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Vibration Analysis of Laminated Viscoelastic Beam: The Finite Difference Method Approach

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ABSTRACT

The damping of structural components and materials is often a significantly overlooked criterion for good structural or system design. The lack of damping in structural components has led to numerous mechanical failures over a seemingly infinite multitude of structures. A method of reducing vibration in a system is to employ a dynamic vibration absorber. When a system is subjected to vibration of variable frequency or to broad band of random excitation, a number of resonances may be excited. It will be impracticable to have so many separate vibration absorber. To control and reduce vibration in such system, a visco-elastic material should be used as an active vibration damper. When laminated the damping characteristic especially for random excitation is improved. In this paper, Finite Difference Method (FDM) was used in analyzing the vibration of laminated visco-elastic beam. The results depicted shows that the damping reduces with increase in the coefficient of friction, while the dynamic deflection increases with the length of the laminated beam for different interfacial pressure. Also, a negative pressure gradient in a visco-elastic cantilever beam tends to increase the level of energy dissipation whereas an enhanced frequency ratio of the driving load tends to reduce the amount of energy dissipation that can be arranged via slip at the laminate interface. The findings confirm that each of these factors can independently be exploited to enhance the level of energy dissipation that can be arranged. It could therefore be stated that the laminates enhance the dissipation of vibration energy via slip damping of structures.

Keywords: *Viscoelastic, Laminates, Damping, Natural Frequency.*

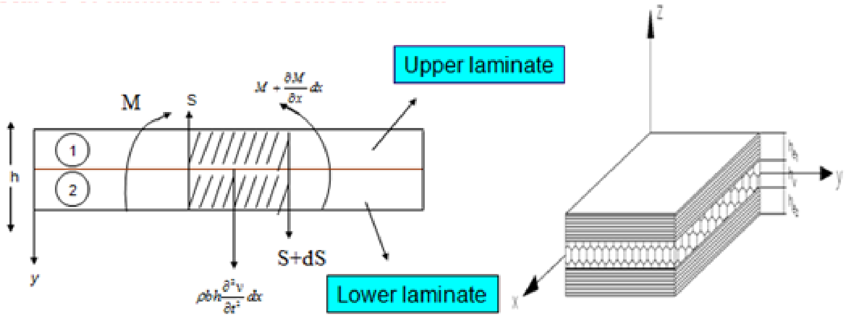
NOMENCLATURE

Ω_1 = Free Fluid Domain
Ω_2 = Beam Surface Domain
Ω_3 = Elastic Seabed Domain
$\sum$ = Beam Modulus of Rigidity
I = Moment of Inertia
P = Density of Pipe Material
P_f = Fluid Density
P_w = Density Of Sea Water
W = Transverse Beam Displacement
u = Longitudinal Displacement
U = Hydrodynamic Pressure Distribution Function
P_x (x) = Differential Of Fluid Velocity
M = Sum of Masses Of Pipe And Fluid
M_f = Mass of Flowing Fluid
M_p = Mass of Beam
T_o = Pre-Stress + Tension From Steel Catenary Riser
F_1 (t), F_2 (t) = External Forces in the Transverse and Longitudinal Direction
τ_1, τ_2 = Damping Force per Unit Velocity in the Transverse and Axial Directions Respectively
C_D = Hydrodynamic Drag Coefficient
L = Length of the Beam
P = Pressure
C = Specific Heat Transfer
K_b = Seabed Modulus of Deformation
U = Velocity of Flowing Fluid
A_p = Surface Area of Pipe
μ_p = Sliding Friction Coefficient at Pipeline and Underlay Interface
R = External Radius of the Pipeline
G = Acceleration Due To Free Fall
h = Height of Pipeline below Mean Sea Surface
H = Height of Sediment Layer on the Pipe
Δ(c) = Coefficient of Area Deformation
α = Coefficient of Thermal Expansivity
T = Temperature, Temperature Gradient and Temp Difference Respect
A_b = Original Cross Sectional Area of Beam
A = Change In the Surface Area of Beam

