacturing process, the rectangular fins are widely used. Also, for ordinary fins problem, the thermal properties of the fin thermal conductivity is assumed to be constant. However, if a large temperature difference exists within the fin, typically, between the tip and the base of the fin (such as heat pipe, space radiator etc), the thermal conductivity is temperature-dependent. These facts attest that for many engineering applications, the thermal conductivity is temperature-dependent. Therefore, while analyzing the fin under such situations, effects of the temperature-dependent thermal properties must be taken into consideration. In carrying out such analysis, the thermal conductivity may be modeled for such and other many engineering applications by a power law and by linear dependency on temperature. [1, 2]. Such condition of dependency of thermal conductivity and heat transfer coefficient on temperature renders the problem highly non-linear and difficult to solve using exact methods. It is also very realistic to consider the temperature-dependent internal heat generation in the fin (electric current-carrying conductor, nuclear rods or any other heat generating components of thermal systems).

The research area of temperature-dependent thermal conductivity and heat transfer coefficient has been receiving intense attention in most literature. Over the past few decades, the solutions of the highly non-linear differential equations have been constructed using different techniques. Aziz and Enamul-Huq [3] applied regular perturbation expansion to study a pure convection fin with temperature-dependent thermal conductivity. Aziz [4] extended the previous analysis to include a uniform internal heat generation in the fin. In some years later, Campo and Spaulding [5] applied the method of successive approximation to predict the thermal behavior of uniform circumferential fins. Chiu and Chen [6] and Arslanturk [7] adopted the Adomian Decomposition Method (ADM) to obtain the temperature distribution in a pure convection fin with variable thermal conductivity. The same problem was also solved by Ganji [8] with the aid of the homotopy perturbation method originally proposed by He [9]. Chowdhury and Hashim [10] applied the Adomian decomposition method to evaluate the temperature distribution of straight rectangular fin wth temperature dependent surface flux for all possible types of heat transfer. In the following year, Rajabi [11] employed Homotopy perturbation method (HPM) to calculate the efficiency of straight fins with temperature-dependent thermal conductivity. A year later, Mustapha [12] adopted Homotopy analysis method (HAM) to find the efficiency of straight fins with temperature-dependent thermal conductivity. Also, Coskun and Atay [13] utilized variational iteration method (VIM) for the analysis of convective straight and radial fins with temperature-dependent thermal conductivity while Languri *et al.* [14] applied both variation iteration and homotopy perturbation methods for the evaluation of efficiency of straight fins with temperature-dependent thermal conductivity. Coskun and Atay [15] applied variational iteration method to analyze the efficiency of convective straight fins with temperature-dependent thermal conductivity. In the same year, Atay and Coskum [16] employed variation iteration and finite element methods to carry out a comparative analysis of power-law-fin type problems. Domairry and Fazeli [17] used Homotopy analysis method to determine the efficiency of straight fins with temperature-dependent thermal conductivity. Chowdhury *et al.* [18] investigated a rectangular fin with power law surface heat flux and made a comparative assessment of results predicted by HAM, HPM, and ADM. Khani *et al*. [19] used Adomian decomposition method (ADM) to provide series solution to fin problem with a temperature-dependent thermal conductivity while Khani and Aziz (2010) considered a trapezoidal fin with both the thermal conductivity and the convection heat transfer coefficient varying as functions of temperature and reported an analytic solution generated using the homotopy analysis method (HAM). Moitsheki *et al.* [20] applied the Lie symmetry analysis to provide exact solutions of the fin problem with a power-law temperature-dependent thermal conductivity. Also, in the same year, Hosseini *et al.* [21] applied homotopy analysis method to provide an approximate but accurate solution of heat transfer in fin with temperature-dependent internal heat generation and thermal conductivity. To the best of the author's knowledge, very few studies were actually directed to the analysis of heat transfer in fins with temperature-dependent thermal properties while the study of fin with temperature-dependent internal heat generation, thermal conductivity, and heat transfer coefficient are very limited or scarcely carried out in literature. Furthermore, differential transform method (DTM) solves the differential equations without linearization, discretization or no approximation, linearization restrictive assumptions or perturbation, the complexity of expansion of derivatives and computation of derivatives symbolically. This method was applied by Joneidi *et al.* [22], Moradi and Ahmadikia [23] and Moradi [24] presented an analytical solution for fin with temperature dependent thermal coefficient. The method was also used by Mosayebidorcheh *et al.* [25], Ghasemi *et al.* [26], Sandri *et al.* [27], Ganji and Dogonchi [28] also applied the DTM to solve the fin problem but the search for a particular value that will satisfy second the boundary condition necessitated the use of Maple software and such could result in additional computational cost in the generation of solution to the problem. This drawback is not only peculiar to DTM, other approximate analytical methods such as HPM, HAM, ADM and VIM also required additional computational cost and time for the determination of auxiliary parameters. Also, most of the approximate methods give accurate predictions only when the nonlinearities are weak, the fail to predict accurately for strong nonlinear models. Also, the methods often involved complex mathematical analysis leading to an analytic expression involving a large number terms and when such methods as HPM, HAM, ADM and VIM are routinely implemented, they can sometimes lead to erroneous results [29, 30]. In practice, approximate analytical solutions with a large number of terms are not convenient for use by designers and engineers. However, simple yet accurate expressions are required to determine the fin temperature distribution, efficiency, effectiveness and the optimum parameter. Hence, this work presents a simple but very powerful approximate method of solution, the Galerkin's method of weighted residual. The method of weighted residuals provides a very powerful, novel and accurate approximate analytical solution procedure that is applicable to a wide variety of linear and non-linear problems and thus makes it unnecessary to search for variational formulations in order to apply the finite element method for the problems. In other to reduce the computation cost and task in the analysis of such problem, this work presents a simple but very powerful approximate method of solution, the Galerkin's method of weighted residual. The results obtained by the method (for solving the problem under investigation) were compared with the previous studies and a very good agreement was established.