1. INTRODUCTION

The damping of structural components and materials is often a significantly overlooked criterion for good mechanical design. The lack of damping in structural components has led to numerous mechanical failures over a seemingly infinite multitude of structures. Scientific work in the area of damping can be traced back to the early 1930s. Foppl, Zener, and Davidenkoff investigated the damping of metals [10]. The first major advancements in the application and use of materials which could applied as a treatment, surface or embedded, which enhance the damping characteristics of structures and components did not occur until the 1950's. During this time, many scientists were investigating the properties and mechanical behavior of polymeric materials [10]. Ross and Kerwin who were among to treat viscoelastics formulated an analytical method of layered damping treatments [27]. Also during this period Mycklestad had the first publication investigating the complex modulus modeling of damping materials [10][21]. A significant spur in this direction of research was from the aerospace industry. Aircraft manufacturers and designers were seeking a way to reduce vibration and noise transmission through aircraft fuselage panels, without a significant increase in weight. Additionally, as time neared the mid 1960's, NASA funded research for thin, lightweight films which could be used for the same purpose of damping in rocket housings. Visco-elastic materials are generally polymers, which allow a wide range of different compositions resulting in different material properties and behaviour. Thus, visco-elastic damping materials can be developed and tailored fairly efficiently for a specific application. It is imperative to indicate that while the study into aerospace structures may not only be subject to conservative loads but also to non-conservative loads as well, little or no work has been found on the stability of laminated beams subject to subtangential loading using the dynamic criterion. Thus, this paper studied how the stability of laminated composite beams is affected by subtangential loads. Because of the inherent complexity of composite materials, fiber-reinforced laminated structures can be difficult to manufacture according to their exact design specifications, resulting in unwanted uncertainties. Reddy established that during the manufacturing of laminates, material defects such as interlaminar voids, delamination, incorrect orientation, damaged fibers, and variation in thickness may be introduced [25]. Damping as a mechanism for controlling undesirable effects of vibration has received considerable attentions in the literatures over the years, right from the period of Goodman and Klump, who were credited for their research into slip damping as a means of controlling vibration [6]. Among the earliest studies on vibration suppression during the 1940s, were those based on the use of rubber-like materials. The main characteristic of these materials were their ability to undergo large and reversible deformation, although the assumption of total recovery was only an approximation, since it is known that natural rubbers do not satisfy this condition. Nevertheless, it is known that rubber-like materials can achieve elastic properties after going through a chemical process known as cross-linking. In fact, the study of these materials, although not for vibration suppression, was made a decade earlier. Meyer proposed a statistical theory of rubber elasticity, which in conjunction with equilibrium thermodynamics was used to relate the force causing extension to molecular parameters [18]. More practical and simple models for rubber-like materials are also present in the literature. The concept of a complex elastic modulus for visco-elastic materials has been used to analyze the behaviour of low- and high-damping rubber-like materials. The former is characterized for having modulus and loss factor that vary slowly with frequency, and consequently may be considered as constant over the frequency range of interest in vibration problems. This type of damping is also usually referred to as hysteretic damping. The later are for materials with modulus varying proportionally with the frequency, but with loss factor independent of it. The need to better represent materials exhibiting strain rate effects led to the development of new mathematical representations of linear viscoelastic behaviour. Two alternative forms used to represent the stress-strain relations were the integral representation and the differential operator method. The use of hereditary integrals has the advantage of describing the time dependence more generally, and also of being able to represent the actual measured VE material properties. These integrals are commonly expressed in terms of the Boltzmann superposition principle, and its representation favors the incorporation of temperature effects. However, the mathematics involved in the stress analysis in such approach is a major drawback. On the otherhand, the use of differential operator methods has led to quite simple mathematical processes compared to the integral approach. The difficulty in analyzing the problem, due to the inclusion of the time variable in the differential equations, can be minimized for problems in which the boundary conditions and the temperature do not change with time. These assumptions lead to the elimination of the time variable by employing the Laplace transform. Lee used this concept, which was known as the elastic-viscoelastic analogy or the correspondence principle [11]. Phenomena such as creep and stress relaxation are common to many viscoelastic materials. However, experiments involving them are not sufficient to provide complete information about the mechanical behaviour of viscoelastic solids, since they are applicable only for long time loadings. The investigation of material behaviour for very short time loading can be made through the use of oscillatory loadings. It was observed in such experiments that the strain lags in phase behind stress by an angle, tangent of which is equal to the loss factor of the material. It can be said then that, under harmonic oscillation, the relationship between stress and strain takes the form of a complex modulus. Bert pointed out that the complex modulus approach was linear and simple to use compared to methods using viscous damping [1]. Some disadvantages are the difficulty in simulating the multiple-frequency forced vibration and high-order systems as well as the fact that these models might not give satisfactory results for the transient solution. Another aspect is the noncausal behaviour of such models. Following a work by Milne on the noncausality of hysteretic damping models, Jones showed that the noncausality could be reduced if the storage modulus and the loss factor frequency dependency are represented accurately [19][10]. Myklestad addressed the case in which the damping of materials, possessing a particular shape for the hysteresis loop, is modeled by multiplying the modulus of elasticity by the complex number e2bi, where 2b is named the complex damping factor [21]. He shows that, under free and forced vibration and for small values of b, the complex damping model is equivalent to the viscous damping model for a single mass-spring system with a slight difference in the phase. Despite the advantages in the numerical calculation, the complex damping gives a natural frequency higher than expected for the free-vibration case, and when applied to a forced-vibration problem, results in peak amplitude at the natural frequency, which does not conform to observation. Johnson and Kienholz developed one of the most used design methods for damping design, the modal strain energy (MSE) method [9]. This method was initially applied to three- layer laminates containing a viscoelastic layer. In this approach it is assumed that the damped structure can be represented in terms of the real normal modes of the associated undamped system if appropriate damping terms are inserted into the uncoupled equations of motion. The method, however, is not very good for frequency-dependent applications, although an approximate solution is possible by doing an empirical correction. Fan et al. applied the complex modulus concept to analyze elastic beams supported by a viscoelastic material [3]. The premise of their work is to represent the viscoelastic support by equivalent complex stiffness coefficients, and then decouple the system through the complex modes to find modal equations that are identical to the equations of a single-degree-of-freedom system. They also investigate the forced vibration response under a harmonic load, and they show that there exists an optimal support length that minimizes the transverse amplitude. However, to achieve a self-adjoint system, the viscoelastic support on the top and bottom of the beam must have the same characteristics, according to Sullivan and Johnson [32]. Their purpose was to use the correspondence principle to determine the numerical values of material constants. The parameters are found by comparing analytical and experimental curves obtained from static loads. Henwood gives a time domain formulation in which the hysteretic damping matrix is approximated by a viscous matrix [8]. He avoids the causality problems usually common when applying the correspondence principle by using an already established time domain method for viscous damping. He revisited the method addressed by Crandall for a one-degree- of-freedom and extended it to a general structure. He shows that both models have good agreement for loss factor values smaller than 0:4. Gandhi and Munsky investigated active constrained layer (ACL) treatments with position and velocity feedback, subjected to a maximum allowable voltage through the piezoelectric layer [4]. They analyzed the system in the frequency domain only, since the VEM is described using the complex modulus approach. They drew many conclusions concerning the vibration attenuation due to position and velocity feedback, but their analysis considered only the fundamental mode, which was a result of using the complex modulus. Various researchers began to apply different approaches to modeling viscoelastic behaviour during the 1980s, with the main intention of correctly representing the damping in structures modeled by finite element methods. The first of them was known as the Modal Strain Energy Method (MSE), as mentioned before. This approach assumes that the damped structures can be represented in terms of the normal modes of the undamped system. This method has proved to be very useful for low damping applications, but its main disadvantage is the necessity of empirical corrections to accommodate frequency-dependent material properties. In order to cope with the deficiencies of the MSE method, authors started to develop more elaborate models with the objective to simplify, in an accurate way, the time-domain analysis of viscoelastic damped structures. It was then that finite element compatible models based on the theory of the internal variables originated. The concept of IV is not new, dating back to the 1950s, however, only a few decades later these coordinates were fully applied to finite element models. The origins of the internal variable theory are found in the principles of thermodynamics. These coordinates were added with the intention to represent the deformation of irreversible systems, such as viscoelasticity. The ADF model was based on the thermodynamical principles and used a Voigt element to model the internal coordinate. These previous models are first order models that consider a massless internal variable. The only exception is the GHM model that is basically a second order model obtained by assigning mass to the internal coordinates. These methods all have a common trait that they are capable of capturing the frequency dependence, inherent in VE materials, at the cost of adding some extra coordinates to the system, referred to as internal variables. The initial development of this methodology started in 1985 with the work of Golla and Hughes, whose goal was to raise the modeling of viscoelastic structures to a level consistent with finite element models [5]. Their main contribution was to show how to construct damping models for viscoelastic structures, by explicitly calculating the damping matrix, through the use of dissipation coordinates. These coordinates are not the same as the internal variables, as pointed out in, but they will be treated here as been the same {for it has been the case since its development {for both are physically unaccessible. This model was further extended by McTarish and Hughes in several aspects, such as the inclusion of light damping approximations.It was this reason that made their approach to be known as the GHM (Golla-Hughes-McTavish) [17]. Lesieutre and Lee performed PD (proportional-derivative) output control using ADF (Anelastic Displacement Fields) to model viscoelastic behavior [12]. The control of a five-element finite element model of an ACLD beam system was based solely on the feedback of the strain and strain rate of the substrate. They treated the forcing terms as modifications to the stiffness and damping matrices, although that only affected the stiffness matrix, since the internal coordinates are not considered in the feedback, and, therefore, do not appear in the forcing terms. Their FE model has achieved a notorious appreciation and has subsequently been used or mentioned by many researchers. Rusovici et al. developed ADF-based, plane-stress and plane-strain finite elements to model shock propagation and absorption through viscoelastic beams [28]. They show that the ADF-based model is capable of capturing wave propagation phenomena, such as geometric dispersion, and viscoelastic attenuation and dispersion of longitudinal waves in beams. The ADF model was also extended to model the dynamic behavior of nonlinear viscoelastic elastomers. The extension was based on the introduction of nonlinear functions to describe the relaxed and unrelaxed moduli, and the evolution of the anelastic strain. Only simple shear is analyzed, due to the nature of the application, which was motivated by helicopters lag dampers. The nonlinear ADF was also extended to include friction-type elements in order to