2. PROBLEM FORMULATION

Consider a straight fin of temperature-dependent thermal conductivity *k(T)*, length L and thickness δ and temperature-dependent internal heat generation per unit area *q(T)*, exposed on both faces to a convective environment at temperature T_∞ and with heat transfer co-efficient *h* shown in Fig.1, assuming that the heat flow in the fin and its temperatures remain constant with time, the temperature of the medium surrounding the fin is uniform, the fin base temperature is uniform., there is no contact resistance where the base of the fin joins the prime surface, the fin thickness is small compared with its width and length, so that temperature gradients across the fin thickness and heat transfer from the edges of the fin may be neglected. The dimension *x* pertains to the length coordinate which has its origin at the tip of the fin and has a positive orientation from the fin tip to the fin base. Following the model assumptions, the governing differential equation for the problem is shown in equation (1).

$$\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-\frac{h}{A_c}P(T-T_\infty)+q(T)=0$$

(1)

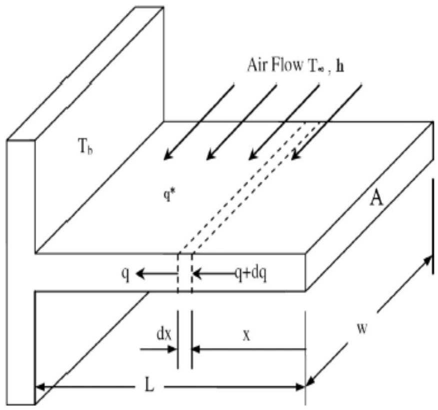


Fig. 1. Schematic of the longitudinal straight fin geometry with the internal heat generation [26]

The boundary conditions are

$$x = 0, \quad \frac{dT}{dx} = 0$$
$$x = L, \quad T = T_b$$

(2)

The temperature-dependent thermal properties and internal heat generation are given by

$$k(T) = k_a[1 + \lambda(T - T_\infty)]$$

(3)

$$q(T) = q_a[1 + \psi(T - T_\infty)]$$

(4)

Substituting equations (3-5) into equation (1), we have

$$\frac{d}{dx}\left[k_a[1 + \lambda(T - T_\infty)]\frac{dT}{dx}\right] - \frac{h_b P(T - T_\infty)}{A_c} + q_a[1 + \psi(T - T_\infty)] = 0$$

(5)

On introducing the following dimensionless parameters into equation (6);

$$X = \frac{x}{b}, \quad \theta = \frac{T - T_\infty}{T_b - T_\infty}, \quad H = \frac{h}{h_b}, \quad K = \frac{k}{k_a}, \quad M^2 = \frac{Ph_b L^2}{A_c k_a},$$
$$Q = \frac{q_a A_c}{h_b P(T_b - T_\infty)}, \quad \gamma = \psi(T_b - T_\infty), \quad \beta = \lambda(T_b - T_\infty)$$

(6)

The dimensionless governing differential equation (7 or 8) was arrived at and the boundary conditions (9)

$$\frac{d}{dX}\left[(1 + \beta\theta)\frac{d\theta}{dX}\right] - M^2\theta + M^2 Q(1 + \gamma\theta) = 0$$

(7)

If we expand equation (7), we have;

$$\frac{d^2\theta}{dX^2} + \beta\theta\frac{d^2\theta}{dX^2} + \beta\left(\frac{d\theta}{dX}\right)^2 - M^2\theta + M^2 Q(1 + \gamma\theta) = 0$$

(8)

The boundary conditions are

$$X = 0, \quad \frac{d\theta}{dX} = 0$$
$$X = 1, \quad \theta = 1$$

(9)

3. METHOD OF SOLUTION

The above non-linear equation (8) defiles the generation of any closed form solution. Therefore, recourse has to be made to either approximation analytical methods, semi-numerical methods or numerical methods of solution. In this work, a simple but very powerful approximate method of solution, the Galerkin's method of weighted residual is used. The procedure of the method is described as follows:

Representing the governing equations by

$$L(\theta) = 0 \quad in \quad \Omega$$

(10)

and

$$\theta \approx \bar{\theta} = \sum_{i=1}^{N} a_i N_i(X)$$

(11)