capture the combined amplitude and frequency dependence. Segalman presented a systematic method for the calculation of damping and stiffness matrices from arbitrary linear viscoelastic models [29]. Yiu reduced the frequency and time dependent material properties to a few material constants through the use of generalized Maxwell constitutive models [37]. He derived a finite element model based on the material constants and elastic stiffness matrix of the viscoelastic components only. The result is a model with constant, real, symmetric and positive semi-definite mass, damping and stiffness matrices. Though his model is presented in standard second-order form, the mass matrix is singular. He also applied some basic substructures techniques to reduce the model, giving as example, a beam with elastic and viscoelastic materials. He shows that the reduced model agrees well for the first three modes. A viscoelastic internal variable constitutive theory was applied to a higher-order elastic beam theory and finite element formulation. The behavior of the viscous material in the beam was approximately modeled as a Maxwell solid. The finite element formulation requires additional sets of nodal variables for each relaxation time constant needed by the Maxwell solid. Recent developments in modeling viscoelastic material behavior with strain variables that are conjugate to the elastic strain measures are combined with advances in modeling through-the-thickness stresses and strains in thick beams. The result is a viscous thick-beam finite element that possesses superior characteristics for transient analysis since its nodal viscous forces are not linearly dependent on the nodal velocities, which is the case when damping matrices are used. Instead, the nodal viscous forces are directly dependent on the material's relaxation spectrum and the history of the nodal variables through a differential form of the constitutive law for a Maxwell solid. The thick beam quasi-static analysis is explored herein as a first step towards developing more complex viscoelastic models for thick plates and shells, and for dynamic analyses. The internal variable constitutive theory is derived directly from the Boltzmann superposition theorem. The mechanical strains and the conjugate internal strains are shown to be related through a system of first-order, ordinary differential equations. The total time-dependent stress is the superposition of its elastic and viscous components. Equations of motion for the solid are derived from the virtual work principle using the total time-dependent stress. Numerical examples for the problems of relaxation, creep, and cyclic creep are carried out for a beam made from an orthotropic Maxwell solid. The feasibility of developing a visco-elastic damping material of structures for vibration damping has received extensive interest. Surface treatment uses high damping materials firmly attached to the surface of structural elements. Understanding the damping characteristics of visco-elastically damped structures is strongly necessary. Vibration damping becomes increasingly important for improved vibration and noise control, fatigue and impact resistance in advanced engineering systems. An excellent review of the literature on the damage characterization problem up to 2006 can be found in Montalvao et al., (2006), Sohn et.al, (2003) and Doebling et al. (1996) [20][31][2]. In Olunloyo V.O.S et al, vibration and noise reduction in structures can significantly enhance dynamic stability. In fact, exploitation of such mechanisms can lead to an improvement of aerodynamic performance in flight motions by aircrafts, hydrodynamic performance in ocean water navigation by ships or floating structures, as well as dynamic behaviour of machine structures in production processes and systems. In this paper slip damping with layered viscoelastic beam-plate structures for dissipation of vibration energy in aircraft, hydrodynamic, and machine structures is investigated analytically [23]. Many research literatures have demonstrated the feasibility of the vibration-based method as found in Steenackers et.al, (2005), Yang et.al (2003), Li and Yang (2006) [30][35][14][36]. The examination of the effect of environmental conditions on structural vibration properties is important in order to reliably apply the vibration-based structural condition monitoring methods and vibration techniques to engineering structures. Modal testing has the potential to provide the basis for rapid, inexpensive vibration characterization of composites structures. Firstly, vibration testing can be used to estimate for composite structures damage detection; Secondly, the same kinds of tests should be very useful in assessing the effectiveness of such kinds of maintenance; thirdly, vibration testing can be used for estimate the vibration characteristic of composite structures. Changes in environmental conditions such as temperature affect structural vibration properties. Many researchers studied the effect of changing environmental conditions on structural vibration properties. The effectiveness of the proposed approach has been evaluated through experimental data obtained out of a viscoelastic sandwich beam. Plunkett and Lee investigated length optimization for a constrained viscoelastic-damping layer theoretically [24]. Lunden obtained an optimum distribution of a constrained damping layer on a beam that minimizes responses [15]. Lifshitz and Leibowitz maximized the system loss factor of a sandwich beam using a sixth-order differential equation and an inequality constrained minimization algorithm [13]. Suweca and Jezequel treated a damping constraint in a structural optimization problem, and evaluated the gradient of the damping constraint using a complex variable sensitivity method [33]. Hajela and Lin used the modal strain energy approach to represent the system loss factor of a beam structure. They obtained optimal design of constrained damping layers using a global criterion method and a genetic algorithm [7]. Yu *et al.* calculated an optimum distribution of a viscoelastic damping layer to maximize the modal loss factor of a plate structure. They obtained an optimum thickness distribution by changing the position of a piece of layer sequentially [38]. Roy and Ganesan investigated partially covered damping layers for various boundary conditions [26]. Trompette and Fatemi determined an optimal distribution of cuts of a constrained viscoelastic layer using a genetic algorithm to maximize the modal damping factor [34]. Nakra reviewed analysis and optimization studies on viscoelastic damping layer for structures [22]. Mercelin *et al*. explored on optimal damping of constrained layer using a finite element analysis and a moving asymptotes method, and a genetic algorithm [16]. Numerical simulations in terms of critical parameters, such as relaxation functions of the structure and piezoelectric devices, are carried out to evaluate system responses, sensing and structural control. Composites are well known to creep, relax and delaminate with time, temperature and moisture dependent rates following well established viscoelastic constitutive relations.