Substitution of the above equation (11) into Equation (10) results in

$$L(\bar{\theta}) \neq 0$$
$$= R\ (residual)$$

(12)

The method of weighted residual requires that the parameters a_1, a_2. . . a_n be determined by satisfying

$$\int_{\Omega} w_i(x) R dx \qquad \text{where} \qquad i = 1, 2, \ldots, n$$

(13)

Where the functions $w_i(x)$ are the n arbitrary weighting functions. There are an infinite number of choices for $w_i(x)$ but four particular functions are most often used. The methods of weighted residual could be Collocation, Sub-domain, Galerkin or Least Squares method depending on the choice of the weighting functions. Among all these methods of weighted residual, the Galerkin method is the most accurate [31]. In the Galerkin's method of weighted residual, the weight function is the same as the trial function.

Since a simple but highly accurate solution is sought, a quadratic trial solution shown in eq. (14) was adopted in this work.

$$\theta = \alpha_1 + \alpha_2 X + \alpha_3 X^2$$

(14)

Eq. (14) could satisfy the boundary conditions in eq. (15) when $\alpha_1 = 1 - \alpha_3, \quad \alpha_2 = 0.$

Thus, the trial function that satisfies the boundary conditions could be written as

$$\theta = 1 - (1 - X^2)\alpha_3 \tag{15}$$

And the weight function is

$$N_i(X) = 1 - X^2 \tag{16}$$

The Galerkin's formulation of the fin equation is

$$\int_0^1 N_i(X)\left[\frac{d^2\theta}{dX^2} + \beta\theta\frac{d\theta}{dX} + \beta\left(\frac{d\theta}{dX}\right)^2 - M^2\theta + M^2Q + M^2Q\gamma\theta\right]dX \tag{17}$$

Substituting the weight function in equation (16) to equation (17), we have;

$$\int_0^1 (1 - X^2)\left[\frac{d^2\theta}{dX^2} + \beta\theta\frac{d\theta}{dX} + \beta\left(\frac{d\theta}{dX}\right)^2 - M^2\theta + M^2Q + M^2Q\gamma\theta\right]dX \tag{18}$$

On substituting the corresponding terms from equation (15) into equation (18), it was found that

$$\alpha_3 = \frac{5(1+\beta) + 2M^2(1-\gamma Q) - \{5(1+\beta) + 2M^2(1-\gamma Q) - 20\beta M^2[1 - Q(1+\gamma)]\}^{\frac{1}{2}}}{4\beta} \tag{19}$$

Substituting equation (19) into equation (15)

$$\theta(X) = 1 - \left\{\frac{5(1+\beta) + 2M^2(1-\gamma Q) - \{5(1+\beta) + 2M^2(1-\gamma Q) - 20\beta M^2[1 - Q(1+\gamma)]\}^{\frac{1}{2}}}{4\beta}\right\}(1 - X^2) \tag{20}$$

3.1 Heat flux of the Fin

The fin base heat flux is given by

$$q_b = A_c k(T)\frac{dT}{dx} \tag{21}$$

Also, substitute equation (21) into equation (16), we have the dimensionless heat transfer rate at the base of the fin (X=1), which is s given by

$$\bar{q}_b = \frac{qL}{k_a A_c (T_b - T_\infty)} = (1 + \beta\theta)\frac{d\theta}{dX} \tag{22}$$

$$\bar{q}_b = \frac{(1+\beta)\{5(1+\beta) + 2M^2(1-\gamma Q) - [5(1+\beta) + 2M^2(1-\gamma Q) - 20\beta M^2[1-}{2\beta} \tag{23}$$

The total heat flux of the fin is given by [34]

$$q_T = \frac{q_b}{A_c h(T - T_b)} \tag{24}$$

Substituting eq. (21) and introducing the dimensionless parameters in eq. (6) into eq. (24), we arrived at

$$q_T = \frac{1}{Bi}\frac{k(\theta)}{h}\frac{d\theta}{dX} = \frac{1}{Bi}(1 + \beta\theta)\frac{d\theta}{dX} \tag{25}$$

3.2 Fin efficiency

The amount of heat dissipated from the entire fin is found by using Newton's law of cooling as

$$Q_f = \int_0^1 Ph(T - T_\infty)dX \tag{26}$$

The maximum heat dissipated is obtained if the fin base temperature is kept throughout the fin

$$Q_{\max} = PhL(T_b - T_\infty) \tag{27}$$

Fin efficiency is defined as the ratio of the fin heat transfer rate to the rate that would be if the entire fin were at the base temperature and is given by

$$\eta = \frac{Q_f}{Q_{\max}} = \frac{\int_0^L Ph(T - T_\infty)dx}{PhL(T_b - T_\infty)} \tag{28}$$

Therefore, the fin efficiency in dimensionless variables is given by

$$\eta = \int_0^1 \theta dX \tag{29}$$

On substituting equations (20) into equation (29), we have equations (30) and (35) respectively.

$$\eta = 1 - \frac{2}{3}\left\{\frac{5(1+\beta) + 2M^2(1-\gamma Q) - \{5(1+\beta) + 2M^2(1-\gamma Q) - 20\beta M^2[1 - Q(1-}{4\beta}\right. \tag{30}$$

It is very important to point out that the thermo-geometric parameter or the fin performance factor, M could be written in terms of Biot number, Bi and the aspect ratio, a_r as shown in eq. (31).

$$M^2 = \frac{Ph_b L^2}{A_c k_a} = \frac{(2L)h_b L^2}{(L\delta)k_a} = \frac{2h_b \delta L^2}{\delta^2 k_a} = \frac{2h_b \delta}{k_a}\left(\frac{L}{\delta}\right)^2 = 2Bia_r^2$$

$$Where \quad Bi = \frac{h_b \delta}{k_a},\ a_r = \frac{L}{\delta}$$

(31)

From equation (31), it implies that $M = a_r\sqrt{2Bi}$

(32)

3.3 Fin Effectiveness

The *removal number* or *fin effectiveness* is the ratio of the fin dissipation (equal, in the steady state, to the heat passing through the base of the fin by conduction) to the heat passing through the fin footprint of the base or prime surface if the fin were not present [32]. Following the definition, the effectiveness of the fin could be expressed mathematically as

$$\varepsilon = \frac{Q_f}{Q_{fb}}$$

(33)

Where Q_{fb} is the amount of heat dissipation from the area of the fin base and is given by

$$Q_{fb} = Ph_b \frac{\delta}{2}(T_b - T_\infty)$$

(34)

Substituting eq. (25) and (34) into eq. (33), equation (35) was arrived at

$$\varepsilon = \frac{Q_f}{Q_{fb}} = \frac{\int_0^L 2Ph(T - T_\infty)}{Ph\delta(T_b - T_\infty)}$$

(35)

Therefore, the fin effectiveness in dimensionless variables is given by equation (36)

$$\varepsilon = 2a_r \int_0^1 \theta \, dX$$

(36)

From equation (36), equation (37) was found to be

$$\varepsilon = 2a_r - \frac{4a_r}{3}\left\{\frac{5(1+\beta) + 2M^2(1-\gamma Q) - \{5(1+\beta) + 2M^2(1-\gamma Q) - 20\beta M^2[1-Q(1+\gamma)]\}^{\frac{1}{2}}}{4\beta}\right\} \quad when \ \ n = 0$$

(37)

In order to establish the validity of the solution of the Galerkin's method, an exact solution was generated to the linear problem of the heat transfer in the straight fin (with constant thermal properties) with and without internal heat generation using an analytical method and also, Galerkin's method was applied to the same problem

The exact solution of straight fin with constant thermal properties and no internal heat generation is given in equation (38) while equation (39) gives the solution of Galerkin's method.

$$\theta(X) = \frac{Cosh(MX)}{Cosh(M)}$$

(38)

$$\theta(X) = 1 - \left\{\frac{5M^2}{2(5+2M^2)}\right\}(1 - X^2)$$

(39)

The exact solution of the straight fin with constant thermal properties with internal heat generation is given in equation (40) while equation (41) gives the solution of Galerkin's method.

$$\theta(X) = \frac{Cosh(MX)}{Cosh(M)} + Q\left(1 - \frac{Cosh\ (MX)}{Cosh\ (M)}\right)$$

(40)

$$\theta(X) = 1 - \left\{\frac{5M^2(1-Q)}{2(5+2M^2)}\right\}(1 - X^2)$$

(41)

4. RESULTS AND DISCUSSION

Figs. 2a and 2b show the variation of dimensionless temperature with dimensionless length and also the effect of the thermogeometric parameter on the straight fin with an insulated tip. From the figure, as the thermogeometric parameter increases, the rate of heat transfer (the convective heat transfer) through the fin increases as the temperature in the fin drops faster (becomes steeper reflecting high base heat flow rates) as depicted in the figures. It can be inferred from the results that the ratio of convective heat transfer to conductive heat transfer at the base of the fin (h_b/k_b) has much effect on the temperature distribution, rate of heat transfer at the base of the fin, efficiency and effectiveness of the fin. As h_b increases (or k_b decreases), the ratio h_b/k_b increases at the base of the fin and consequently the temperature along the fin, especially at the tip of the fin decreases i.e. the tip end temperature decrease as M increases. The profile has steepest temperature gradient at *M=1.0*, but its much higher value gotten from the lower value of thermal conductivity than the other values of *M* in the profiles produces a lower heat-transfer rate. This shows that the thermal performance or efficiency of the fin is favored at low values of thermogeometric parameter since the aim (high effective use of the fin) is to minimize the temperature decrease along the fin length, where the best possible scenario is when $T=T_b$ everywhere. It must be pointed out that equation (31) shows the direct relationship between the thermogeometric parameter, M and the Biot number, Bi which directly depends on the fin length. A small value of M corresponds to a relatively short and thick fin of poor thermal conductivity and a high value of M implies a long fin or fin with a low value of thermal conductivity. Since the thermal performance or efficiency of the fin is favored at low values of thermogeometric fin parameter, very long fins are to be avoided in practice. A compromise is reached for one-dimensional analysis of fins $0 < Bi < 0.1$. When the Biot number is greater than 0.1, two-dimensional analysis of the fin is recommended as one-dimensional analysis predicts unreliable results for such limit.