2. FREE BODY DIAGRAM



4. Model formulation

From the free body diagram and the force balance in Fig.1, the governing equation of motion for the two laminates are:

For Upper Laminate

$$\rho_1 bh_1\frac{\partial^2 v_1}{\partial t^2} + E_1I_1\left(\frac{\partial^4 v_1}{\partial x^4} + c\frac{\partial^5 v_1}{\partial x^4 \partial t}\right) - \frac{\mu_1 bh_1}{2}\frac{\partial p}{\partial x} = 0 \quad (1)$$

For Lower Laminate

$$\rho_2 bh_2\frac{\partial^2 v_2}{\partial t^2} + E_2I_2\left(\frac{\partial^4 v_2}{\partial x^4} + c\frac{\partial^5 v_2}{\partial x^4 \partial t}\right) - \frac{\mu_2 bh_2}{2}\frac{\partial p}{\partial x} = 0 \quad (2)$$

equations (1) and (2) could be written as:

$$\frac{\partial^4 v_1}{\partial x^4} + c\frac{\partial^5 v_1}{\partial x^4 \partial t} + \frac{\rho b\, h_1}{E_1I_1}\frac{\partial^2 v_1}{\partial t^2} - \frac{\mu bh_1}{2EI_1}\frac{\partial p}{\partial x} = 0 \quad (3)$$

$$\frac{\partial^4 v_1}{\partial x^4} + c\frac{\partial^5 v_1}{\partial x^4 \partial t} + \frac{\rho bh_2}{E_2I_2}\frac{\partial^2 v_1}{\partial t^2} - \frac{\mu bh_1}{2EI_2}\frac{\partial p}{\partial x} = 0 \quad (4)$$

where $p(x) = p_0(1 + \frac{\Sigma x}{L})$ (5)

The initial and boundary conditions are:

Initial Condition

$t = 0,\ v_1 = 0,\ v_2 = 0$

Boundary Condition

$x = 0,\ v = 0$

From equation (5), $\frac{\partial p}{\partial x} = \frac{P_0 \varepsilon}{L}$

So equation (3) and (4) could be written as:

$$\frac{\partial^4 v_1}{\partial x^4} + c\frac{\partial^5 v_1}{\partial x^4 \partial t} + \frac{\rho bh_1}{E_1I_1}\frac{\partial^2 v_2}{\partial t^2} - \frac{\mu bh_1 \varepsilon P_0}{2E_1I_1L} = 0 \quad (6)$$

$$\frac{\partial^4 v_2}{\partial x^4} + c\frac{\partial^5 v_2}{\partial x^4 \partial t} + \frac{\rho bh_2}{E_2I_2}\frac{\partial^2 v_2}{\partial t^2} - \frac{\mu bh_2 \varepsilon P_0}{2E_2I_2L} = 0 \quad (7)$$

Assume

$$\alpha_1 = \frac{\rho bh_1}{E_1I_1},\ \alpha_2 = \frac{\rho bh_2}{E_2I_2},\ \beta_1 = \frac{\mu bh_1 P_0}{2E_1I_1L},\ \beta_2 = \frac{\mu bh_2 P_0}{2E_2I_2L} \quad (8)$$

Hence, equation (7) and (8) becomes

$$\frac{\partial^4 V_1}{\partial x^4} + c\frac{\partial^5 V_1}{\partial x^4 \partial t} + \alpha_1\frac{\partial^2 V_1}{\partial t^2} - \beta_1 \varepsilon = 0 \quad (9)$$

$$\frac{\partial^4 V_2}{\partial x^4} + c\frac{\partial^5 V_2}{\partial x^4 \partial t} + \alpha_2\frac{\partial^2 V_1}{\partial t^2} - \beta_2 \varepsilon = 0 \quad (10)$$

where V_1 and V_2 are the vertical deflection
The continuity condition $v_1 = v_2 = v$
Thus, equation (9) or (10) could then be written as:

$$\frac{\partial^4 V}{\partial x^4} + c\frac{\partial^5 V}{\partial x^4 \partial t} + \alpha\frac{\partial^2 V}{\partial t^2} - \beta\varepsilon \qquad (11)$$

Initial condition
$V(x,0) = V_0$
Boundary conditions
$V(0,t) = V(L,t) = 0$

$$\frac{\partial^2 V(0,t)}{\partial x^2} = \frac{\partial^2 V(L,t)}{\partial x^2} = 0$$

Method of Solution

The developed model (11) was solved using Finite Difference Method.
Hence, the discretized scheme for each term of equation (11) is given as:

$$\frac{\partial^4 V}{\partial x^4} = \frac{V^n_{i+2} - V^n_{i+1} + 6V^n_i - 4V^n_{i-1} + V^n_{i-2}}{(\triangle x)^4} \qquad (12)$$

$$\frac{\partial^5 V}{\partial x^4 \partial t} = \frac{V^n_{i+2} - 4V^{n+1}_{i+1} + 6V^{n+1}_i - 4V^n_{i-1} + V^n_{i-2} - V^{n-1}_{i+2} + 4V^{n-1}_{i+1} - 6V^{n-1}_i + 4V^{n-1}_{i-1} - V^{n-1}_{i-2}}{2\triangle t(\triangle x)^4} \qquad (13)$$

$$\frac{\partial^2 V}{\partial t^2} = \frac{V^n_{i+1} - 2V^n_i + V^n_{i-1}}{(\triangle t)^2} \qquad (14)$$

substituting equation (12) into (11), we have

$$\frac{V^n_{i+2} - V^n_{i+1} + 6V^n_i - 4V^n_{i-1} + V^n_{i-2}}{(\triangle x)^4} + c\left[\frac{V^{n+1}_{i+2} - 4V^{n+1}_{i+2} + 6V^{n+1}_i - 4V^{n+1}_{i-1} + V^{n+1}_{i-2} - V^{n-1}_{i+2} + 4V^{n-1}_{i+1} - 6V^{n-1}_i + 4V^{n-1}_{i-1} - V^{n-1}_{i-2}}{2\triangle t(\triangle x)^4}\right] + \alpha\left[\frac{V^n_{i+1} - 2V^n_i + V^n_{i-1}}{(\triangle t)^2}\right] - \beta\varepsilon = 0 \qquad (15)$$

$$c\left(\frac{V^{n+1} - 4V^{n+1}_{i+1} + 6V^{n+1}_i - 4V^{n+1}_{i-1} + V^{n+1}_{i-2}}{2\triangle t(\triangle x)^4}\right)$$

$$= c\left(\frac{V^{n-1}_{i+2} - 4V^{n-1}_{i+1} + 6V^{n-1}_i - 4V^{n-1}_{i-1} + V^{n-1}_{i-2}}{2\triangle t(\triangle x)^4}\right) - \left(\frac{V^n_{i+2} - V^n_{i+1} + 6V^n_i - 4V^n_{i-2} + V^n_{i-2}}{(\triangle x)^4}\right) - \alpha\left(\frac{V^n_{i+1} - 2V^n_i + V^n_{i-1}}{(\triangle t)^2}\right) + \beta\varepsilon$$