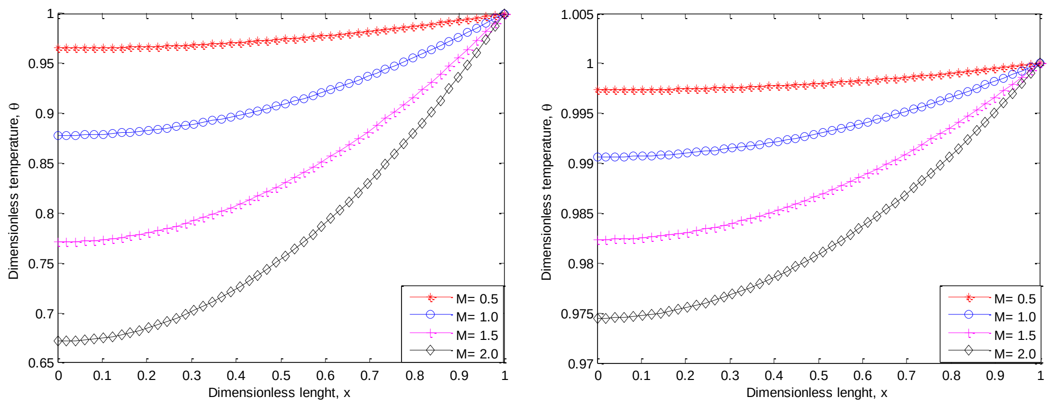
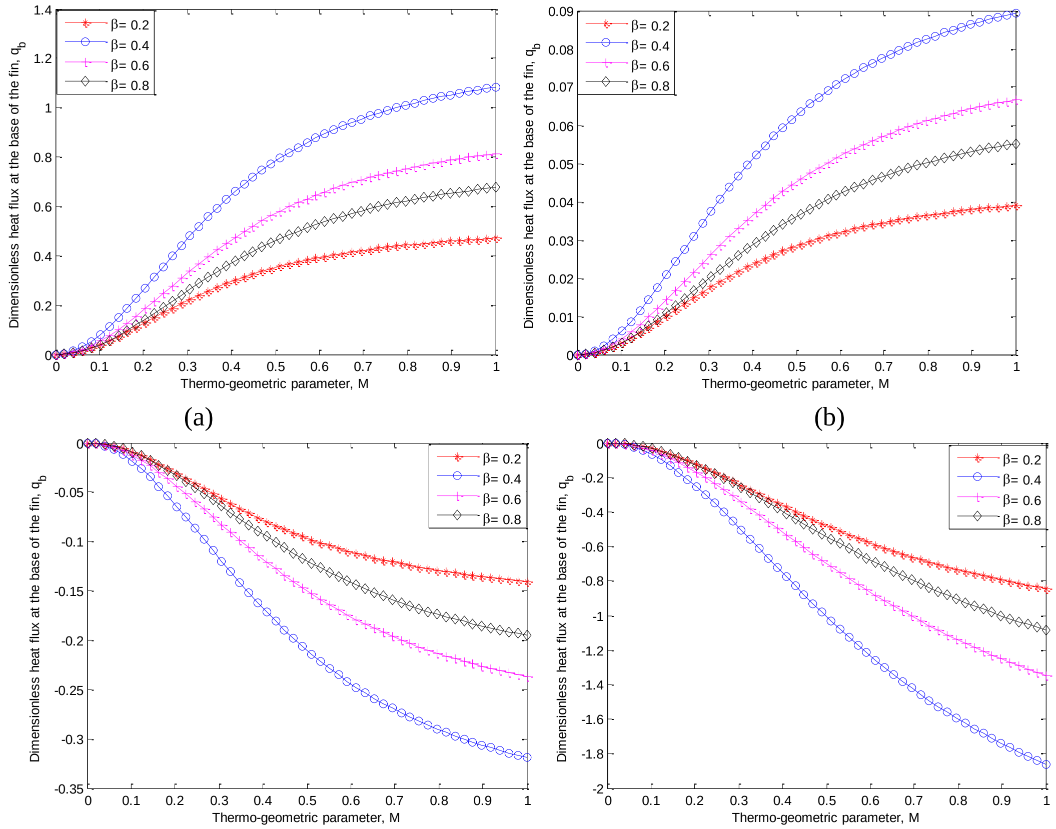


Fig. 2. Effects of thermo-geometric parameter on the temperature distribution in the fin when (a) Q=0.4, γ=0.2, β=0.8 (b) Q=0.8, γ=0.2, β=0.8

One of the major important analyses in the fin problem is the determination of the rate of heat transfer at the base of the fin. Figs. 8a-d show the effects of no-linear or thermal conductivity term on the dimensionless heat transfer rate at the base of the fin. Also, the figures depict the variation of the rate of heat transfer with the thermo-geometric parameter. From the figures, it could be deducted that the thermal conductivity, heat transfer coefficients, and internal heat generation terms have direct and significant effects on the rate of heat transfer at the base of the fin. The temperature gradient at the fin base is such that the fin is extracting heat from the prime surface and dissipating this energy together with the internally generated energy in the fin to the environment. However, a high value or an excessive internal heat generation results in an undesirable situation where some of this energy cannot escape to the sink and instead ends up flowing into the prime surface and the fin tends to gain heat rather than losing it. This scenario defeats the purpose of the fin as shown in Fig. 8d . Thus, the operational parameters must be carefully chosen to ensure that the fin retains its primary purpose of removing heat from the primary surface [30].