The initial and boundary conditions are discretized

Initial Condition

$V^0_i = 0$

Boundary Condition

$V^n_0 = V^n_N = 0 \qquad V^n_1 = V^n_0$

$V^n_N = V^n_{N-1}$

5. RESULTS AND DISCUSSION

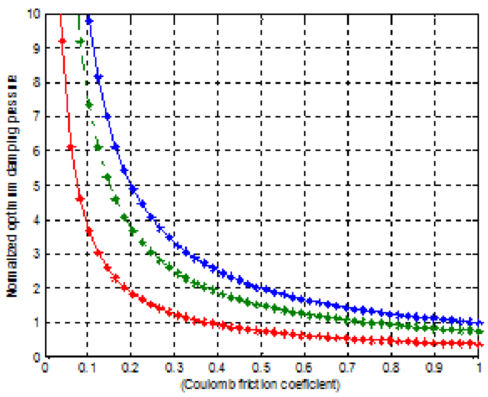


Fig.(a)

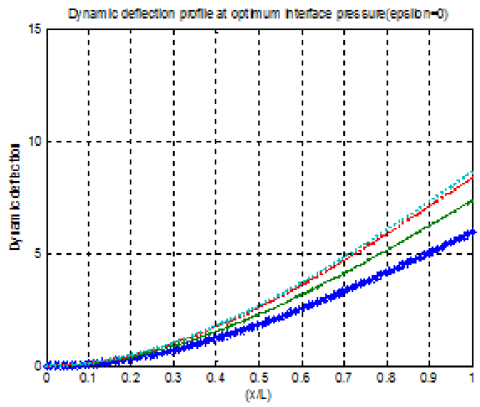
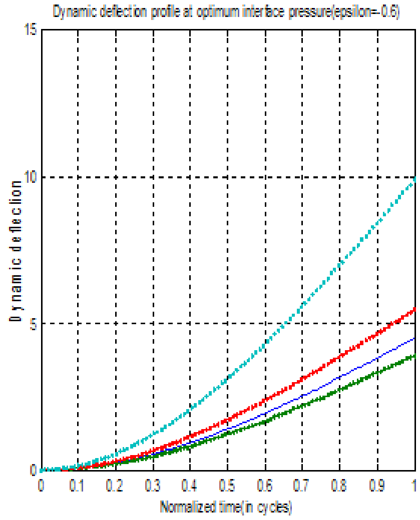
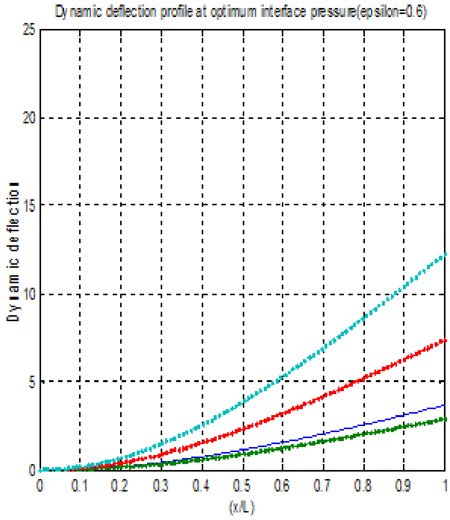


Fig (b)

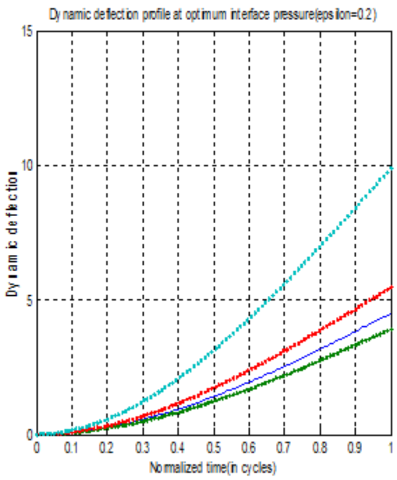
Dynamic deflection profile at optimum interface pressure (epsilon=0.6)



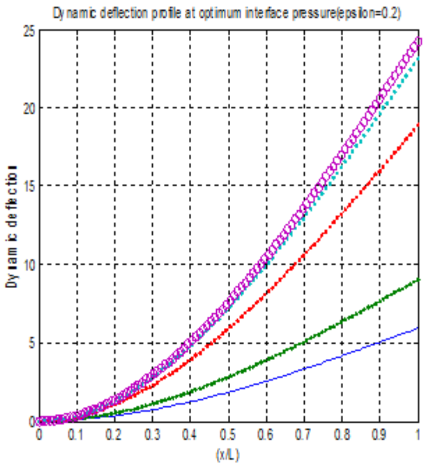
Dynamic deflection profile at optimum interface pressure (epsilon=0.6)