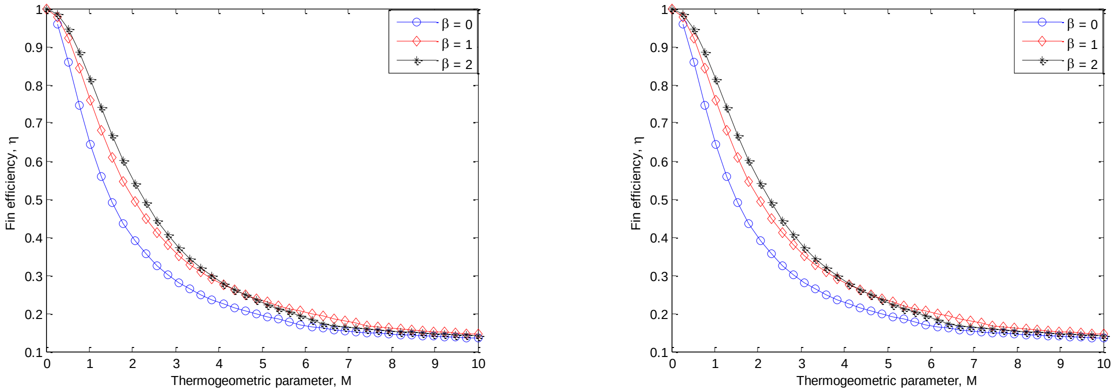


(a) (b)

(c) (d)

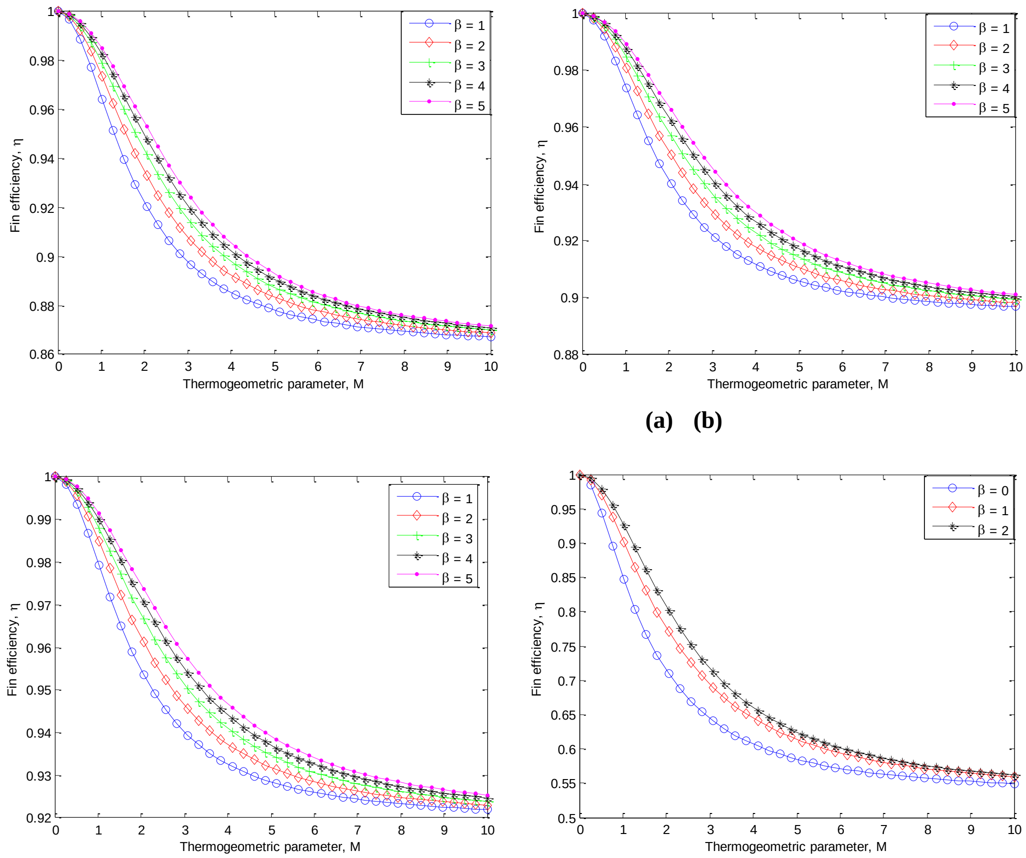
Fig. 3. Effects of thermal conductivity parameter of the fin when (a) Q=0.4, γ=0.2, β=0.8 (b) Q=0.8, γ=0.2, β=0.8 (c) Q=0.8, γ=0.4, β=0.8 (d) Q=0.8, γ=0.8, β=0.8

Fig. 4a and 4b show the effects of thermal conductivity parameter on the efficiency of the fin when there is no internal heat generation in the fin and at constant heat transfer coefficient while Figs. 5a-d show that the fin efficiency decreases monotonically (for different thermal conductivity and at a constant heat transfer coefficient) with increasing thermogeometric parameter. Also, it shows the variation of fin efficiency with thermogeometric in longitudinal convecting fin with insulated tip. From the figures, it is shown that as the thermogeometric parameter increases, the efficiency of the fin decreases. When the fin parameter equals to zero, the fin efficiency is 100%, which implies that there is no conduction resistance or no presence of fin at all. As the convective heat transfer coefficient to thermal conductivity ratio approaches zero, the temperature at every point in the fin is equal to the temperature of the base. The figures also depict that there are steep drops in the efficiency for $0 < M < 4$ after which the slopes of the curves decrease and is almost zero for $M > 8$. The inverse variation in the fin efficiency with the thermo-geometric parameter is due to the fact that as more material is attached to the prime surface, the resistance to heat flow increases thereby reducing the fin efficiency. Upon further increase in the fin thermo-geometric parameter, the effect of reducing the resistance becomes visible in the sense that the fin efficiency starts to normalize. Therefore, high efficiency of the fin could be achieved by using small values of thermogeometric parameter, which could be realized using a fin of small length or by using a material of better thermal conductivity. Moreover, the results depict that care must be taken in the choice of length of fin used during applications. This is because, the thermogeometric parameter (which increases as the fin length increases) tends to infinity, as the fin efficiency tends to zero. The fin to a large extent of its length will remain at ambient! This consequently results in weak conduction limit. The extended area is largely useless in the heat transfer process and hence inefficient. Therefore, very long fins are to be avoided in practice.



(a) (b)

Fig. 4. Effects of thermal conductivity parameter on the efficiency of the fin when (a) n=0, Q=0, γ=0.2 (b) n=0, Q=0, γ=0.8



(a) (b)

(c) (d)

Fig. 5. Effects of thermal conductivity parameter on the efficiency of the fin when (a) n=0, Q=0.6,γ=0.2 (b) n=0, Q=0.5,γ=0.6 (c) n=0, Q=0.6,γ=0.4 (d) n=0, Q=0.2,γ=0.2

Also, as shown in the figures, the fin efficiency is unity in the limit $M \rightarrow 0$. In this limit, the actual heat transfer rate from the fin is zero! This fin parameter (the thermo-geometric parameter) plays a very important role in determining the amount of heat transfer from the fin as it accounts for the effects of a decrease in temperature on the heat transfer from the fin. Since the fin temperature drops along the fin length, the fin efficiency decreases with increase in fin length. Therefore, in practice, required fin length should be properly determined because the fin length that causes the fin efficiency to drop below 60% usually cannot be justified economically and should be avoided.