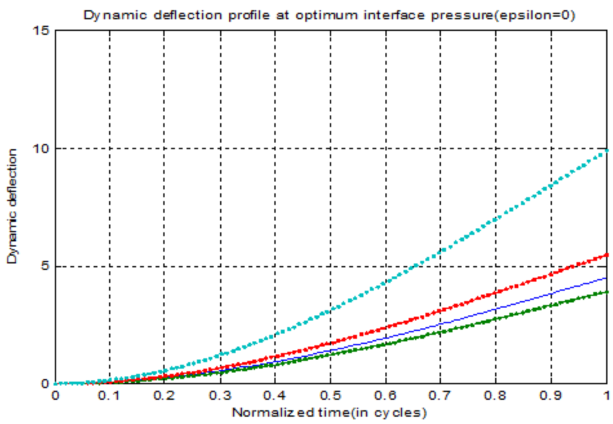
Dynamic deflection profile at interface pressure of 0.2 (epsilon=0.2)



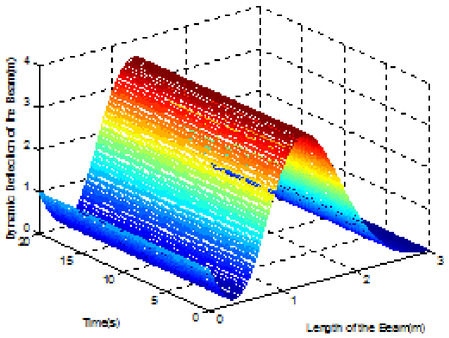
Dynamic deflection profile at interface pressure of 0.2 (epsilon=0.2)



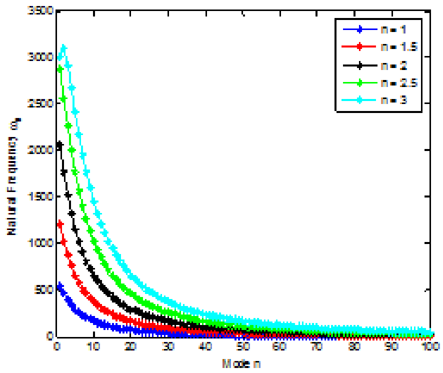
Dynamic deflection profile at interface pressure of 0 (epsilon=0)



DYNAMIC DEFLECTION OF LAMINATED BEAM WITH TIME(t) AND LENGTH OF THE BEA(m). 3D DIAGRAM



NATURAL FREQUENCY(ω) OF THE LAMINATED BEAM AGAINS THE MODE(n)



From the results shown above, the damping reduces with increase in the coefficient of friction, while the dynamic deflection increases with the length of the laminated beam for different interfacial pressure. It was well known that a negative pressure gradient in a cantilever beam tends to increase the level of energy dissipation whereas an enhanced frequency ratio of the driving load tends to reduce the amount of energy dissipation that can be arranged via slip at the laminate interface. These observations however assume that both upper and lower laminates are of the same thickness and are made from the same material. When such restrictions are removed, two new effects arise and are the subject of this paper. Our findings in fact confirm that each of these factors can independently be exploited to enhance the level of energy dissipation that can be arranged. In other words such increases can be arranged either by using different materials for the upper and lower laminates in a prescribed fashion or by retaining the same material for both laminates but varying the individual ratios of the laminate thicknesses in a defined manner. Another deduction from the present work is that for effective energy dissipation, it is better to simultaneously play with choice of the laminate materials and their thickness ratios rather than thinker with any one of them by itself. In fact there are many instances when choice of material alone eclipses whatever gains can be made from playing with interfacial pressure gradient. This underscores the quest in the search for composites in the construction of such laminates. The strategy here is to exploit the advantage of composite structures to dissipate vibration energy via slip damping especially in aerodynamic structures where the effect of weight of structural member becomes significant.

6.0 CONCLUSION AND FUTURE WORK

The paper presented the problem of using a layered structural member as a mechanism for dissipating unwanted vibration or noise has been revisited, be it in an aerodynamic or machine structure. Earlier work had established that some of the factors influencing the level of energy dissipation include the nature of the pressure distribution profile at the interface of the laminates as well as the nature of the external force to which the structure is subjected. The conclusion therefore, is that for maximum energy dissipation, laminates of different materials and of different thicknesses is required. This makes the use of composites beams inevitable. These results can be positively exploited in the design of aerodynamic and machine structures. Hence, it will assist the product designer consider the use of laminated metal material in place of traditional sheet metal thereby enabling various practical modeling techniques to be used both as a damping prediction and design optimization tool. This complexity offers more design flexibility as the thickness and type of the damping core as well as the constraining layers can be altered to optimize effectiveness of the laminated metal product.

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