Validation of results and thermal stability analysis of the fin

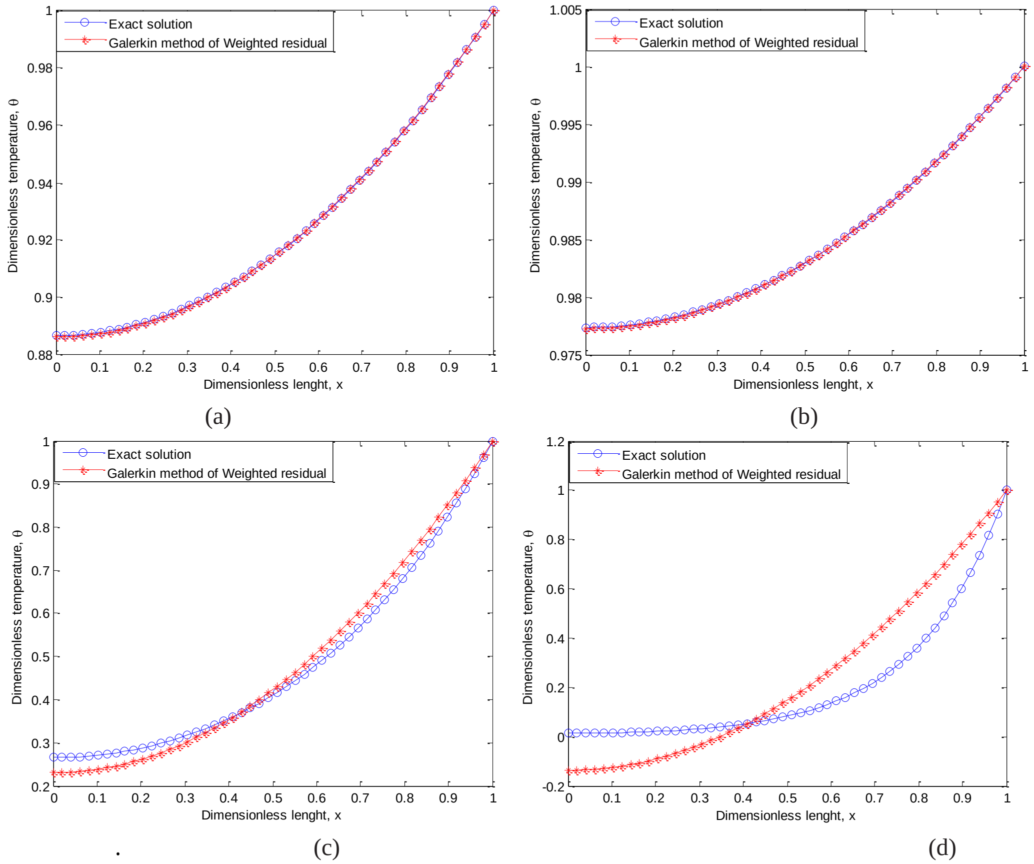


Fig. 6. Dimensionless temperature distribution in the fin when (a) n=0, M=0.5, β=0, Q=0 (b) n=0, M=0.5, β=0, Q=0.8 (a) n=0, M=2, β=0, Q=0 (b) n=0, M=5, β=0, Q=0

The approximate analytical method of solution was validated by the exact solution in Figs. 6a and 6b for the linear thermal model of the fin. From the figures, it is depicted that the Galerkin's method is highly accurate and agrees very well with the exact solution in the range of thermal stability of the fin for the linear problem.

It is very important to study the effects of the thermo-geometric parameter, M on the thermal stability of the fin. Fig. 6, it is obvious that as M increases to a certain value, the dimensionless temperature distribution results in a negative value (which shows thermal instability) at x=0, contradicting the assumption (Fig. 6d). From the analysis, the limiting value of M for thermal for thermal instability to the initially in the fin of constant thermal properties without internal heat generation is $\sqrt{2}$. However, when the temperature-dependent properties and internal heat generation in the fin are considered, the value of M for the thermal stability range increases.

5. CONCLUSION

In this work, we analyzed the heat transfer in a longitudinal rectangular fin with temperature-dependent thermal properties and internal heat generation using Galerkin's method of weighted residual. The dimensionless temperature distribution falls monotonically along fin length for all various thermogeometric, thermal conductivity and convective heat transfer parameters. For larger values of the thermogeometric parameter *M*, the more the heat convected from the fin through its length and the more thermal energy is efficiently transferred into the environment through the fin length. In the situation of negligible heat loss from the fin tip (insulated tip) to the environment, the fin temperature decreases along the fin length also, and the temperature decreasing rate is the same around fin base area. The solution was validated by the exact solution for the linear. The developed heat transfer models were used to investigate the effects of thermo-geometric and thermal conductivity (non-linear) parameters on the temperature distribution, heat transfer and thermal performance of the longitudinal rectangular fin. It can be concluded that the ratio of convective heat transfer to conductive heat transfer at the base of the fin (h_b/k_b) has much effect on the temperature distribution, rate of heat transfer at the base of the fin, efficiency and effectiveness of the fin. As h_b increases (or k_b decreases), the ratio h_b/k_b increases at the base of the fin and consequently the temperature along the fin, especially at the tip of the fin decreases. Also, the total heat transfer, the fin effectiveness, and the fin efficiency are shown to be enhanced when using a fin material with a high thermal conductivity. This depicts that the higher the thermal conductivity of the fin material, the better its efficiency. These results serve as a basis for comparison of any other method of analysis of the problem. In addition, they provide a platform for improvement in the design of fin in heat transfer equipment such as air–land–space vehicles and their power sources, in chemical, refrigeration, and cryogenic processes, in electrical and electronic equipment, in conventional furnaces and gas turbines, in the design of firebox for the generation of steam power from fossil fuels, in process heat dissipators and waste heat boilers, and in nuclear-fuel modules, steam power plants, automobiles radiators etc.

NOMENCLATURE

A cross-sectional area of the fins, m^2
Bi Biot number
h heat transfer coefficient, $Wm^{-2}k^{-1}$
h_b heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
H dimensionless heat transfer coefficient at the base of the fin, $Wm^{-2}k^{-1}$
k thermal conductivity of the fin material, $Wm^{-1}k^{-1}$
k_b thermal conductivity of the fin material at the base of the fin, $Wm^{-1}k^{-1}$
K dimensionless thermal conductivity of the fin material, $Wm^{-1}k^{-1}$
L Length of the fin, m
M dimensionless thermo-geometric fin parameter
m^2 thermo-geometric fin parameter m^{-1}
P perimeter of the fin, m
T Temperature, K
T_∞ ambient temperature, K
T_b Temperature at the base of the fin, K
x fin axial distance, m
X dimensionless length of the fin
Q dimensionless heat transfer
q_i the uniform internal heat generation in W/m^3

Greek Symbols

β thermal conductivity parameter or non-linear parameter
δ thickness of the fin, m
δ_b fin thickness at its base.
γ dimensionless internal heat generation parameter
θ dimensionless temperature
θ_b dimensionless temperature at the base of the fin
η efficiency of the fin
ε effectiveness of the fin
a_r aspect ratio

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